2.001 - MECHANICS AND MATERIALS I
Lecture #1
9/6/2006
Prof. Carol Livermore

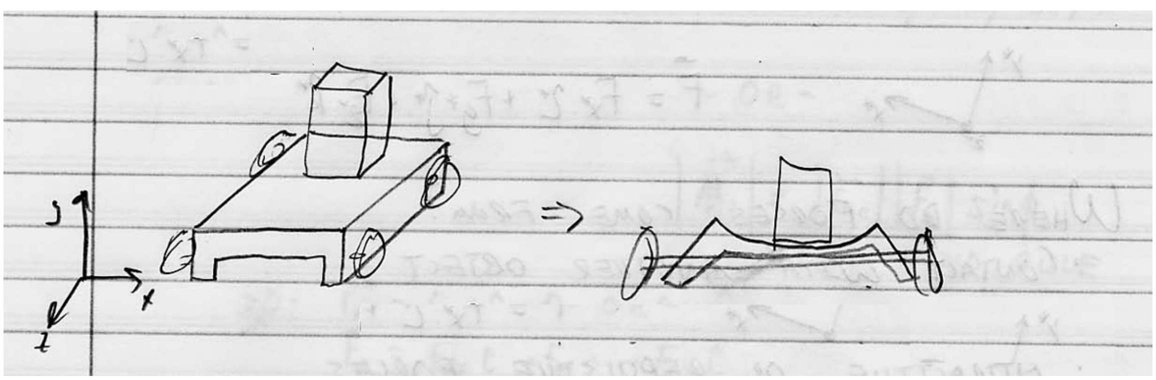
A first course in mechanics for understanding and designing complicated systems.

PLAN FOR THE DAY:

1. Syllabus
2. Review vectors, forces, and moments
3. Equilibrium
4. Recitation sections

BASIC INGREDIENTS OF 2.001

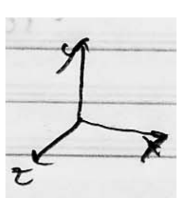
1. Forces, Moments, and Equilibrium → Statics (No acceleration)
2. Displacements, Deformations, Compatibility (How displacements fit together)
3. Constitutive Laws: Relationships between forces and deformations

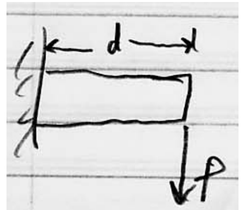


Use 2.001 to predict this problem.

HOW TO SOLVE A PROBLEM
RULES:

1. SI inits *only*.
2. Good sketches.
   - Labeled coordinate system.

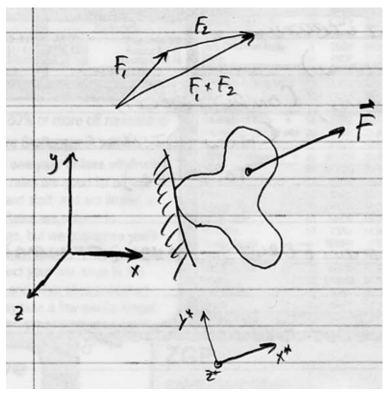




- Labeled dimensions and forces.

3. Show your thought process.
4. Follow all conventions.
5. Solve everything symbolically until the end.
6. Check answer ⇒ Does it make sense?
   - Check trends.
   - Check units.
7. Convert to SI, plug in to get answers.

VECTORS, FORCES, AND MOMENTS
Forces are vectors (Magnitude and Direction)



$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

OR

$$\vec{F} = F_{x^*}\hat{i}^* + F_{y^*}\hat{j}^* + F_{z^*}\hat{k}^*$$

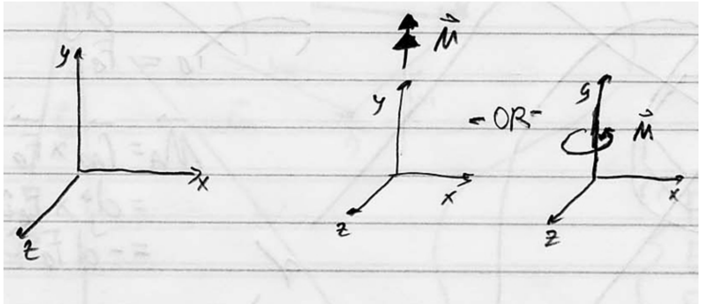
Where do forces come from?

- Contact with another object
- Gravity
- Attractive of repulsive forces

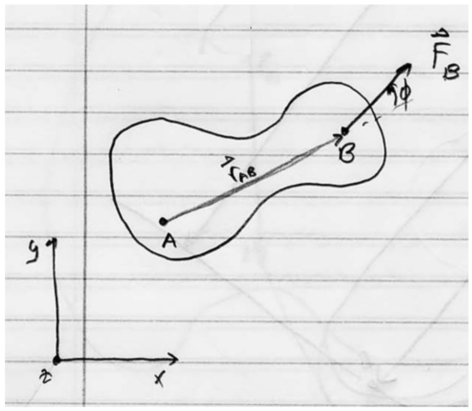
Moments are vectors. (Moment is another word for torque.)

- Forces at a distance from a point
- Couple

Moments are vectors.
$\vec{M} = M_y \hat{j}$.



EX:



2D:

$$\vec{F_B} = F_{Bx}\hat{i} + F_{By}\hat{j}$$
$$r\vec{_{AB}} = r_{AB_x}\hat{i} + r_{AB_y}\hat{j}$$

$$\begin{aligned}\vec{M_A} = r\vec{_{AB}} \times \vec{F_B} &= \\ &= (r_{AB_x}\hat{i} + r_{AB_y}\hat{j}) \times (F_{B_x}\hat{i} + F_{B_y}\hat{j}) \\ &= (r_{AB_x}F_{B_y} - r_{AB_y}F_{B_x})\hat{k}\end{aligned}$$

OR

$$\left|\vec{M_A}\right| = |r\vec{_{AB}}| \left|\vec{F_B}\right| \sin\phi \text{ using Right Hand Rule}$$

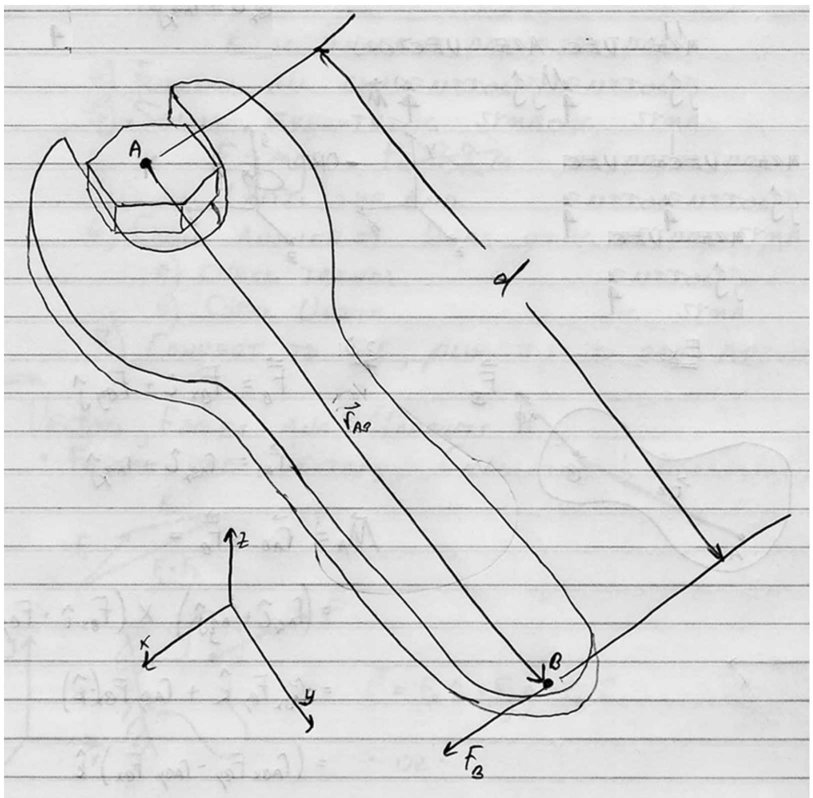
3D:

$$r\vec{_{AB}} = r_{AB_x}\hat{i} + r_{AB_y}\hat{j} + r_{AB_z}\hat{k}$$
$$\vec{F_B} = F_{B_x}\hat{i} + F_{B_y}\hat{j} + F_{B_z}\hat{k}$$

$$\vec{M_A} = r\vec{_{AB}} \times \vec{F_B} = (r_{AB_x}F_{B_y} - r_{AB_y}F_{B_x})\hat{k} + (r_{AB_z}F_{B_x} - r_{AB_x}F_{B_z})\hat{j} + (r_{AB_y}F_{B_z} - r_{AB_z}F_{B_y})\hat{i}$$

EX: Wrench

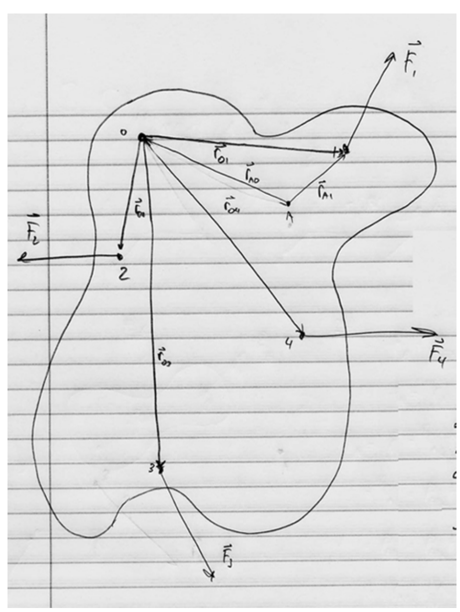
$$r_{\vec{AB}} = d\hat{j}$$
$$\vec{F_B} = F_B\hat{i}$$
$$\vec{M_A} = r_{\vec{AB}} \times \vec{F_B} = d\hat{j} \times F_B\hat{i} = -dF_B\hat{k}$$



Equations of Static Equilibrium

$$\Sigma\vec{F} = 0 \quad \begin{matrix} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{matrix}$$

$$\Sigma\vec{M_0} = 0 \quad \begin{matrix} \Sigma M_0x = 0 \\ \Sigma M_0y = 0 \\ \Sigma M_0z = 0 \end{matrix}$$



$$\sum_{i=1}^{4} \vec{F} = 0$$

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 0$$

$$F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$$

$$F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

$$F_{1z} + F_{2z} + F_{3z} + F_{4z} = 0$$

$$\sum \vec{M_o} = 0$$

$$\Rightarrow \vec{r_{01}} \times \vec{F_1} + \vec{r_{02}} \times \vec{F_2} + \vec{r_{03}} \times \vec{F_3} + \vec{r_{04}} \times \vec{F_4} = 0$$

a. Expand in $\hat{\imath}, \hat{\jmath}, \hat{k}$.
b. Group.
c. Get 3 equations (x,y,z).

Try a new point? Try A.

$$\vec{r_{A1}} = \vec{r_{A0}} + \vec{r_{01}}$$

$$\ldots$$

$$\sum \vec{M_A} = 0$$

$$\vec{r_{A1}} \times \vec{F_1} + \vec{r_{A2}} \times \vec{F_2} + \vec{r_{A3}} \times \vec{F_3} + \vec{r_{A4}} \times \vec{F_4} = 0$$

$$(\vec{r_{A0}} + \vec{r_{01}}) \times \vec{F_1} + (\vec{r_{A0}} + \vec{r_{02}}) \times \vec{F_2} + (\vec{r_{A0}} + \vec{r_{03}}) \times \vec{F_3} + (\vec{r_{A0}} + \vec{r_{04}}) \times \vec{F_4} = 0$$

$$\vec{r_{A0}} \times (\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}) + (\vec{r_0}1 \times \vec{F_1} + \vec{r_0}2 \times \vec{F_2} + \vec{r_0}3 \times \vec{F_3} + \vec{r_0}4 \times \vec{F_4}) = 0$$

$$\vec{r_{A0}} \times (\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}) = 0$$

$$\sum \vec{F} = 0$$

$$(\vec{r_0}1 \times \vec{F_1} + \vec{r_0}2 \times \vec{F_2} + \vec{r_0}3 \times \vec{F_3} + \vec{r_0}4 \times \vec{F_4}) = 0$$

$$\sum \vec{M_0} = 0$$

So taking moment about a new point gives no additional info.