MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
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2.29 NUMERICAL FLUID MECHANICS— SPRING 2007

Quiz 1

Time 1 hour and 15 min, Totally 25 points
Thursday 11 a.m. 03/22/07, Focused on Lecture 1 to 11

Problem 1 (6 points):

State which of the following statements are true and which are false. You do not have to justify your answer.

1. The number of significant digits achievable by a specific floating point representation is not dependent on exponent length.
2. If $f = ax^2y^{-2}$, then the relative error of f is more sensitive to relative error of x, compared to relative error of y.
3. Bi-section method is capable of predicting the maximum number of iterations required for a specific error level ahead in time.
4. If the Newton-Raphson's method converges for a root finding problem, then the absolute error in each step will be less than the square of absolute error in previous step.
5. The Jacobi iterative method for a linear problem will always converge for a positive definite matrix.
6. The numerical stability of Gaussian elimination is guaranteed provided that we do full pivoting and equilibration.

Problem 2 (3 points):

The steady state potential flow past a cylinder is given by below formula.

$$\phi = U_\infty (r + \frac{a^2}{r})\cos\theta$$

Here r,θ are cylindrical coordinates, a is cylinder radius and U_∞ is the far field uniform velocity.

1. Derive the expression for the velocity field.
2. Ignore the gravity and derive the expression for the pressure field assuming zero far field pressure.

Problem 3 (2 points):

In a special floating point representation we have 3 bytes, with base 2:

Mantissa length 15 bits
Mantissa sign 1 bit
Exponent length 7 bits
Exponent sign 1 bit

Now answer below questions:

1. What numerical range is covered by this floating point? How many significant digits do we have?
2. What is the smallest nonzero number?
3. What is the largest relative error due to rounding of the mantissa?
4. What is the largest absolute error due to rounding of the mantissa?

Problem 4 (2 points):

Find one of the roots of the following equation with your method of choice.

$$f(x) = x + \frac{1}{2} - \tan x$$

The relative error in the root, between consecutive steps, should be less than 10^{-6}.

Problem 5 (8 points):

Consider the following system of equations:

$$Ax = b, \quad A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 4 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$$

1. Cholesky factorize A (Note that A is positive definite).
2. Find an LU factorization form for A.
3. Use LU factorization of A to find x.
4. Compute the x by two iterations of successive over-relaxation scheme. Use relaxation parameter $\omega = 1.5$ and initial guess of zero.

Problem 6 (4 points):

Consider the below (x,y) pairs:

$$x = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad y = f(x) = \begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix}$$

1. Find the Lagrange polynomial for above points.
2. Interpolate that polynomial at x=-1.
3. Find the ordered polynomial for above points with Newton's formula.
4. Interpolate the ordered polynomial at x=-1.