Prof. M. A. Dahleh   Out 2/23/1994

Problem Set No. 2   Due 3/07/1994

Reading: Chapter 4.

Problem 1:

Do the following problems from Ljung's book:

  a. 6G.3

  b. 6G.4

  c. 6E.1

  d. 6E.2

  e. 6E.3 Also, compute the minimal variance. What can you say about estimates (6.69) and (6.70), p. 158, of the textbook?

  f. 6T.2 Apply the result of this exercise to lemma 6.1. How restrictive is the technical assumption on the correlation function?

Problem 2:

Consider the following system: $y = Gu + v$, where G is an unknown plant to be identified from the data record u, y. A practical situation that often arises is when both the input process u and the output noise process v are corrupted by a periodic signal, for instance a 60 Hz hum.

After taking 6.435, you have decided to use a correlation method to estimate the plant G. Analyse the performance of your estimate and comment on your decision.

In order to simplify the analysis, you can assume that both u and v are periodic sequences with exactly one common frequency.

Problem 3:

The objective of this problem is to enhance your understanding of the properties of the ETFE estimate of the transfer function and the role of smoothing in improving the ETFE estimate.

Given the following process:

$$y = Gu + He \quad G = \frac{q^{-1}}{1 - .6q^{-1}} \quad H = \frac{1 + .5q^{-1}}{1 - .2q^{-1}}.$$

Let the input be given by the periodic signal

$$u = \sum_{k=1}^{4} \cos\left(\frac{2\pi}{4}k\right)$$

and let e be white gaussian noise with mean 0 and variance 1.

a. Generate M data records of lengths $N_1 < N_2 < ... < N_M$ for this system. The number and lengths of these records are left to your judgement, stamina, and patience.

b. For each record, use ETFE to obtain an estimate of G. Compute an estimate of the variance at one frequency of your choice, and an estimate of the correlation of any two frequencies. Show a plot of these quantities as a function of $N_k, 1 \leq k \leq M$. Do the results confirm with the theoretical properties of ETFE?

c. Use SPA with a window of length 20 to obtain a spectral estimate of G. For one frequency of your choice, repeat the questions in part b. Comment on the observed differences between the plots in b and c. Also Comment on the tradeoff between the window length and the length of the data record.

Problem 4:

The objective of this problem is to study nonparametric identification in the presence of output feedback.

Consider the closed loop system shown below. The signal r is the reference input assumed to be a WGN of mean 0 and variance 1. The signal v is the process noise, with a power spectrum given by $\Phi_v = |H(e^{iw})|^2$ where H is given as in problem 2. The signals r and v are uncorrelated. The plant G and the controller K are fixed to be:

$$G = \frac{q^{-1}}{1 - .5q^{-1}} \quad K = .5$$

Assume that you can measure both u and y as shown in the diagram.

a. Generate a data record of length $N = 1000$ for the above system.

b. Use the correlation method directly on y, u to estimate the plant G. Hint: computationally, use (6.51). Estimate the noise spectrum.

c. Explain your results theoretically. Hint: derive the equations satsified by the different correlation functions. You are not asked to perform any asymptotic analysis.

d. Come up with a scheme that gets rid of the bias in the above results (Hint: Use r directly)