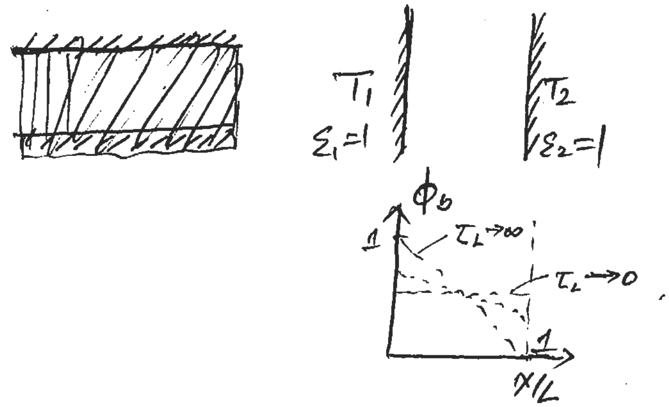
Review of last lecture

(1)



(2) approximate solutions

$\tau_L << \phi$ optically thin

$\tau_L >> 1$ optically thick

$$\mu \frac{dI_\eta}{d\xi} = -I_\eta + I_{b\eta}.$$

linear expansion.

but simply replace derivative term by $\frac{dI_\eta}{d\xi} \approx \frac{dI_{b\eta}}{d\xi}$

$$I_\eta = I_{b\eta} - \mu \frac{dI_{b\eta}}{d\xi}$$

$$q = \int_0^\infty d\eta \int_{4\pi} I_\eta \cos\theta \, d\Omega$$

$$= \int_0^\infty d\eta \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \int_0^\pi [\overbrace{\cancel{I_{b\eta}}}^{0} - \cos\theta \frac{dI_{b\eta}}{d\xi}] \cos\theta \sin\theta \, d\theta$$

$$\frac{1}{3}\cos^3\theta \Big|_0^\pi = -\frac{2}{3}$$

$$= -\frac{4\pi}{3} \int_0^\infty \frac{dI_{b\eta}}{d\xi} = -\frac{4\pi}{3} \frac{de_b}{d\xi} = -\frac{4}{3K_e} \frac{de_b}{dx}$$

$$= -\frac{c}{3k_e} \frac{du_b}{dx} = -\frac{c}{3k_e} \cdot \underbrace{\frac{du_b}{dT}}_{v} \frac{dT}{dx} = -\frac{1}{3} c v \frac{1}{K_e} \frac{dT}{dx}$$

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Approximate Solutions:

$\tau_L \ll 1$ optically thin

$$\cancel{\frac{dI_\eta}{dz}} \quad q = (J_1 - J_2)\underbrace{(1-\tau_L)}_{\text{linearly attenuated}}$$

$\tau_L \gg 1$ optically thick:

$$\mu \frac{dI_\eta}{d\xi} = -I_\eta + I_{b\eta}$$

linear expansion at $I_{b\eta}$: $\quad I_\eta = I_{b\eta} + \beta \frac{dI_{b\eta}}{d\xi} + \cdots$

1st order solution $\quad \beta = -\mu$

$$I_\eta = I_{b\eta} - \mu \frac{dI_{b\eta}}{d\xi}.$$

$$q = \int_0^\infty \cancel{\int_0^{2\pi} d\varphi \left[\int_0^1 I_\lambda^+ \mu \, d\mu - \int_0^1 I \ldots\right]}$$

$$= \int_0^\infty d\eta \int_{4\pi} I_\lambda \mu \, d\Omega$$

$$= \int_0^\infty d\eta \int_0^{2\pi} d\varphi \int_1^{-1} \left[\underbrace{\cancel{I_{b\eta}}}_{0} - \mu \frac{dI_{b\eta}}{d\xi}\right] \mu \, d(-\mu)$$

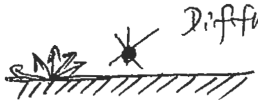
$$= +2\pi \int_1^{-1} \mu^2 \left(\int_0^\infty \frac{dI_{b\eta}}{d\xi} d\eta\right) d\mu$$

$$= -\frac{4\pi}{3} \frac{dI_b}{d\xi} = -\frac{1}{3\kappa_e} \frac{d(4\pi I_b)}{dz} = -\frac{c}{3\kappa_e} \frac{d\overbrace{u_b}^{\text{energy density}}}{dz} \quad \frac{du_b}{dT}\frac{dT}{dz}$$

$$= -\frac{4}{3\kappa_e} \frac{d(\sigma T^4)}{dz} = -\frac{c\, c_v}{3\kappa_e} \frac{dT}{dz}.$$

General $q = -\frac{4\pi}{3k_e}\nabla I_b$ — Rosseland diffusion approximation.

Boundary Conditions:



Diffusion approximation right to the wall

Intensity ~~flux~~ leaving wall : $I^+ = \varepsilon I_{bw} + I^-(1-\varepsilon)$

$$q = \int_0^{2\pi} d\varphi \left[\int_0^1 I^+ \mu \, d\mu - \int_0^1 I^-(-\mu)\, d\mu \right]$$

$$= 2\pi \left[\varepsilon \cdot \frac{1}{2} I_{bw} - \varepsilon \int_0^1 I^-(-\mu)\mu \, d\mu \right]$$

$$I^-(\mu) = I_b(0) - \mu \frac{dI_b}{dz}$$

$$q = \varepsilon E_{bw} - \varepsilon \, 2\pi \left[\frac{1}{2} I_b(0) - \frac{1}{3} \frac{dI_b}{dz} \right]$$

$$q = \varepsilon \left[E_{bw} - E_b(0) \right] + \frac{\varepsilon q}{2}$$

$$q = \frac{\left[E_{bw} - E_b(0) \right]}{\frac{1}{\varepsilon} - \frac{1}{2}}$$

$$q(L) = \frac{E_b(L) - E_{bw2}}{\frac{1}{\varepsilon_2} - \frac{1}{2}}$$

If radiative equilibrium q = constant

$$E_b(0) - E_b(L) = \frac{3k_e L}{4} q$$

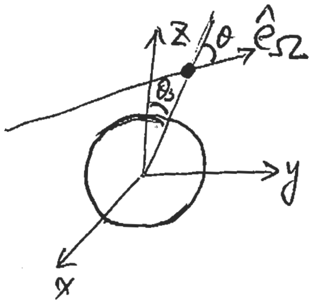
$$\Rightarrow \quad q = \frac{E_{bw1} - E_{bw2}}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 + \frac{3}{4}\tau_L}$$

Comments
(1) higher order expansion
(2)

Spherical & Cylindrical Coordinates:

$$\frac{dI_\eta}{ds} = \hat{e}_\Omega \cdot \nabla_r I_\eta$$

$$\frac{1}{K_e} \hat{e}_\Omega \cdot \nabla_r I_\eta = -I_\eta + \underbrace{(1-\omega_\eta) I_{b\eta} + \frac{\omega_\eta}{4\pi} \int \Phi(\Omega' \to \Omega) I(\Omega') d\Omega'}_{S}$$

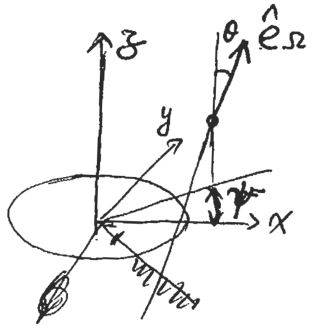


$\mu = \cos\theta$

$$\mu \frac{dI_\eta}{dz} + \frac{1-\mu^2}{z} \frac{dI_\eta}{d\mu} = S_\eta - I_\eta .$$

↑ local

Cylindrical Coordinate:



$$\sin\theta \left[\cos\psi \frac{\partial I_\eta}{\partial z} - \frac{\sin\psi}{z} \frac{\partial I_\eta}{\partial \psi}\right] = S_\eta - I_\eta .$$

Diffusion approximation: $\frac{\partial I_\eta}{\partial z}, \frac{\partial I_\eta}{\partial \psi} \to \frac{\partial I_{b\eta}}{\partial z}, \frac{\partial I_{b\eta}}{\partial \psi}$.

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Numerical Solution: Discrete Ordinate Method

$$q = \int_{4\pi} I \cos\theta \, d\Omega \quad \cancel{= \int_0^{2\pi} d\varphi \int_0^{\pi} I \mu \, d\mu}$$

$$2\pi \left[\int_0^1 I^+ \mu \, d\mu - \int_0^1 I^-(-\mu) \mu \, d\mu \right]$$

↑ axisymmetric