LAGRANGEAN EXAMPLE

Prof. Stephen Graves

Production quantities for metal stamping (from David B. Kletter's SM thesis, *Determining Production Lot Sizes and Safety Stocks for an Automobile Stamping Plant*, 1994) : A metal stamping plant for an automobile manufacturer will produce 100's of parts on dozens of stamping lines to satisfy part demand from the assembly plants. In running the stamping plant, a key planning decision is the assignment of parts to stamping lines. Once this assignment is made, then inventory control policies need to be established for the family of parts assigned to each line. A common control policy is to determine the relative production frequency (and/or batch size) for each part and the necessary buffer stock to assure a high level of service for the assembly plants.

The following describes how to find the production batch sizes.

$$\min \sum_{i=1}^{n} \frac{S_i D_i}{Q_i} + \frac{H_i Q_i}{2}$$

$$s.t. \quad \sum_{i=1}^{n} \frac{T_i D_i}{Q_i} = K$$

Q_i = production quantity or lot size

S_i = setup cost

H_i = holding cost

T_i = setup time

D_i = annual demand

K = available capacity for setups

Introduce Lagrange Multiplier: λ

$$L(Q,\lambda) = \sum_{i=1}^{n} \frac{S_i D_i}{Q_i} + \frac{H_i Q_i}{2} + \lambda \left(\sum_{i=1}^{n} \frac{T_i D_i}{Q_i} - K \right)$$

Approach:

- for given λ, find Q_i that minimize L (Q, λ);
- then choose λ such that Q_i satisfy original constraint.

Look at first-order conditions:

$$\frac{\partial L}{\partial Q_i} = 0 \quad for\ i = 1, \ldots n$$

$$\frac{\partial L}{\partial \lambda} = 0$$

$$\frac{\partial L}{\partial Q_i} = -\frac{S_i D_i}{Q_i^2} + \frac{H_i}{2} - \frac{\lambda T_i D_i}{Q_i^2} = 0$$

$$\Rightarrow Q_i^2 = \frac{2}{H_i}(S_i + \lambda T_i) D_i$$

$$Q_i^* = \sqrt{\frac{2}{H_i}(S_i + \lambda T_i) D_i}$$

Now choose λ such that

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \frac{T_i D_i}{Q_i^*} - K = 0$$

$$\sum_{i=1}^{n} \frac{T_i D_i}{\sqrt{\frac{2}{H_i}(S_i + \lambda T_i) D_i}} - K = 0$$

Solve by univariate search.

Note that

$$\frac{\partial L}{\partial K} = -\lambda \quad \Rightarrow$$

λ is a shadow price on the capacity, and equals the amount we would pay for additional capacity, on the margin.

This approach is very useful for removing "troublesome" constraints.

Lagrange Example

	Limit	Total	Item 1	Item 2	Item 3	
Setup cost($)			25	40	50	
Demand (per yr)			10000	20000	5000	
Hold. cost($/un-yr)			1	1	1	
Setup time (hrs)			20	10	25	
EOQ			707.11	1264.91	707.11	
EOQ setup time/year		617.73	282.84	158.11	176.78	
EOQ Cost		2679.12	707.11	1264.91	707.11	
Avail. Setup Time	400	400	160.77	122.91	116.32	
Constrained cost		2898.16	822.98	1305.23	769.95	
		Annual	Quantity	Quantity	Quantity	
		Setup	Item 1	Item 2	Item 3	total cost
		Time				
Lagrange multiplier	0	617.73	707.11	1264.91	707.11	2679.12
	1	496.58	948.68	1414.21	866.03	2732.34
	10	250.97	2121.32	2366.43	1732.05	3710.15
	5	326.39	1581.14	1897.37	1322.88	3169.42
	2.5	405.19	1224.74	1612.45	1060.66	2884.89
	3.75	359.27	1414.21	1760.68	1198.96	3026.59
	3.125	380.09	1322.88	1688.19	1131.92	2955.22
	2.81	392.11	1274.36	1650.45	1096.59	2919.57
	2.65	398.69	1249.00	1630.95	1078.19	2901.61
	2.57	402.11	1236.12	1621.11	1068.88	2892.68
	2.61	400.39	1242.58	1626.04	1073.55	2897.14
	2.63	399.54	1245.79	1628.50	1075.87	2899.38
	2.62	399.96	1244.19	1627.27	1074.71	2898.26
	2.615	400.18	1243.38	1626.65	1074.13	2897.70
	2.6175	400.07	1243.78	1626.96	1074.42	2897.98
	2.61875	400.02	1243.99	1627.11	1074.56	2898.12

Microsoft Excel Answer Report
Worksheet: Lagrange example

Target Cell (Min)

Cell	Name	Original Value	Final Value
C19	Constrained cost Total	2679.12	2898.16

Adjustable Cells

Cell	Name	Original Value	Final Value
D4	Quantity Item 1	707.11	1244.05
E4	Quantity Item 2	1264.91	1627.16
F4	Quantity Item 3	707.11	1074.61

Constraint

Cell	Name	Cell Value	Formula	Status	Slack
C17	Avail. Setup Time Total	400	C17<=B17	Binding	0
D4	Quantity Item 1	1244.05	D4>=0	Not Binding	1244.05
E4	Quantity Item 2	1627.16	E4>=0	Not Binding	1627.16
F4	Quantity Item 3	1074.61	F4>=0	Not Binding	1074.61

Microsoft Excel Sensitivity Report
Worksheet: Lagrange example

Changing Cells

Cell	Name	Final Value	Reduced Gradient
D4	Quantity Item 1	1244.05	0.00
E4	Quantity Item 2	1627.16	0.00
F4	Quantity Item 3	1074.61	0.00

Constraints

Cell	Name	Final Value	Lagrange Multiplier
C17	Avail. Setup Time	400	-2.61915