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Navier Stokes Equation

$$\rho \frac{Dv}{Dt} = \rho\left\{\frac{\partial v}{\partial t} + v\cdot\nabla v\right\} = -\nabla p + \mu\nabla^2 v + \rho\mathbf{g}$$

Euler Equation; inviscid flow

$$\rho\left\{\frac{\partial v}{\partial t} + v\cdot\nabla v\right\} = -\nabla p + \rho\mathbf{g}$$

$$v\cdot\nabla v = \nabla\left(\tfrac{1}{2} v\cdot v\right) - v\times\left[\nabla\times v\right]$$

Barotropic flow; $\rho = f(p)$ only

scalar potential for conservative body force (e.g. $\Pi = \rho g z$)

$$\frac{\partial v}{\partial t} + \nabla\left\{\tfrac{1}{2} v\cdot v + \int\frac{dp}{\rho} + \Pi\right\} = v\times\boldsymbol{\omega}$$

steady flow $\frac{\partial v}{\partial t} = 0$

$\boldsymbol{\omega} = 0$

Steady Irrotational Inviscid Flow

$$B = \tfrac{1}{2} v\cdot v + \int\frac{dp}{\rho} + \Pi = \text{constant everywhere}$$

$\boldsymbol{\omega} \neq 0$

Steady Rotational Inviscid Flow

$$\nabla\left\{\tfrac{1}{2} v\cdot v + \int\frac{dp}{\rho} + \Pi\right\} = \nabla B = v\times\boldsymbol{\omega}$$

where $B = \tfrac{1}{2} v\cdot v + \int\frac{dp}{\rho} + \Pi = \text{constant along a Lamb surface}$

Integrate along a streamline

Unsteady Bernoulli equation *along a streamline*

$$\int_1^2 \frac{\partial v}{\partial t}\cdot d\mathbf{r} + \left(\tfrac{1}{2}v_2^2 - \tfrac{1}{2}v_1^2\right) + \int_1^2\frac{dp}{\rho} + \left(\Pi_2 - \Pi_1\right) = 0$$

Integrate along a streamline & assume Incompressible fluid

Incompressible fluid

steady flow

Steady Bernoulli Equation along a streamline for an inviscid flow of an incompressible fluid

$$\left(\tfrac{1}{2}v_2^2 - \tfrac{1}{2}v_1^2\right) + \frac{1}{\rho}\left(p_2 - p_1\right) + g\left(z_2 - z_1\right) = 0$$