2.25 ADVANCED FLUID MECHANICS **Fall 2004**

QUIZ 1

THURSDAY, OCTOBER 14, 2004, 7:00-9:00 P.M.

OPEN QUIZ WHEN TOLD AT 7:00 PM

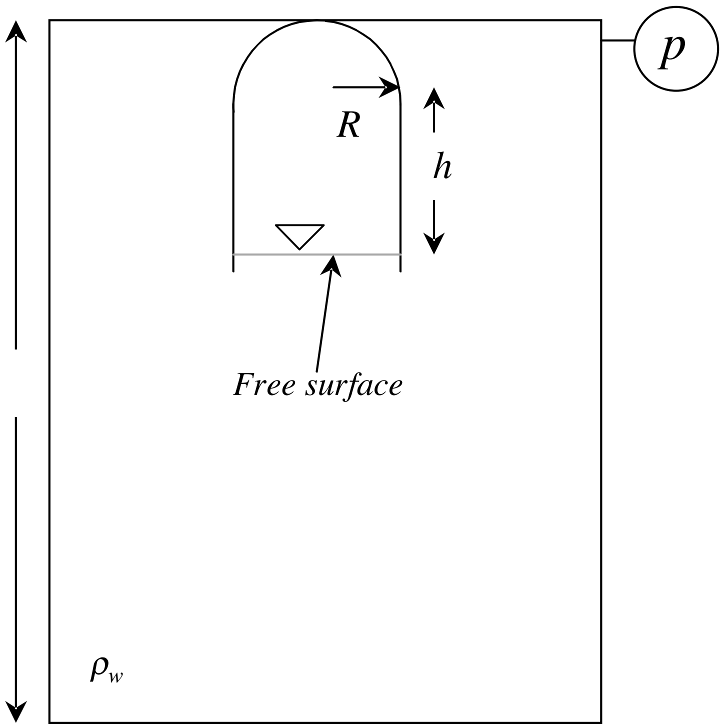
THERE ARE TWO PROBLEMS
OF EQUAL WEIGHT

Please answer both questions in the same book
Clearly showing where one question ends and another begins!

Question 1 Hydrostatic Tank Indicators

It is possible to use hydrostatic systems as tank level indicators and pressure valves (the most common example you may be familiar with is the "float valve" resting on the surface of a cistern in a flushing toilet). Here we consider two separate examples of hydrostatic indicators:

1A. A float level/pressure indicator



This device can be used as an independent indicator of pressure in a rigid tank.

It consists of an air-filled cylinder with a hemispherical top end and a free surface at the lower end. The radius of the cylinder is R and the total dry mass of the device assembly is m. The solid walls of the valve are rigid but very thin and you may neglect the volume of fluid displaced by them.

Typical initial values are R = 1 cm, h = 5 cm, m = 15 grams (including the air). The tank is undeformable, of depth L = 1m and is filled completely with water with no air space at the top. The initial absolute pressure measured by the pressure indicator at the top of the tank is atmospheric; $p_a = 1.0 \times 10^5$ N/m^2.

(a) The float indicator initially sits as shown against the top of the tank. Find the value of the reaction force exerted by the vessel on the float valve.

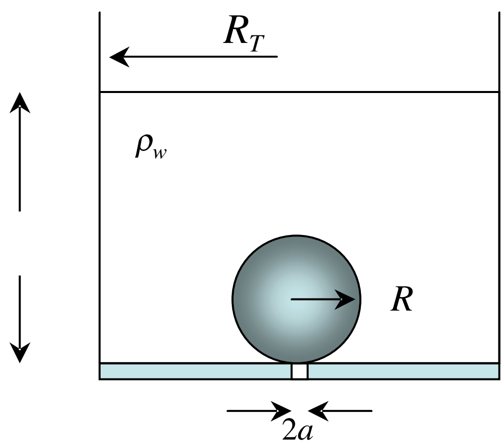
(b) The supply pressure in the tank now increases *very slowly*. At what pressure does the float valve become neutrally buoyant?

(c) The pressure increases suddenly from the initial state of part (a) to a value of $1.5p_a$. You may assume that this increase is rapid enough that it occurs adiabatically. Find an expression for the new buoyant force and also the new equilibrium position of the float valve. What is the height h_f of the deformable diaphragm when the float reaches its new equilibrium position.

⇒ Some of you may be familiar with the toy called a *Cartesian Diver*. This is exactly how it functions. It can be placed in a filled soda bottle for example and its position varied by someone squeezing the walls of the bottle.

1.B) A Minimum Tank Level Indicator

Now consider a rigid and nondeformable sphere of radius R made from a material of specific gravity X (i.e. such that $\rho_{sphere} = X\rho_{water}$). The sphere is completely submerged in a tank of water with depth L as shown in the figure below. The sphere is placed over a small hole of radius a in the bottom of the tank that is open to atmosphere.



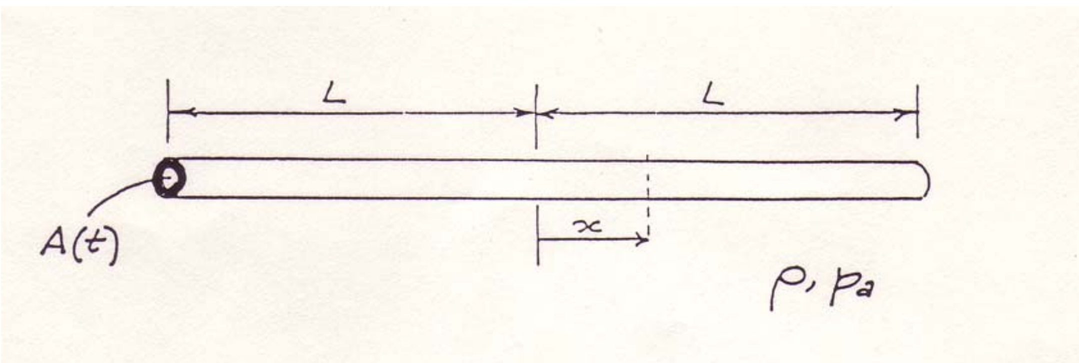
(i) By performing an appropriate force balance on the system, using a clearly drawn free-body diagram show the forces acting on a suitable control volume (*e.g.* you may want to consider pressure forces, buoyancy, weight etc..). Find a general inequality specifying for which range of parameters (X, R, a, L) that the sphere will remain sitting on the bottom of the tank plugging the hole.

(ii) If the tank is of depth L = 80 cm, the sphere is of radius R = 2 cm, and the hole is of radius a = 0.2 cm, determine the **minimum** specific density X_{min} for which the sphere will remain at the bottom of the tank. For this given density of sphere, explain briefly in words what will happen if the level of water in the tank drops below a height of 80 cm.

(iii) Assuming that the ball lifts off the bottom and subsequently stays completely out of the way of the hole when the level drops below the critical value of part (ii), find a general expression for the time taken for the tank (of finite radius R_T) to drain completely from an initial height L_0. You may assume viscous effects are negligible. If you make any additional simplifications, clearly show and justify them.

Problem 2

A small tube of length $2L$ is submerged in a pool of liquid (density ρ, pressure p_a). The tube is open at both ends, and filled with liquid. However, the tube is made of piezoelectric material, and its cross-sectional area A (which is uniform over the tube's length) can be controlled by the application of an electric voltage.



Suppose that, by the application of a suitable voltage, the tube's area is reduced in time according to a specified function $A(t)$, which is monotonically decreasing. Assuming that the flow inside the tube is incompressible and inviscid, obtain, in terms of the given quantities and the function $A(t)$ and its derivatives, expressions for

(a) the flow speed u at a station x in the tube, and

(b) the pressure at the tube's centerpoint, $x = 0$

(c) Are your results in (a) and (b) valid for increasing as well as decreasing $A(t)$?