Poroelasticity in **1D Confined Compression** ($\varepsilon_{22} = \varepsilon_{33} = 0$)

Assumptions:
Solid: Hookean (linear elastic, or viscoelastic), homogeneous, isotropic, incompressible
Liquid: Inviscid (fluid friction negligible or μ=0, but there is solid-liquid friction), incompressible

Constitutive relation
$$\sigma_{11}^{tot} = (2G+\lambda)\varepsilon_{11} - p = H\varepsilon_{11} - p \quad \text{with} \quad \varepsilon_{11} = \frac{\partial u_1}{\partial x_1} \qquad (1)$$

Conservation of mass
$$U_1 = -\frac{\partial u_1}{\partial t} + U_0 = \phi(v_f - v_s) = \frac{A_f}{A_{tot}}(v_f - v_s) \qquad (2)$$

Conservation of momentum
$$\frac{\partial \sigma_{11}}{\partial x_1} = 0 \qquad (3)$$

Fluid-structure interaction: Darcy's law
$$U_1 = -k\frac{\partial p}{\partial x_1} \qquad (4)$$

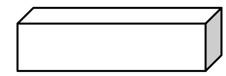
Diffusion equation
$$\frac{\partial u_1}{\partial t} = Hk\frac{\partial^2 u_1}{\partial x_1^2}$$

For stress relaxation
$$u_1(x_1,t) = u_0\left(1-\frac{x_1}{L}\right) - \sum_n \frac{2u_0}{n\pi}\sin\left(\frac{n\pi x_1}{L}\right)\exp\left(-\frac{n^2\pi^2 Hk}{L^2}t\right)$$

Cell-Membrane Mechanics

- Stretching

$$N = \int_{-h/2}^{h/2} \sigma_{11} dx_3 = h\sigma_{11} = \frac{Eh}{1-\nu}\varepsilon_{11} = \frac{Eh}{2(1-\nu)}\frac{\Delta A}{A_0} = K_e \frac{\Delta A}{A_0} h$$

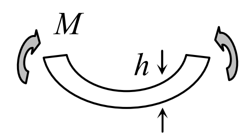


$$K_e = \frac{Eh}{2(1-\nu)}$$

Typical K_e (N/m): Lipid bilayer ~ 0.1, red blood cell ~ 0.45, water /air ~ 0.07
$N_{rupture}$(N/m) ~ 0.04-0.06

- Bending

$$M = \int_{-h/2}^{h/2} \sigma_{11} x_3 dx_3 = -\frac{Eh^3}{12(1-\nu^2)}\frac{\partial^2 u_3}{\partial x_1^2} = K_B \frac{\partial^2 u_3}{\partial x_1^2}$$



$$K_B = \frac{Eh^3}{12(1-\nu^2)}$$

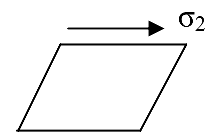
Typical K_B (*Nm*): 10^{-18}-10^{-19}

- Shear

$$N_{21} = \sigma_{21} h = 2hG\varepsilon_{21} = K_s \varepsilon_{21}$$

$$K_s = 2hG$$

Typical K_s (N/m): 10^{-6} (red blood cell)



- Composite Equation

General: $P - K_B \nabla^4 u_3 + N\nabla^2 u_3 = 0$

1D: $p + N\frac{\partial^2 u_3}{\partial x_1^2} - K_B \frac{\partial^4 u_3}{\partial x_1^4} = 0$

$$\frac{bending}{tension} \sim \frac{K_B}{NL^2}$$

Filipod Extension

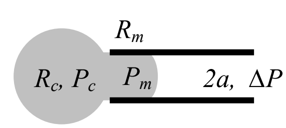
$$\frac{K_B}{NL^2} \sim 10^{-3}$$

$$\Delta p \sim N\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \sim \frac{2N}{R}$$

Neutrophil aspiration

At max Δp (stable shape)

$$P_m + \Delta P = \frac{2N}{a}; P_c = \frac{2N}{R_c}$$



If $P_m = P_c$

$$\Delta P^* = \frac{2N}{a}\left(1 - \frac{a}{R_c}\right)$$

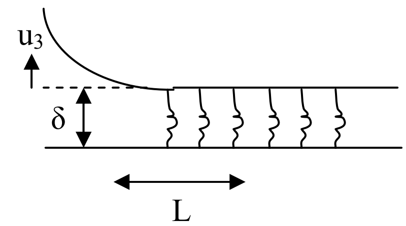
Cell Peeling

$$\frac{K_B}{NL^2} \sim 10^2$$

$$p \sim K_B \frac{u_3}{L^4} \sim \rho_A u_3 \kappa$$

ρ_A = receptors/unit area

κ = spring constant = bond stiffness



Cytoskeletal Mechanics

Structural elements:
- actin: microfilaments
- tubulin: microtubules
- intermediate filaments

Scaling & dimensional analysis

$$\frac{\partial}{\partial x} \sim \frac{1}{L} \qquad \frac{\partial}{\partial t} \sim \frac{1}{\tau} \sim \omega \qquad \int f dx \sim fL \qquad \int f dt \sim f\tau$$

Ignore first order dimensionless constants

Governing equations — Solid (Hookean)

- Force balance (inertia neglected)

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0; F = \sigma A$$

- Constitutive equations

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2G\varepsilon_{ij}$$

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2G\varepsilon_{ij} - p$$

Note: $\delta_{ij}=1$ if i=j, and $\delta_{ij}=0$ if i≠j

- Energy

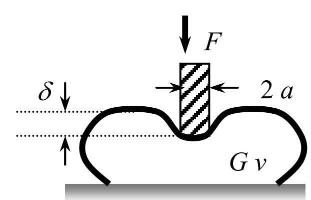
$$U = \int \frac{1}{2} \underline{\underline{\sigma}} \cdot \underline{\underline{\varepsilon}} d\forall \sim E\varepsilon^2 \forall$$

Experimental measurements

- Cell indentation

$$F \sim \sigma a^2 \sim G\varepsilon a^2 \sim G\frac{\delta}{a}a^2 \sim G\delta a$$

$$U \sim F\delta \sim E\varepsilon^2\forall \sim G\varepsilon^2 a^3 \sim G\left(\frac{\delta}{a}\right)^2 a^3$$



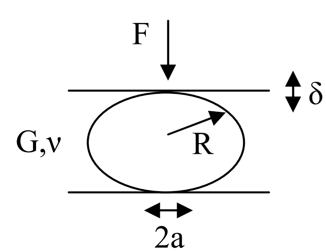
- Cell Squashing

$$U \sim F\delta \sim G\varepsilon^2 a^3$$

$$F \sim G\delta a$$

$$a^2 + (R-\delta)^2 = R^2 \Rightarrow a \sim \sqrt{R\delta}$$

$$F \sim G\delta^{3/2} R^{1/2}$$



- Magnetic twisting cytometry (relate torque to bead rotation angle to get $G^* = G' + iG''$)
- Passive Particle Methods (particle tracking, Brownian motion)

Microstructural models of the cell:

- Cellular solids : $E_n = E_f \varphi^2$
- Tensegrity: $E_n \sim P\varphi$ (P = press stress in tensile elements as ε→0)
- Biopolymer: $E_n \sim \frac{l_p K_b}{a^5}\varphi^{5/2}$ at maximum cross link density

  $E_n \sim \frac{l_p K_b}{l^3 a^2}\varphi$ at low cross link density