NODICITY

One of the important ways that physical system behavior *differs* between domains is the way elements may be connected.

Electric circuit elements may be connected in series or in parallel

- **— networks of arbitrary structure may be assembled**

This not true of all domains

Electric circuit elements have a special property called *nodicity*

NODICITY

***Nodicity* means that a network element (or sub-network) has behavior analogous to a "node" in an electric circuit.**

Consider a "delta" network of resistors

Assuming linear inductors, the relation between currents and voltages may be written as

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & G_2+G_3 & -G_3 \\ -G_1 & -G_3 & G_1+G_3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

where G_i is conductance

$$i_i = G_i\, e_i$$

$$G_i = \frac{1}{R_i}$$

Note that this conductance matrix is singular

— the sum down each column is zero.

i.e., the sum of currents flowing into the network is zero

$$i_1 + i_2 + i_3 = 0$$

As a result, the three constitutive equations describing the currents may be re-arranged as follows

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_2+G_3 \end{bmatrix} \begin{bmatrix} e_1-e_3 \\ e_2-e_3 \end{bmatrix}$$

and

$$i_3 = -(i_1 + i_3)$$

POINTS TO NOTE:

- **any one effort may be used as reference**
- **the reference effort *may vary arbitrarily***
- **the constitutive equations depend only on input *effort differences***

Thus the "delta" network has the properties of a circuit node.

Kirchhoff's current law applies

This constraint arises from charge continuity

It applies when any or all of the resistors are replaced by inductors or capacitors

It applies when any or all of the element constitutive equations are non-linear.

In fact Kirchhoff's current law applies to *any* sub-network of an electric circuit

(i.e., any *cut set* of the network)

That is, networks of electric elements (electric circuits) are *nodic*.

NON-NODIC NETWORKS

Network models in other domains may not be nodic.

TWO ASPECTS:

1. choice of reference may not be arbitrary

example:

in the constitutive equation of a translational mass

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

velocity *must* be referenced to an inertial (non-accelerating) frame

— if not, the constitutive equations must be modified to include coriolis and/or centrifugal accelerations

example:

in the ideal gas equation

$$PV = mRT$$

- temperature must be referenced to absolute zero
- pressure must be referenced to vacuum
- volume must be referenced to absolute zero volume
- mass must be referenced to absolute zero mass

note that linearized approximations may *appear* nodic

example:

linearize the ideal gas equation

$$P\Delta V + V\Delta P \approx \Delta mRT + mr\Delta T$$

a fixed mass of gas at constant temperature may be described as

$$-\Delta V \approx \left(\frac{V}{P}\right) \Delta P = C_{fluid}\Delta P$$

in this case pressure may be referenced to any convenient value, e.g., ambient pressure

i.e., gauge pressure may be used

But this is only an approximation

In contrast, the nodicity of electric circuits is fundamental

THE SECOND ASPECT:

2. constitutive equations may not depend input effort differences

example

Stefan-Boltzmann Law of radiative heat transfer

$$\dot{Q} = \sigma(T_1{}^4 - T_2{}^4)$$

$\dot{Q}$: heat flow rate

σ: radiative heat transfer coefficient.

T_1 and T_2: absolute temperatures.

example

choked (supersonic) flow through an orifice:

$$\dot{N} = C_d A_t \left(\frac{2}{\gamma+1}\right)^{1/(\gamma-1)} \sqrt{\frac{2\gamma}{\gamma+1} \rho_u P_u}$$

$\dot{N}$: mass flow rate

C_d: discharge coefficient

A_t: area at orifice throat

γ: ratio of specific heats

ρ_u: upstream density

P_u: upstream pressure

(source: Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik, 1994.)

NODICITY

a formal definition

An element (or subsystem) is nodic if efforts and flows at its ports satisfy two conditions:

(1) FLOW CONTINUITY:

The (signed) sum of flows into the element is zero.

i.e., a generalization of Kirchhoff's current law applies

(2) EFFORT RELATIVITY:

The element's constitutive equations depend only on a difference of efforts.

If the same effort is added to all inputs, the output is unchanged.

WHY DOES NODICITY MATTER?

The analogy between network elements in different domains is not complete

arbitrary connections of non-nodic elements may be impossible

(or have no physical meaning)

example:

a “delta” or “wye” network of electrical capacitors may be assembled

if we assume an electrical capacitor is analogous to a gas-filled pressure vessel

what physical system corresponds to a “delta” or “wye” network of pressure vessels?

example:

a “bridged-tee” network of inductors can be assembled

if we assume an inductor is analogous to a translational mass

what physical system corresponds to a “bridged-tee” network of masses?