2.004 Dynamics and Control II
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*

Spring Term 2008

Problem Set 2

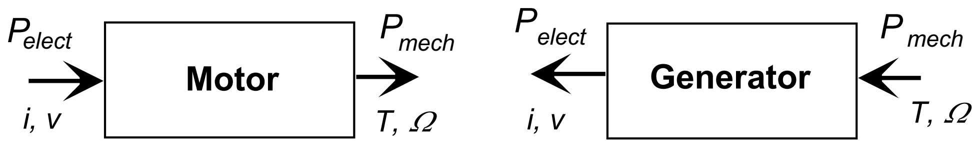
Assigned: Feb. 15, 2008 Due: Feb. 22, 2008

Reading:

- Nise, Secs. 2.1 - 2.4.
- Nise, Secs. 5.1 - 5.2.

Problem 1: Getting ready for Lab. 2 - The DC Motor.
Many servo control systems us a *permanent magnet DC motor* as the actuator. In our laboratory set-up we have a precision Maxon motor to drive the rotational plant. We will characterize this motor in Lab. 2.

A DC motor is an *energy transducer*, that is it transforms energy in one domain to another. The motor converts, electrical power into mechanical rotational power, as depicted schematically below. As you will discover in Lab. 2, the same device "driven backward" can also transform mechanical power into electrical power, that is it can act as an electrical generator by rotating the shaft.



(a) Assuming that in an ideal dc motor:

(1) The electrical current i_m and the torque produced T_m have a linear relationship, that is

$$T_m = K_m i_m$$

and the voltage v_b, known as the *back emf*, and the shaft angular velocity Ω are also linearly related

$$v_b = K_v \Omega_m$$

where K_m and K_e are the "motor torque constant" and "back emf constant" respectively.

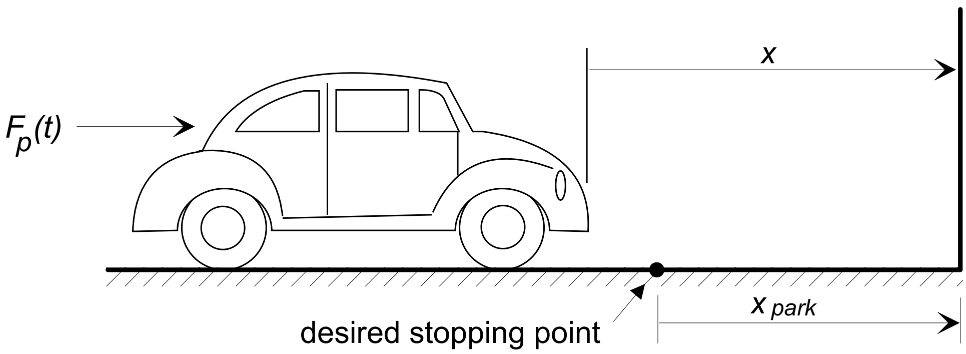
(2) and that the energy transduction is lossless, i.e. $\text{Power}_{in} = \text{Power}_{out}$,

show how K_m and K_e must be related. You will be measuring these quantities in Lab 2.

(b) The motor used in the lab is the Maxon model 148867. The data sheet is attached; the data for the 148867 is contained in the second column. Lines 14 and 15 specify the torque constant and back-emf constants. By converting to a consistent set of units, show that the energy transduction is lossless for this motor.

Note: In practice a motor is not lossless because it has copper electrical coils inside, with finite electrical resistance. If this resistance is R there will be a dissipation $P = i^2R$ in the form of heat, and the terminal voltage will not be v_e because of the ohmic voltage drop across the resistance R. Similarly if there is any friction in the bearings there will be mechanical power dissipation.
Despite this, the *process* of electro-mechanical energy transduction is lossless – as we will see in class in a few weeks. We separate out the ideal situation, and handle the dissipative elements separately.

Problem 2: In the first two lectures we used the example of a simple automobile cruise-control to demonstrate the concepts of closed-loop proportional control. Now imagine that we have the same automobile, but this time we are developing the new *AutoParker* car parking system that will stop a car at a fixed distance from the end wall in a garage. A mini radar system is used to monitor the distance from the wall as the car approaches, and to slow it to a stop at a distance x_{park}. As before, the control system controls the propulsive force $F_p(t)$. If the car is too close the engine will generate a reverse force and move the car back. A control on the dashboard allows the driver to select the distance x_{park}.



Using an approach similar to that we used in class (with the same basic mass/friction linear model):

(a) Derive a transfer function $G_c(s)$ for the open-loop car, relating the *distance travelled*, to the engine command ϕ as we described in class.

(b) Draw a block diagram of the feedback control system using the quantities depicted above. Take care to get the signs correct.

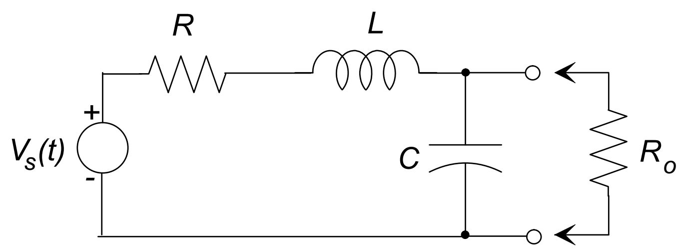
(c) Derive the closed-loop transfer function for the position controller with a proportional controller with gain K_p.

(d) Using whatever method you like determine whether there will be a steady-state error in the response $x(t)$ to a constant command x_{park}.

(e) If your answer to (d) is that there is no steady-state error (hint!), think about why there is a steady-state error for the same car under velocity control (as we discussed in class), but not for the position control. Write down your reasons, arguing from from a physical point of view, considering the error and the forces acting on the car.

Problem 3: Nise, Chapter 5, Problem 4.

Problem 4: Use impedance reduction (parallel/series combination) to find the transfer function of the ripple-filter used in an electronic power supply as shown below. Assume the filter is connected to a resistive load R_o as shown. (The output is the voltage across R_o.)



Problem 5: Does the input impedance uniquely characterize a system? Answer the question by determining the input impedance of the two electrical circuits below: