Massachusetts Institute of Technology

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Fields, Forces and Flows in Biological Systems, Spring 2007

__Problem Set #10.__ · **Issued:** May 4th (Friday)
· **Due date:** May 11th (Friday)

Problem 1: Electrokinetic Pumping (40 points)

Consider a glass capillary, with length L and the inner radius R, connecting the two electrolyte reservoirs. The cathode, as shown in the figure below, is contained within a closed container, tightly sealed. The anode is immersed within the reservoir facing the atmospheric pressure. The glass surface of the capillary has negative surface charges that can be characterized by the (negative) zeta potential ζ. An electric potential is applied between the two electrodes. Debye length is κ^{-1}, and the viscosity and dielectric constant are ε and μ, respectively. Ignore electrolysis at the electrodes, and assume that $\kappa R \gg 1$.
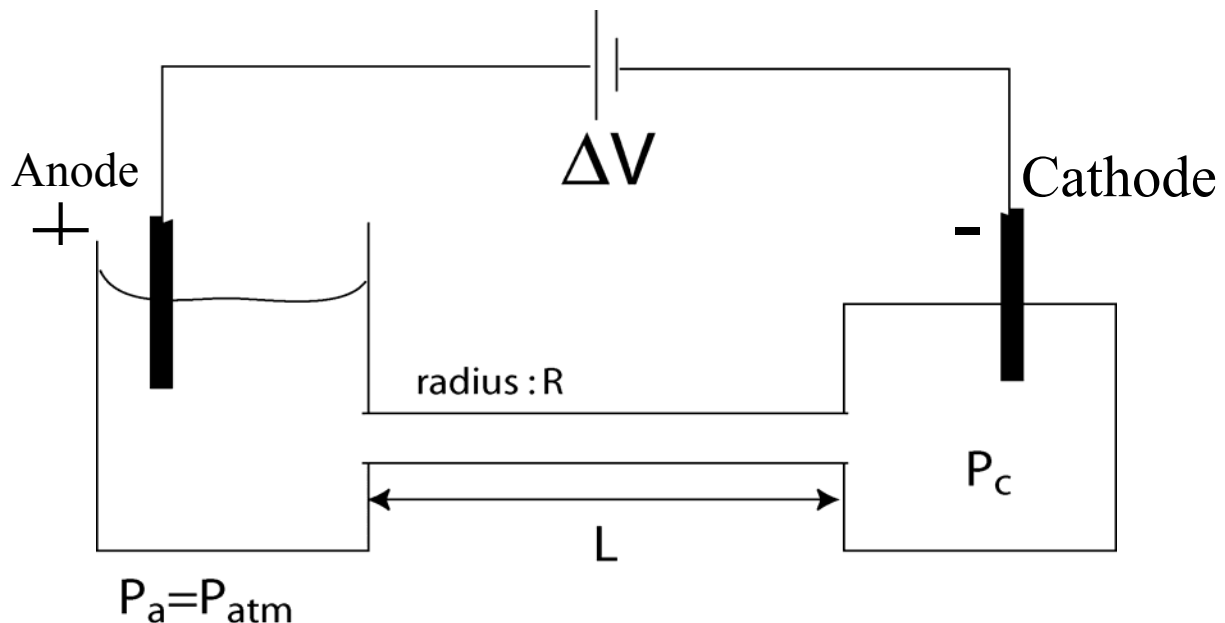
(a) Find the expression for (steady-state) pressure difference between the cathodic and anodic reservoir ($\Delta P = P_c - P_a$) at the steady state.

(b) Determine the flow velocity profile $v_z(r)$ within the capillary at the steady state. Sketch the flow streamline along the capillary (over the entire length of the capillary). Use the cylindrical coordinate system with the center line of the capillary being r=0 (z axis).

(c) Calculate ΔP for R=10μm, 0.1μm, and 0.01μm. Use ζ=-100mV, ΔV=1000V, L=5cm, σ=1Ω^{-1}m^{-1} and use the values for water for dielectric constant and viscosity.

(d) Four different experimental situations are compared in the following. Identify the case that will generate the highest pressure, and explain why.
A: One capillary (with the radius R and length L) bridging the reservoirs
B: One capillary (with the radius R and length 2L) bridging the reservoirs
C: Four capillaries (with the radius 0.5R and length L) bridging the reservoirs
D: One capillary (with the radius 0.4R and length 2L) bridging the reservoirs

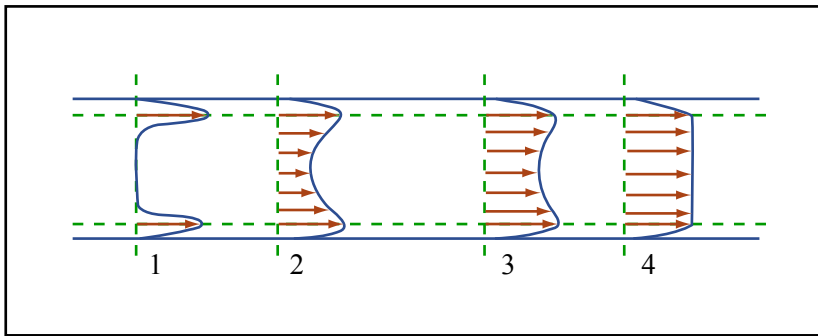**Problem 2. Time scale for the onset of electroosmotic flow. (15 points)**

In the lecture, we all learned that the electroosmotic flow is generated by the motion of surface-bound Debye layer charges. Initially only the surface fluid layer moves (as shown in figure 1 above), but then its momentum is transferred to the entire fluid column, yielding a flat flow profile as shown in 4 above. Assuming the small Debye length limit ($\kappa R \gg 1$) and assuming that the buffer solution in the capillary is water, *estimate the approximate time scale* for the transition from 1 to 4 (a scaling result will suffice). What are the values of these time scales when R=10μm, 100nm and 1nm? (Hint: This is indeed a fluid dynamics problem. Write down the Navier-Stokes equation and think about scaling arguments.)

Problem 3: Capillary On-line Preconcentration of proteins (45 points)
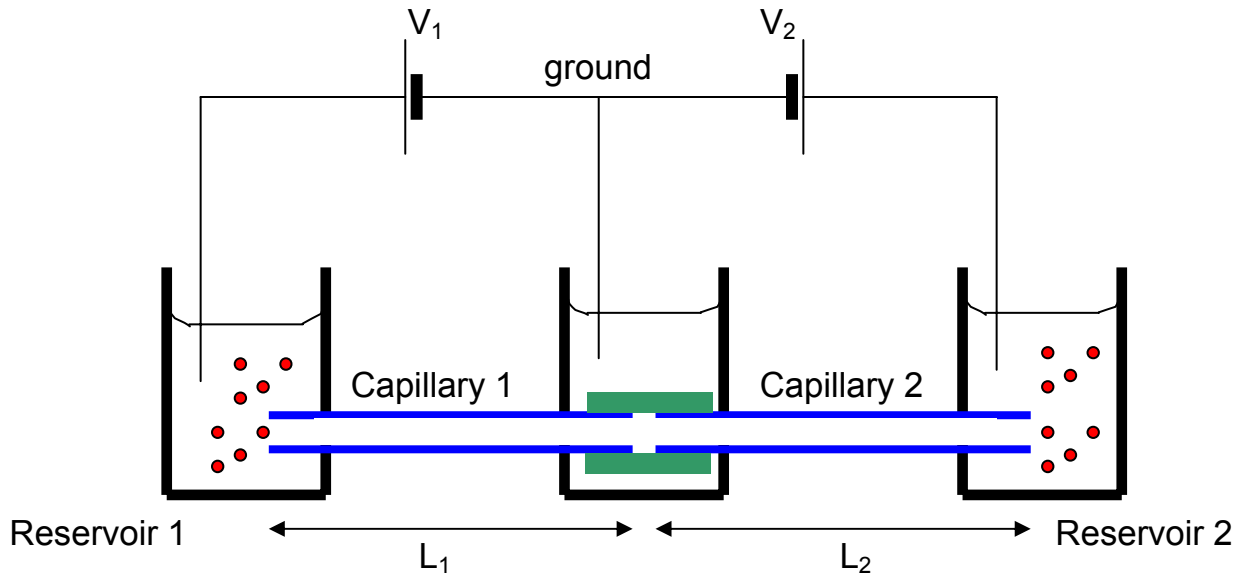
Consider the above capillary system, where two different capillaries with the same radius R but different lengths L_1 and L_2 (blue) are joined by a porous membrane (green) which conducts electrical (ionic) currents freely but has negligible water permeability. Then a different potential (V_1 and V_2) are applied to the two reservoirs, which contain positively charged proteins (red dots) with the electrophoretic mobility u_{ep}. The capillary surface has (negative) zeta potential of ζ at the buffer condition used in this experiment, and the Debye length thickness κ^{-1} is much smaller than any other size parameters of the system ($\kappa R \gg 1$).

(a)   Show that a hydraulic press ΔP (above atmospheric pressure of reservoir 1 and 2) will be developed within the capillary junction, at the steady state. Get the expression for ΔP in terms of other parameters. Explain why there is a pressure developed at the junction.

$$P = P_{ATM} + \Delta P$$

Impermeable to fluid flow
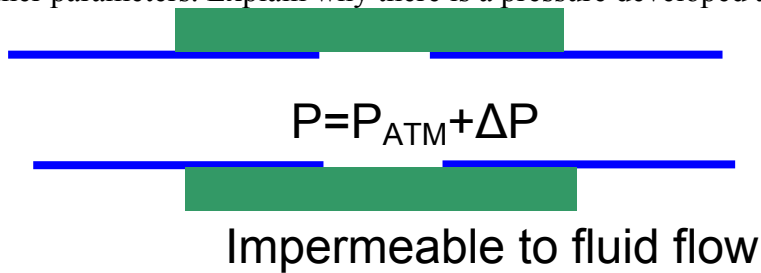
(b)   Determine the stead-state flow velocity (averaged over the cross section of the capillary) of the system, as a function of other parameters.

(c)   Assume $V_1 > V_2$. For a given values of L_1, L_2, u_{ep}, and V_1, determine the conditions for V_2, which will allow the proteins to be continuously focused and concentrated at the capillary junction. This could be a concentration scheme for protein analysis in capillary electrophoresis.