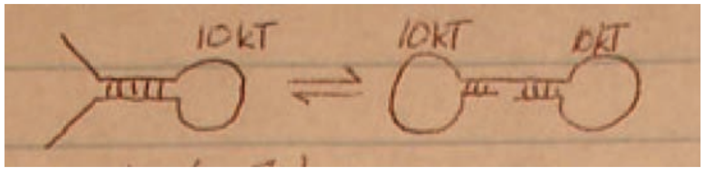
20.310 **09/26/06**

HW#2 Problem #5

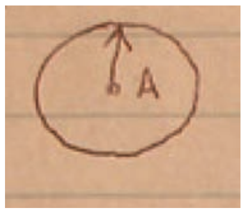


loop of 6-7 bases requires 7-8 k_BT
loops are **unfavorable** ← a cost
backbone interactions are favorable (benefits)

$P_1/P_2 = \exp(\_\_\_\_\_\_\_)$

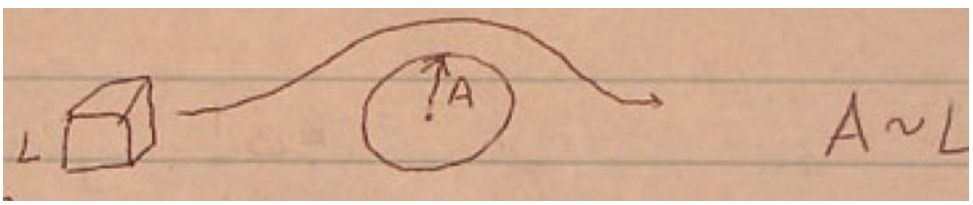
Archimedes

Force = $f_{drag} \cdot v$



$$F = c\mu A v$$
$$F_{sphere} = 6\pi\mu r v$$

$$\text{Reynolds's number} = \frac{\text{inertial forces}}{\text{viscous forces}}$$



$$F = ma = m\frac{dv}{dt} = m\frac{v^2}{A}$$

*characteristic time $A/v = \tau$

* $a = dv/dt = v/(A/v) = v^2/A$

mass = $\rho L^3 = m$ where ρ = density
inertial forces = $(L^3\rho)(v^2/A)$

$$\text{Reynolds's number} = \frac{inertial}{viscous} = \frac{L^3\rho v^2}{A}\frac{1}{\mu v A} = \frac{\rho v L}{\mu} \qquad \surd \qquad (L \sim A)$$

Reynolds's number for a sailboat

v = 10 m/s, L = 10m, $\rho = 10^3$ kg/m^3, $\mu = 10^{-3}$ Pa·s

$$\text{Re\#} = \frac{\rho v L}{\mu} = \frac{10^3 \cdot 10 \cdot 10}{10^{-3}} \sim 10^8$$ HIGH! Turbulent.

Einstein Relation

$$\boxed{\begin{array}{l}\gamma D = k_B T \\ F_{drag} D = k_B T\end{array}}$$

$V(t) = V_0 + \dfrac{F \cdot t}{m}$ where V_0 is the "random kick" term

$$F = ma = m\frac{dv}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \frac{F}{m} \quad \Rightarrow \quad dv \cong \frac{F}{m}t$$

$\Delta x = V_0 \Delta t + \dfrac{1}{2}\dfrac{F}{m}\Delta t^2$ but $V_0\,\Delta t \rightarrow \text{Ø}$ over long time (random kick)

$$\langle x \rangle = \frac{F(\Delta t)^2}{2m}$$

Define a drift velocity:

$\dfrac{\langle x \rangle}{\Delta t} = \dfrac{F\Delta t}{2m} = \dfrac{F}{\gamma}$ where $\gamma = \dfrac{2m}{\Delta t}$ ($\gamma = 6\pi\mu r$ for a sphere)and $F = \gamma V = \gamma \dfrac{\langle x \rangle}{\Delta t}$

$D = \dfrac{L^2}{2\Delta t}$ ($D \rightarrow$ diffusion) $\quad \gamma = \dfrac{2m}{\Delta t}$

$\gamma D = \dfrac{L^2}{2\Delta t} \cdot \dfrac{2m}{\Delta t} = \dfrac{mL^2}{\Delta t^2} = mV^2 = k_B T \qquad \dfrac{1}{2}mV^2 = \dfrac{1}{2}k_B T$

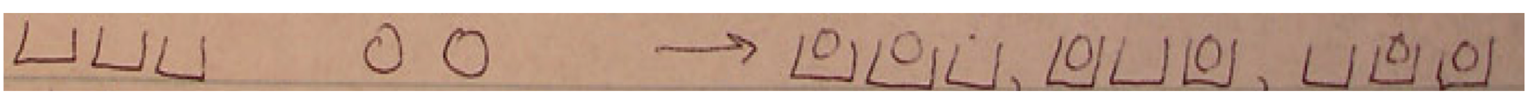
Therefore, $\boxed{\gamma D = k_B T}$ Einstein Relation

Entropy

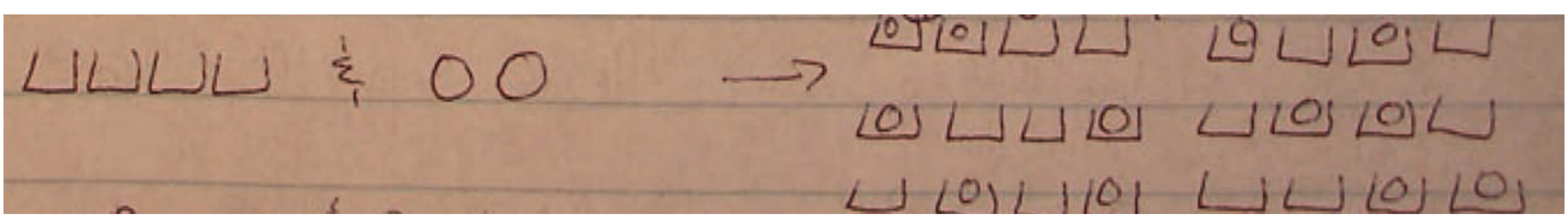
$S = k_B \ln W$

	<u>W</u>
2 boxes and 2 balls	1 way

3 boxes and 2 balls	3 ways



4 boxes and 2 balls	6 ways



5 boxes and 2 balls $\rightarrow \dfrac{5!}{(5-2)!\,2!} = \dfrac{5!}{3!\,2!}$

10 boxes and 2 balls $\rightarrow$ 45 states