2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

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Solution of Problem Set 8

Assigned: April 11, 2008 Due: April 18, 2008

Problem 1: Nise Problem 7-1 (p. 357 5th Ed.).

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Problem 2: Nise Problem 7-15 (p. 358 5th Ed.).

Collapsing the inner loop and multiplying it by $\frac{1000}{s}$ yields the following equivalent forward-path transfer function, corresponding to a *type 1* system:

$$G_e(s) = \frac{10^5(s+2)}{s(s^2+1005s+2000)}$$

$\boxed{\textbf{MATLAB Command – line}:}$

```
>> G1=feedback(tf(100*[1 2],[1 5 0]),10)

Transfer function:
    100 s + 200
-------------------
s^2 + 1005 s + 2000

>> G2=series(G1,tf(1000,[1 0]))

Transfer function:
   100000 s + 200000
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s^3 + 1005 s^2 + 2000 s
```

Problem 3: Nise Problem 7-59 (p. 365 5th Ed.).

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Problem 4: Nise Problem 7-60 (p. 365 5th Ed.).

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Problem 5: Nise Problem 6-33 (p. 314 5th Ed.)

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K(s+4)}{s^3+3s^2+(K+2)s+4K}$$

As a necessary condition for the stability, all the coefficients of the denominator should have the same sign which requires $(K+2) > 0, (4K) > 0 \Rightarrow K > 0$. Furthermore, we compute marginal value of K by comparing the denominator with standard format of $(s+a)(s^2+\omega^2) = s^3 + as^2 + \omega^2 s + a\omega^2$. By matching those two third order polynomials we realize that:

$$a = 3$$
$$a\omega^2 = 4K \Rightarrow \omega^2 = \tfrac{4K}{3}$$
$$\omega^2 = (K+2) \Rightarrow K = 6 \Rightarrow \omega^2 = 8 \Rightarrow \omega = 2\sqrt{2}\ \tfrac{rad}{\text{sec}}$$

Hence the range of $0 < K < 6$ keeps the system stable. The system has an undamped oscillation for $K = 6$, corresponding to $\omega = 2\sqrt{2}\ \frac{rad}{\text{sec}}$.

<u>Problem 6:</u> Nise Problem 6-26 (p. 314 5th Ed.).

$$G(s) = \frac{K(s-2)(s+2)}{s^2+3} = \frac{K(s^2-4)}{s^2+3}$$
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K(s^2-4)}{(K+1)s^2+(3-4K)}$$

As a necessary condition for the stability, all the coefficients of the denominator should have the same sign which requires $(K+1)(3-4K) > 0 \Rightarrow -1 < K > \frac{3}{4}$. However, for that range of K the system has only two purely oscillatory poles without any damping. Hence, the system is not truly stable, but only marginally stable.