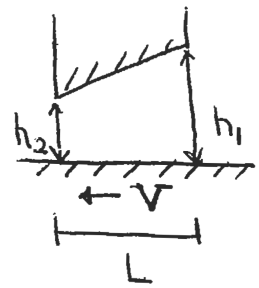
Example from Lecture (11/8/05):

The Slider Block: ρ, μ



$h_2 < h_1 \ll L$

$\left(\frac{h_2 - h_1}{L} \ll 1 \right)$

- Based on lubrication approximation $\left(Re_L \left(\frac{h}{L}\right)^2 \ll 1, \; \left(\frac{h}{L}\right)^2 \ll 1, \; \frac{h^2}{\nu \tau} \ll 1 \right)$, N-S equations reduce to:

$$\left.\begin{aligned} 0 &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2} \\ 0 &= -\frac{\partial p}{\partial y} - \rho g \end{aligned}\right) \xRightarrow{P = p - \rho g y} \boxed{\frac{dP}{dx} = \mu \frac{\partial^2 v_x}{\partial y^2}} \qquad \text{(ex-1)}$$

- Solve for v_x by integrating (ex-1) twice:

$$\boxed{\frac{v_x}{V} = \left(1 - \frac{y}{h(x)}\right) + \frac{h(x)^2}{2\mu V}\left(-\frac{dP}{dx}\right)\left[\frac{y}{h(x)}\left(1 - \frac{y}{h(x)}\right)\right]} \qquad \text{(ex-2)}$$

- Find $\frac{dP}{dx}$ by first considering Q = constant:

$$\frac{Q}{b} = \underbrace{Q'}_{\text{flux per unit depth}} = \int_0^{h(x)} v_x \, dy = \frac{V h(x)}{2} + \frac{h(x)^3}{12\mu}\left(-\frac{dP}{dx}\right)$$

By re-arranging,

$$\int_{P_0}^{P(L)} d = \cancel{P_{atm}} - P_{atm} = 6\mu V \int_0^L \frac{dx}{h^2(x)} - 12\mu Q' \int_0^L \frac{dx}{h^3(x)}$$

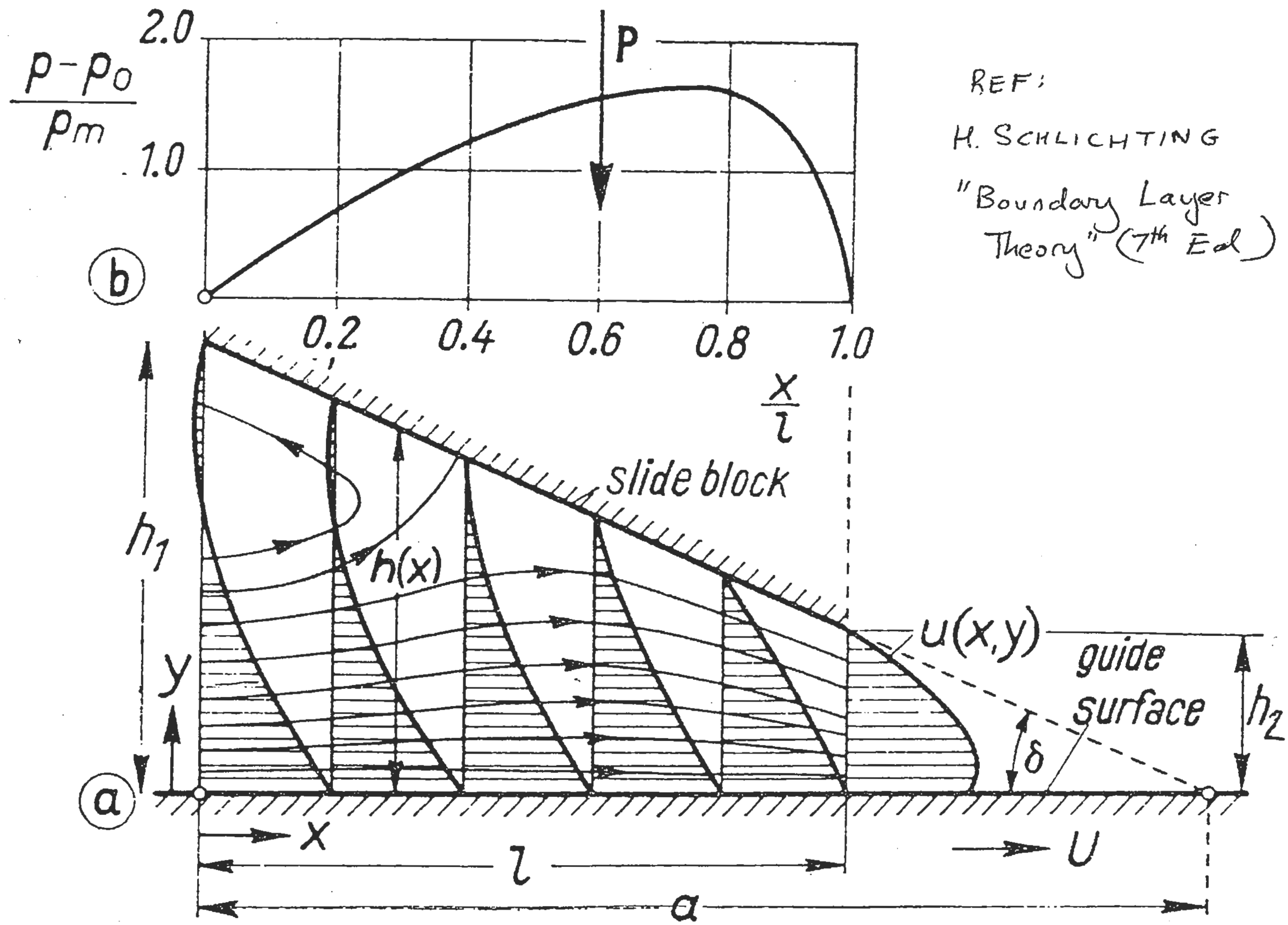
OR

$$Q' = \frac{1}{2} V H \quad \text{where } H = \frac{\int_0^L \frac{dx}{h^2}}{\int_0^L \frac{dx}{h^3}}$$

is a "mean" length.

- Substitute for H in (ex-2) and re-arrange to find $\frac{dP}{dx}$:

$$\boxed{\frac{dP}{dx} = \frac{6\mu V}{h(x)^2}\left[1 - \frac{H}{h(x)}\right]} \quad \text{where} \quad h_2 < H < h_1 \qquad \text{(ex-3)}$$



REF:
H. SCHLICHTING
"Boundary Layer Theory" (7th Ed)

- Evaluate F_y' (Thrust) and F_x' (Drag):

$$F_y' = \int_0^L (P - P_{atm})\,dx = \frac{\mu V}{h_2}\left(\frac{L^2}{h_2}\right)\frac{6}{(k-1)^2}\left[\ln k - \frac{2(k-1)}{2(k+1)}\right]$$

$$F_x' = \int_0^L \mu \frac{\partial V_x}{\partial y}\bigg|_{y=0} dx = \left(\frac{\mu V L}{h_2}\right)\frac{1}{k-1}\left[4\ln k - \frac{6(k-1)}{(k+1)}\right] \qquad \text{(ex-4)}$$

$$\text{where} \quad k = \frac{h_1}{h_2} > 1$$