2.092/2.093

FINITE ELEMENT OF SOLIDS AND FLUIDS I

FALL 2009

Homework 4- solution

Instructor:	Prof. K. J. Bathe	Assigned:	Session 9
TA:	Seounghyun Ham	Due:	Session 10

Problem 1 (30 points):

a) The displacements and reaction forces for the truss structure were evaluated analytically in HW #1. Using, E=1.0×10^{9}N/m^2, A=0.0025m^2, a=2m, and R_5 = 300 kN, we found that:

$$\frac{EA}{a}\begin{bmatrix} \frac{1}{2\sqrt{2}}+1 & 0 & 0 \\ 0 & \frac{1}{2\sqrt{2}}+1 & \frac{1}{2\sqrt{2}} \\ 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}}+1 \end{bmatrix}\begin{bmatrix} U_3 \\ U_5 \\ U_6 \end{bmatrix}=\begin{bmatrix} R_3 \\ R_5 \\ R_6 \end{bmatrix}=\begin{bmatrix} 0 \\ 300 \\ 0 \end{bmatrix} kN.$$

$$\begin{bmatrix} U_3 \\ U_5 \\ U_6 \end{bmatrix}=\begin{bmatrix} 0 \\ 0.1903 \\ -0.0497 \end{bmatrix} m$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_4 \\ R_7 \\ R_8 \end{bmatrix}=\frac{EA}{a}\begin{bmatrix} -1 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 0 & -1 \\ -\frac{1}{2\sqrt{2}} & -1 & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0.1903 \\ -0.0497 \end{bmatrix}=\begin{bmatrix} -62.13 \\ -62.13 \\ 62.13 \\ -237.5 \\ 0 \end{bmatrix} kN$$

For numbering of R_i, see HW #1.

These analytical linear elastic results can now be compared to the results calculated numerically by ADINA.

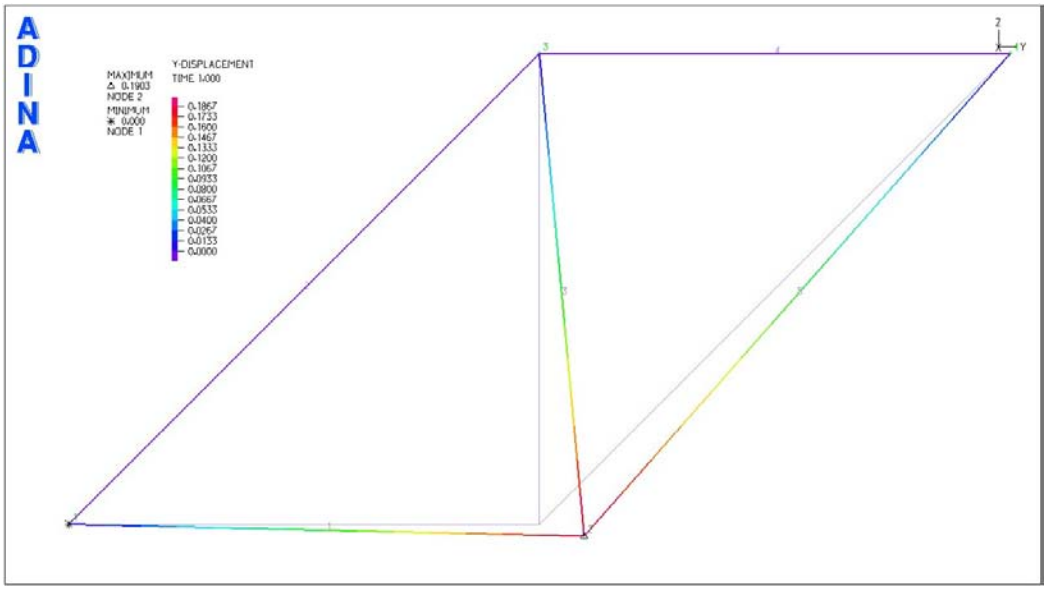


Table 1: Displacements at each node

	Y-DISPLACEMENT	Z-DISPLACEMENT
Node 1	0.0000E+00	0.0000E+00
Node 2	1.9029E-01	-4.9706E-02
Node 3	0.0000E+00	0.0000E+00
Node 4	0.0000E+00	0.0000E+00

Table 2: Reactions at each node

	Y-REACTION	Z-REACTION
Node 1	-2.3749E+05	0.0000E+00
Node 2	3.0000E+05	0.0000E+00
Node 3	0.0000E+00	6.2132E+04
Node 4	-6.2132E+04	-6.2132E+04

Table 3: Element forces

	Force
Element 1	2.3787E+05
Element 2	0.0000E+00
Element 3	6.2132E+04
Element 4	0.0000E+00
Element 5	-8.7868E+04

As expected, the numerical results are exactly the same as the analytical results calculated in HW #1.

b) Nonlinear analysis results

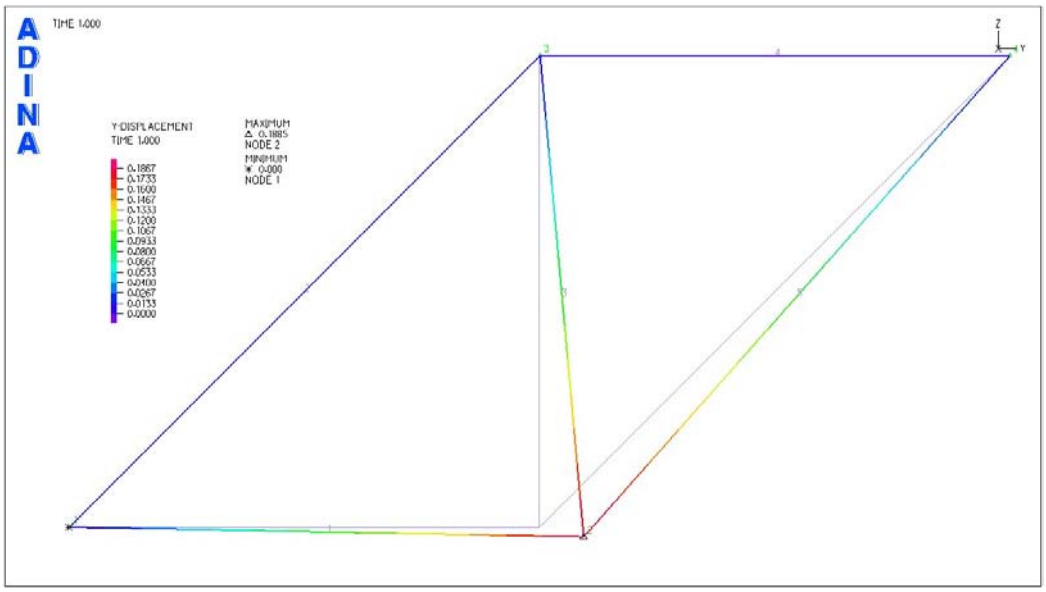


Table 4: Displacements at each node

	Y-DISPLACEMENT	Z-DISPLACEMENT
Node 1	0.0000E+00	0.0000E+00
Node 2	1.8848E-01	-4.0916E-02
Node 3	3.2904E-03	0.0000E+00
Node 4	0.0000E+00	0.0000E+00

Table 5: Reactions at each node

	Y-REACTION	Z-REACTION
Node 1	-2.3749E+05	2.9594E+03
Node 2	3.0000E+05	0.0000E+00
Node 3	0.0000E+00	6.2827E+04
Node 4	-6.2505E+04	-6.5787E+04

Table 6: Element forces

	Force
Element 1	2.3608E+05
Element 2	2.0574E+03
Element 3	6.1626E+04
Element 4	-4.1130E+03
Element 5	-8.7964E+04

c)

Table 7: Comparison between Linear and Nonlinear analyses

	Linear		Nonlinear	
	Y-DISPLACEMENT	Z-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
Node 1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
Node 2	1.9029E-01	-4.9706E-02	1.8848E-01	-4.0916E-02
Node 3	0.0000E+00	0.0000E+00	3.2904E-03	0.0000E+00
Node 4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
	Y-REACTION	Z-REACTION	Y-REACTION	Z-REACTION
Node 1	-2.3749E+05	0.0000E+00	-2.3749E+05	2.9594E+03
Node 2	3.0000E+05	0.0000E+00	3.0000E+05	0.0000E+00
Node 3	0.0000E+00	6.2132E+04	0.0000E+00	6.2827E+04
Node 4	-6.2132E+04	-6.2132E+04	-6.2505E+04	-6.5787E+04
	Element Force		Element Force	
Element 1	2.3787E+05		2.3608E+05	
Element 2	0.0000E+00		2.0574E+03	
Element 3	6.2132E+04		6.1626E+04	
Element 4	0.0000E+00		-4.1130E+03	
Element 5	-8.7868E+04		-8.7964E+04	

Displacement – Linear Vs. Nonlinear

The linear analysis predicted zero lateral (Y-) displacement for node 3; however, the nonlinear analysis predicted a Y-displacement of 3.29e-3m in node 3.

Element Forces – Linear Vs. Nonlinear

The linear analysis predicted zero force in elements 2 and 4. Conversely, the nonlinear analysis predicts a tensile force of 2.06e3N in element 2 and a compressive force of -4.11e3N in element 4.

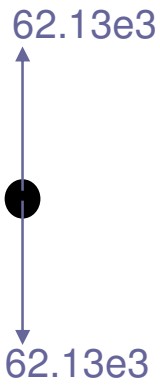
Reaction Force – Linear Vs. Nonlinear

The linear analysis predicted zero vertical (Z-) reaction at node 1; however, the nonlinear analysis predicts a vertical reaction force of 2.96e3N at node 1.

d)

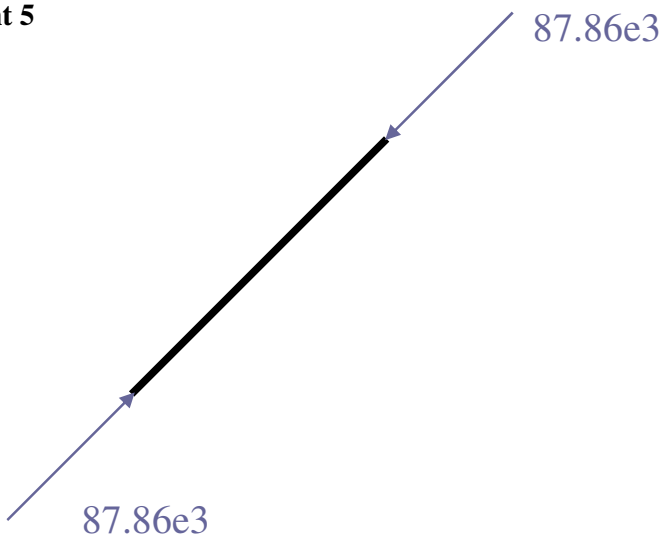
For the linear case

Joint 3

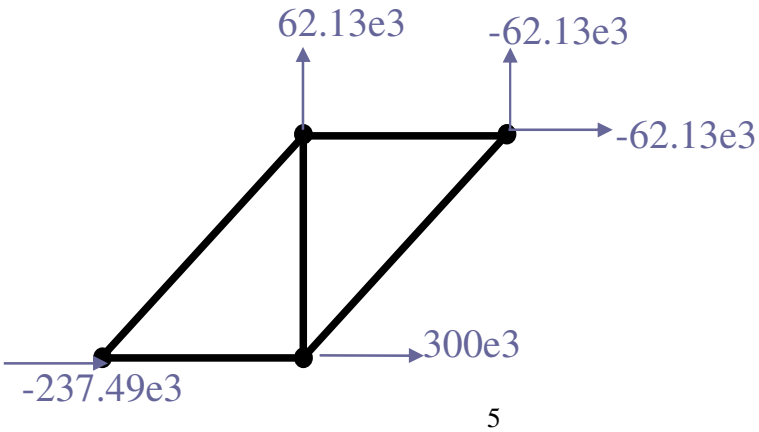


$$\sum \vec{F}_y = -62.13\times10^3 + 62.13\times10^3 = 0$$

Element 5



Complete Structure



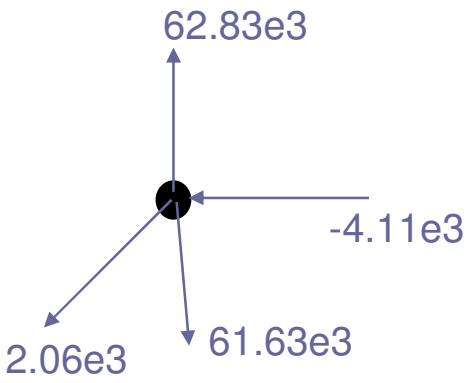
$$\sum \vec{F}_x = -237.49\times10^3 + 300\times10^3 - 62.13\times10^3 = 0$$

$$\sum \vec{F}_y = 62.13\times10^3 - 62.13\times10^3 = 0$$

$$\sum \vec{M}_4 = -62.13\times10^3\times2 - 237.49\times10^3\times2 + 300\times10^3\times2 = 0$$

For the nonlinear case

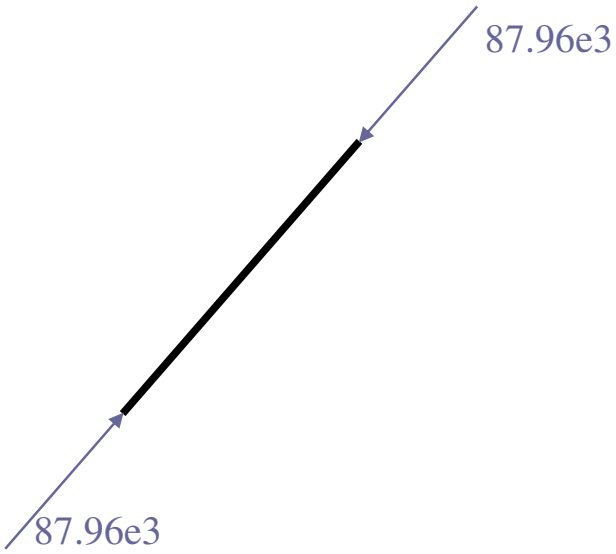
Joint 3



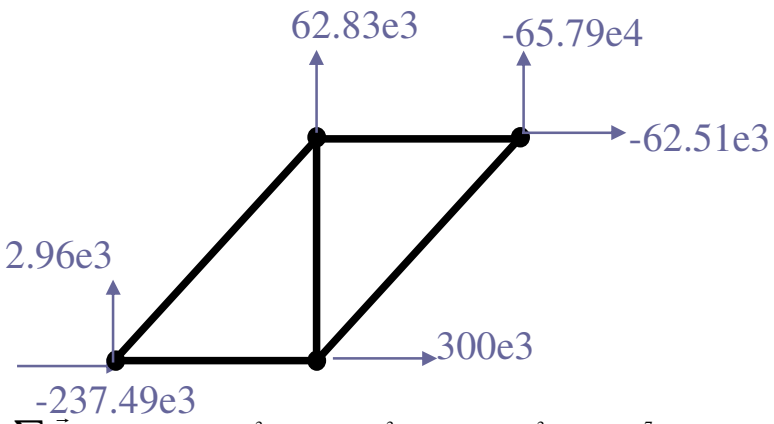
$$\sum \vec{F}_x = -2.06\times10^3 \sin(45.0471^\circ) + 61.63\times10^3 \sin(5.1848^\circ) - 4.11\times10^3 = 1.6698\times10^{-8} \approx 0$$

$$\sum \vec{F}_y = -2.06\times10^3 \cos(45.0471^\circ) - 61.63\times10^3 \cos(5.1848^\circ) + 62.83\times10^3 = -2.2359\times10^{-8} \approx 0$$

Element 5



Complete Structure



$$\sum \vec{F}_x = -237.49\times10^3 + 300\times10^3 - 62.51\times10^3 = 5\times10^{-7} \approx 0$$

$$\sum \vec{F}_y = -65.79\times10^3 + 62.83\times10^3 + 2.96\times10^3 = -5.9\times10^{-8} \approx 0$$

$$\sum \vec{M}_4 = -237.49\times10^3\times2 - 2.96\times10^3\times4 - 62.83\times10^4\times1.9967 + 3\times10^5\times2.0409 = 1.004\times10^{-6} \approx 0$$

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