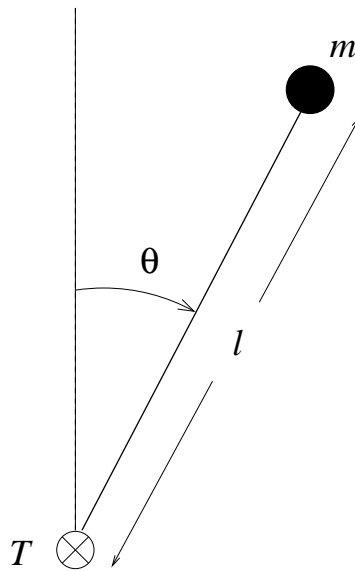


MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering
2.004 Dynamics and Control II
Fall 2007

Problem Set #10

Posted: Friday, Nov. 30, '07

Due: Friday, Dec. 7, '07



1. **Inverted pendulum** Consider the inverted pendulum system shown above. It consists of a point mass m attached to the end of a rigid rod of length l . The rod's mass is negligible. An input torque T is applied to the rod base in order to control the pendulum rotation angle θ .
 - a) Derive the inverted pendulum's equation of motion; then linearize the equation that you derived by assuming that the angle θ is very small ($\theta \ll 1$ rad). Don't forget to include the effect of gravity in your model!
 - b) With the angle θ as output and torque T as input, and using numerical values $l = 1.09$ m, $m = 0.8417$ kg, derive the transfer function of the inverted pendulum system.
 - c) Deduce from the transfer function that the inverted pendulum system is *unstable*. Describe an experiment you could do with everyday objects to verify this result. Note: Your transfer function should have two real poles at $s = \pm 3$. If you cannot quite arrive at this result, assume it to be true and move on to the remaining questions.

- d) We will now perform a sequence of attempts to stabilize and control the inverted pendulum. First we consider proportional (P) control. Draw the block diagram of the proportional control system, indicating clearly and thoroughly the signals flowing over the arrows and the transfer functions of the blocks in your diagram.
- e) Using the root locus technique, argue that with proportional control we cannot stabilize this system; at best, we can make it marginally stable.
- f) Next we consider proportional–derivative (PD) control. Draw the modified block diagram and root locus, and argue that the inverted pendulum system can now be stabilized with appropriate choice of gain.
- g) What is the additional information that the PD controller accounts for to succeed in stabilizing the system where the P controller failed?
- h) Let us now design the PD controller for the following transient specifications: peak time $T_p = 1.61$ sec and overshoot 20%. Interpret these specifications as the locations of the desired closed–loop dominant poles on the s –plane. What is the natural frequency of the desired (compensated) system?
- i) Using geometry on the s –plane (don’t go to MATLAB quite just yet), find the location of the PD controller’s zero that meets our transient specifications.
- j) Now import the system to MATLAB and find the gain K to complete the PD controller design. Note: Define the plant transfer function (two real poles at $s = \pm 3$) separately from the controller transfer function. To avoid confusion with the gain, change the following option in the “Control and Estimation Tools Manager” : in the menu `Edit`→`SISO Tool Preferences...`, select the tab `Options` and under `Tuned parameters` select the `Zero/Pole/Gain` option. This way, MATLAB will display the controller’s transfer function in a manner compatible to our notation in class.
- k) Now get MATLAB to display the step response of the PD–compensated inverted pendulum. Explain why the overshoot is slightly higher and the peak time slightly shorter than our design targets.
- l) What is the steady–state error? Calculate it directly from the closed–loop transfer function of your PD–compensated inverted pendulum and then verify your result using MATLAB.
- m) Construct the (open–loop) Bode plots of the uncompensated inverted pendulum and the PD–compensated inverted pendulum with the gain chosen as in question (j). Comment on the gain and phase margins of the two systems; verify your results using MATLAB.
- n) As a final effort to correct this system, let us attempt to eliminate the

steady-state error by PID compensation without degrading the transient, if at all possible. We maintain the same derivative term that we found from the previous questions (since the desired transient has already been achieved) and cascade an integral controller. Use a safety factor of 10 for the relative size of the integral controller's zero relative to the plant's stable pole. Verify with MATLAB that indeed the PID controller has eliminated the steady-state error.

- o) Now use MATLAB to adjust the gain such that the peak time requirement is met. What goes horribly wrong when you do that? This seems to suggest that the PID controller does not work as advertised in this case, *i.e.* it does not leave the transient response unchanged. Can you explain why this system is different than the PID control example that we saw in class. Lecture 33?
- p) Suggest a strategy to get around this problem with the same PID controller but different gains, now interpreting the peak time and overshoot requirements as upper bounds: "achieve peak time *less than* $T_{p,\max} = 1.61$ sec and overshoot *no more than* 20%." Calculate the final PID controller's gains K_1 , K_2 , K_3 (notation same as in the Nise textbook and Lecture 33).

2. Review questions The purpose of the following questions is to remind you of some fundamental topics and results that we derived in this class. Answer as accurately and succinctly as possible.

- a) What is the relationship between the torque constant K_m and back-emf constant K_v of a DC motor? What is the physical justification for this relationship?
- b) In what sense are a spring and a capacitor analogous as elements in electro-mechanical systems?
- c) Which parameter specifies whether a 2nd-order system is overdamped/critically damped/underdamped/undamped? How are the step responses of these systems different?
- d) Does the Bode magnitude plot of a transfer function with a single zero always indicate a gain of 20dB/decade above the break (cut-off) frequency? Does the Bode phase plot of a transfer function with a single zero always indicate a gain of 90° over a 2 decade bandwidth around the break (cut-off) frequency?
- e) Describe how one may obtain the overshoot in the step response of an underdamped closed-loop system from two system representations: the root-locus and the open-loop Bode plots. Use sketches to aid your description.