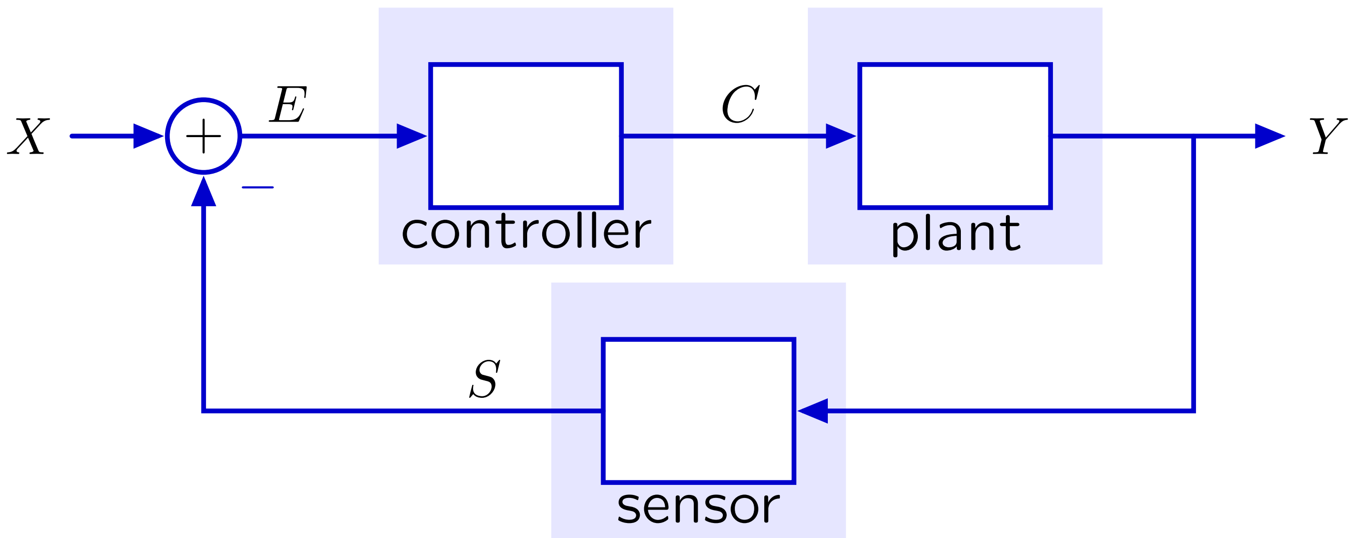
6.003: Signals and Systems

CT Feedback and Control

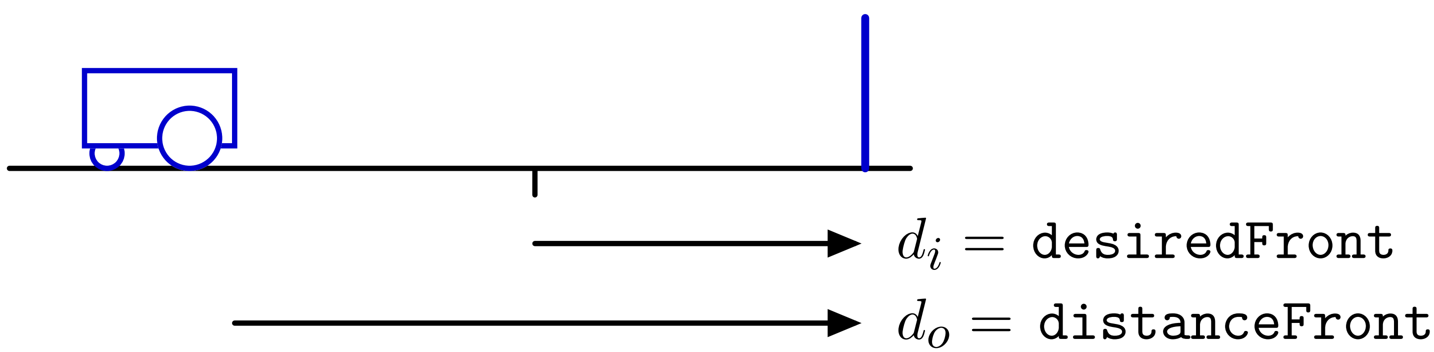
March 16, 2010

Feedback and Control

Feedback: simple, elegant, and robust framework for control.



Last time: robotic driving.



Feedback and Control

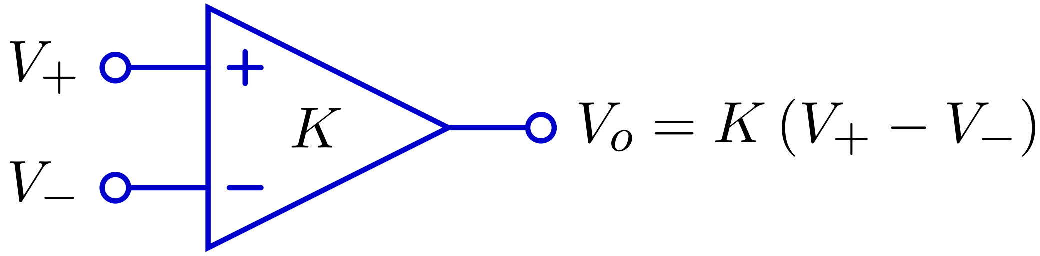
This week: using feedback to enhance performance.

Examples:

- increasing speed and bandwidth
- controlling position instead of speed
- reducing sensitivity to parameter variation
- reducing distortion
- stabilizing unstable systems
  - magnetic levitation
  - inverted pendulum

Op-amps

An "ideal" op-amp has many desireable characteristics.

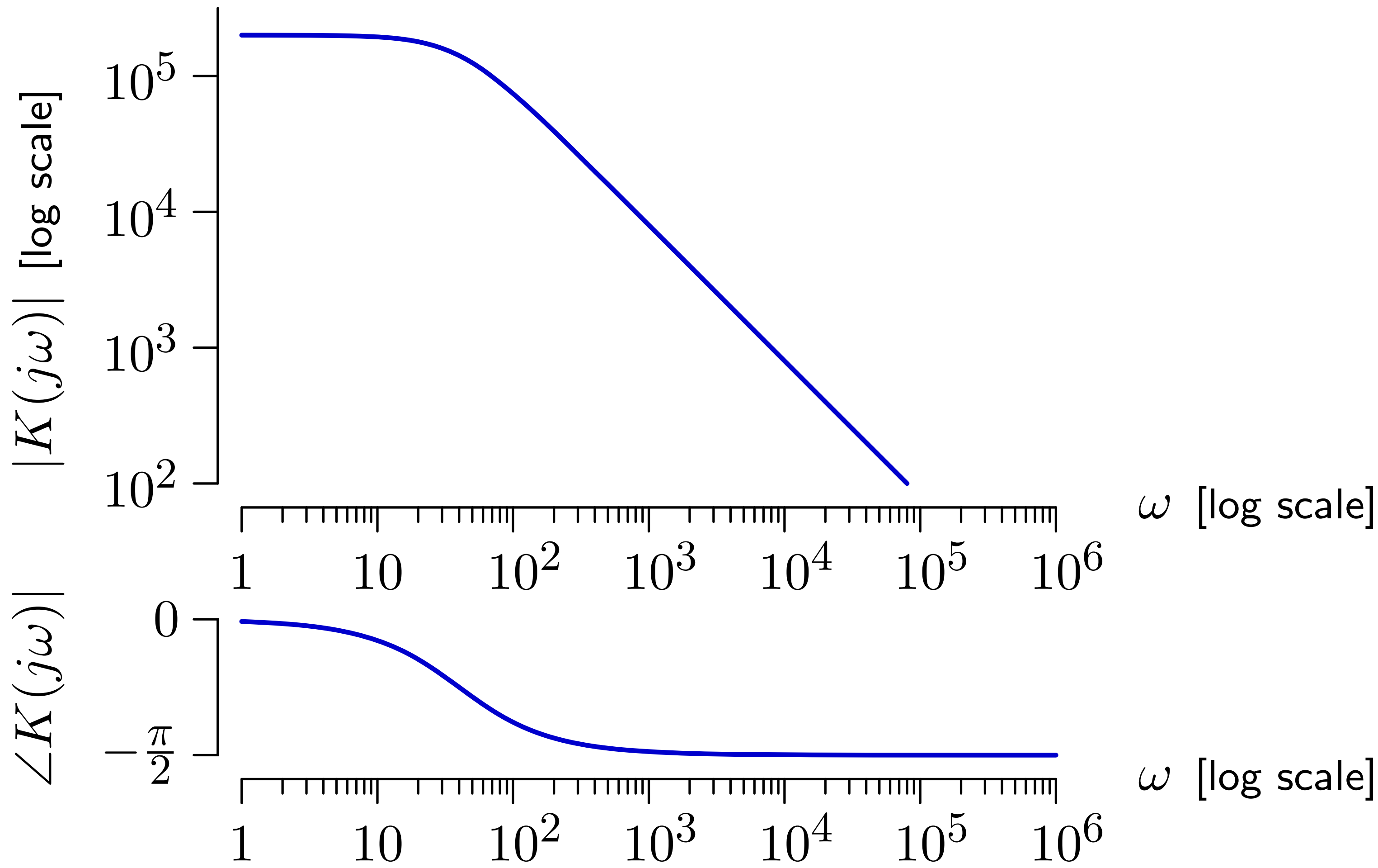


- high speed
- large bandwidth
- high input impedance
- low output impedance
- ...

It is difficult to build a circuit with all of these features.

Op-Amp

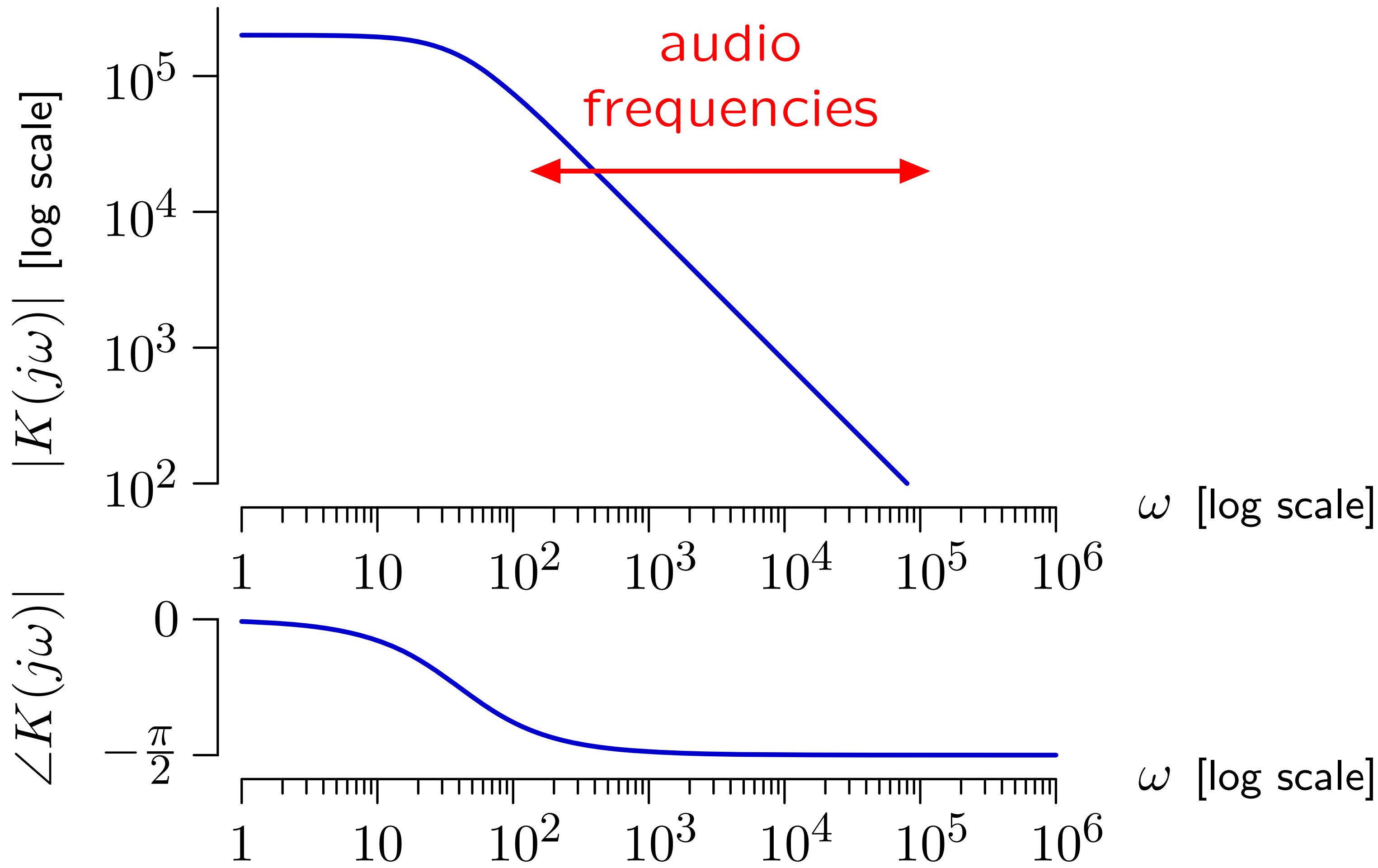
The gain of an op-amp depends on frequency.



Frequency dependence of LM741 op-amp.

Op-Amp

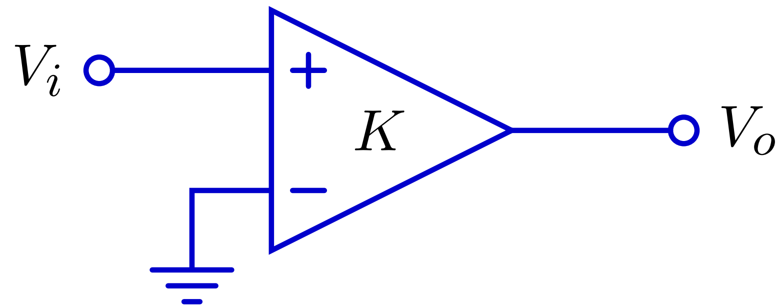
Low-gain at high frequencies limits applications.



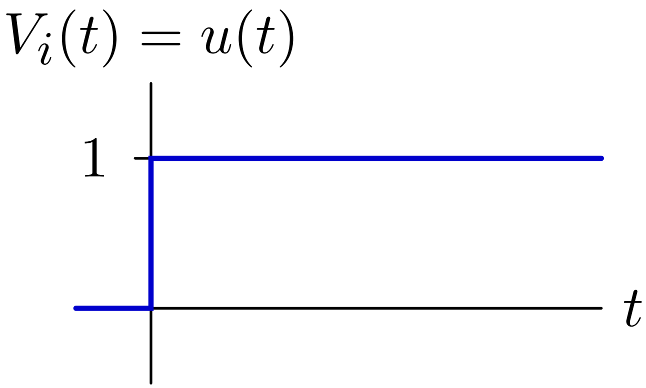
Unacceptable frequency response for an audio amplifier.

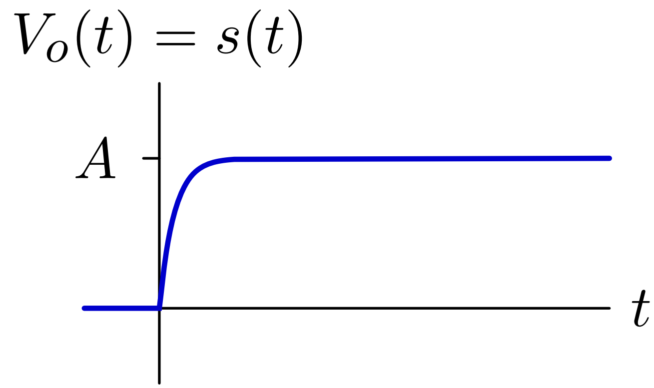
Op-Amp

An ideal op-amp has fast time response.



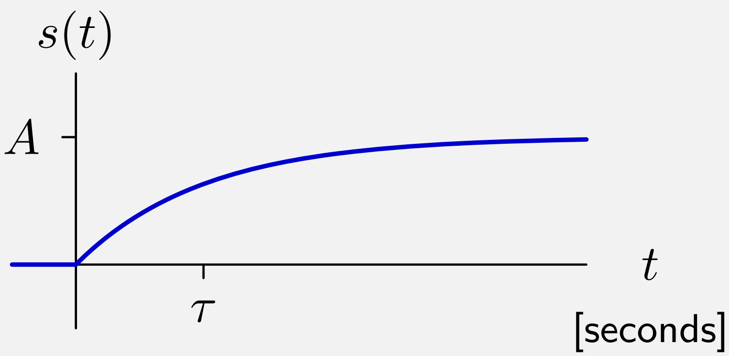
Step response:

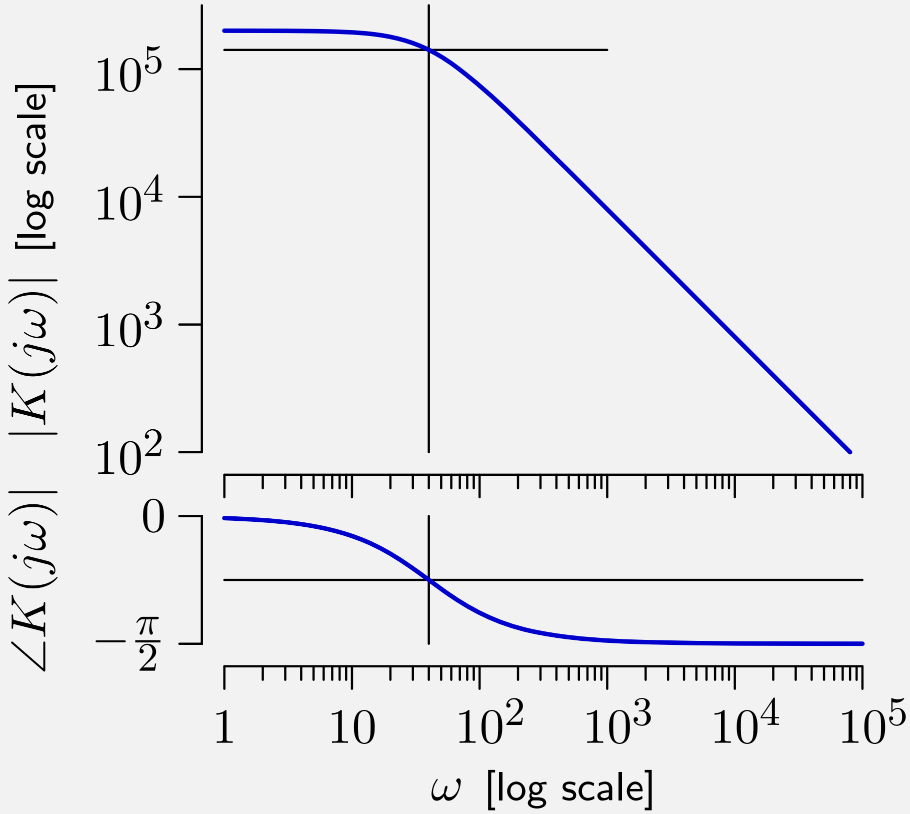




Check Yourself

Determine τ for the unit-step response $s(t)$ of an LM741.





1. 40 s
2. $\frac{40}{2\pi}$ s
3. $\frac{1}{40}$ s
4. $\frac{2\pi}{40}$ s
5. $\frac{1}{2\pi\times 40}$ s

0. none of the above

Check Yourself

Determine the step response of an LM741.

System function:

$$K(s) = \frac{\alpha K_0}{s + \alpha}$$

Impulse response:

$$h(t) = \alpha K_0 e^{-\alpha t} u(t)$$

Step response:

$$s(t) = \int_{-\infty}^{t} h(\tau) d\tau = \int_{0}^{t} \alpha K_0 e^{-\alpha\tau} d\tau = \left. \frac{\alpha K_0 e^{-\alpha\tau}}{-\alpha} \right|_0^t = K_0(1 - e^{-\alpha t}) u(t)$$

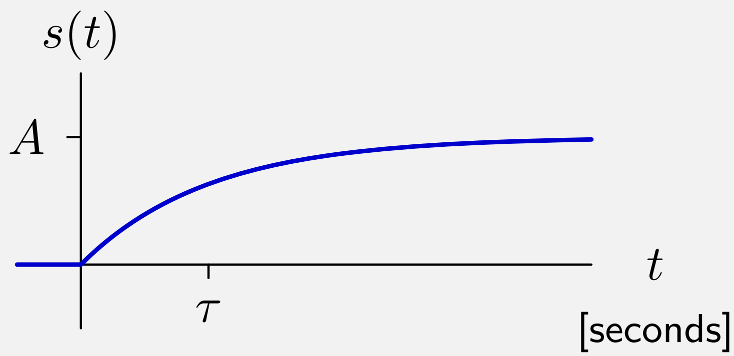
Parameters:

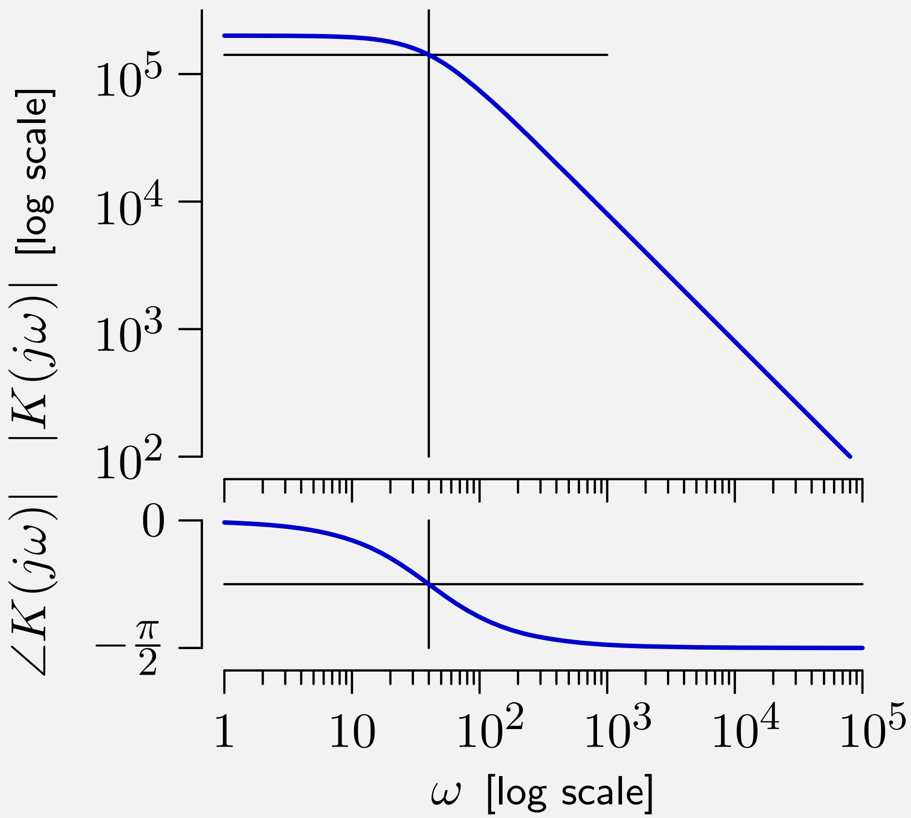
$$A = K_0 = 2 \times 10^5$$

$$\tau = \frac{1}{\alpha} = \frac{1}{40}\ \text{s}$$

Check Yourself

Determine τ for the unit-step response $s(t)$ of an LM741.





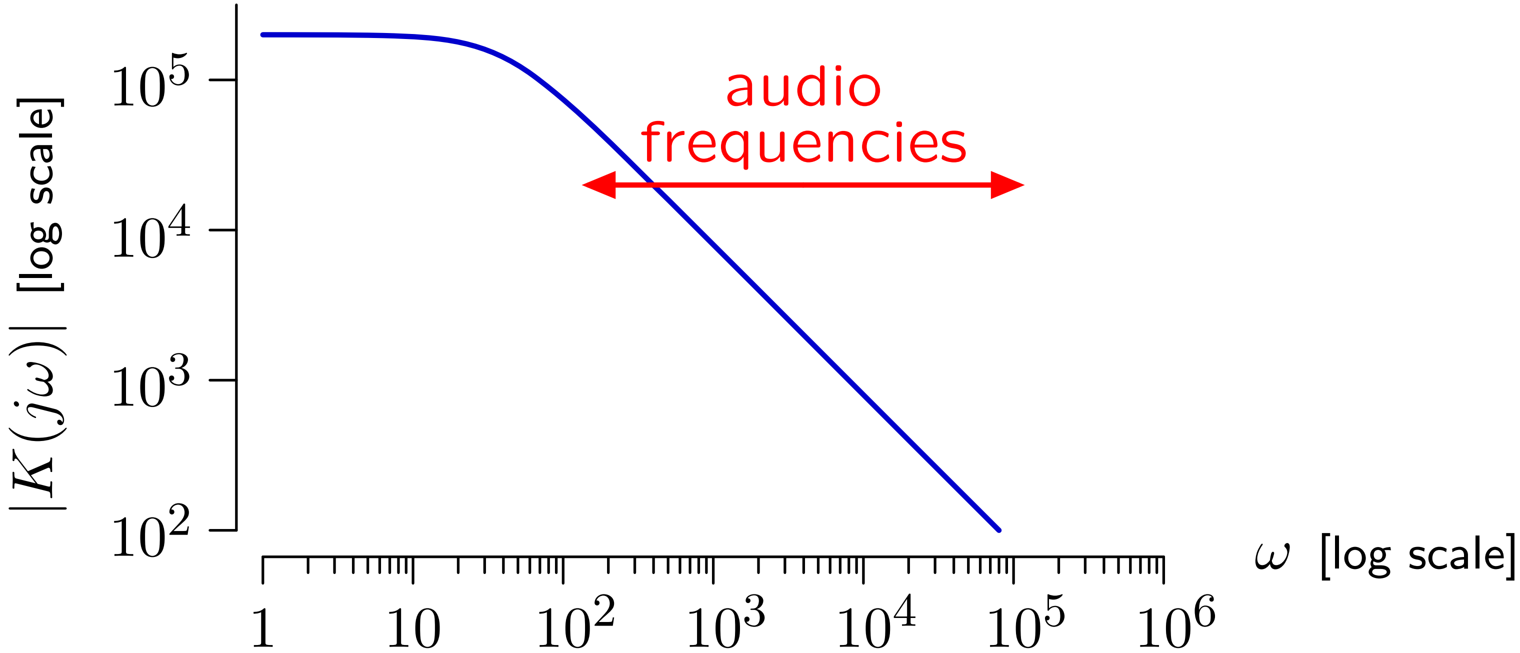
1. 40 s
2. $\frac{40}{2\pi}$ s
3. $\frac{1}{40}$ s
4. $\frac{2\pi}{40}$ s
5. $\frac{1}{2\pi \times 40}$ s

0. none of the above

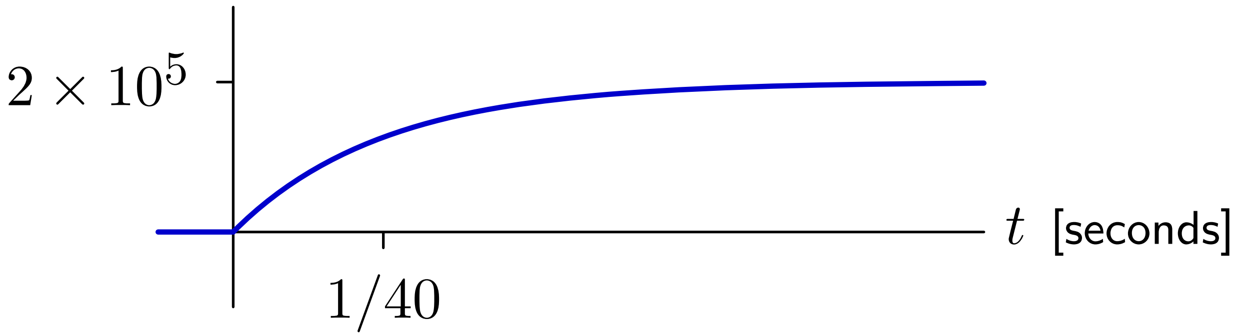
Op-Amp

Performance parameters for real op-amps fall short of the ideal.

Frequency Response: high gain but only at low frequencies.



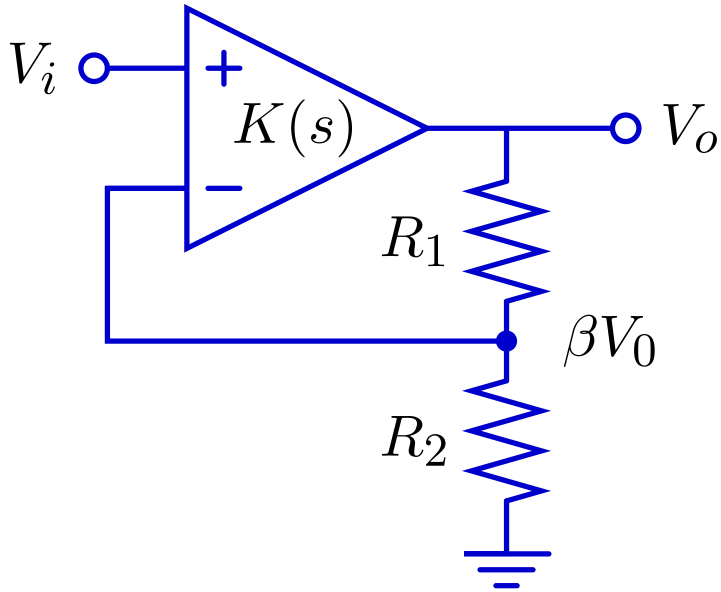
Step Response: slow by electronic standards.



Op-Amp

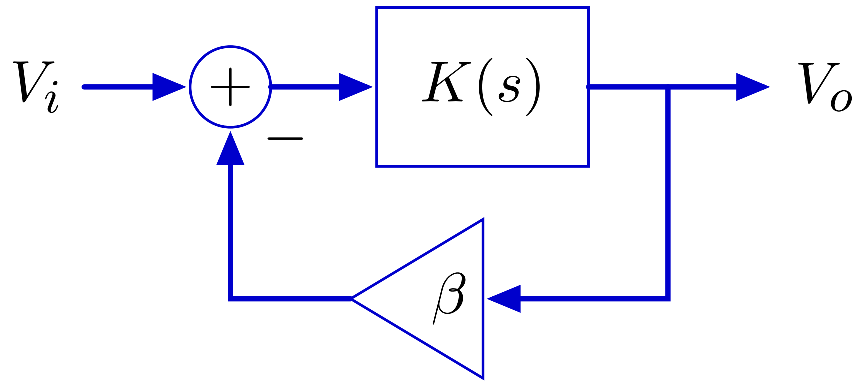
We can use feedback to improve performance of op-amps.

circuit



$$V_- = \beta V_o = \left(\frac{R_2}{R_1 + R_2}\right) V_o$$

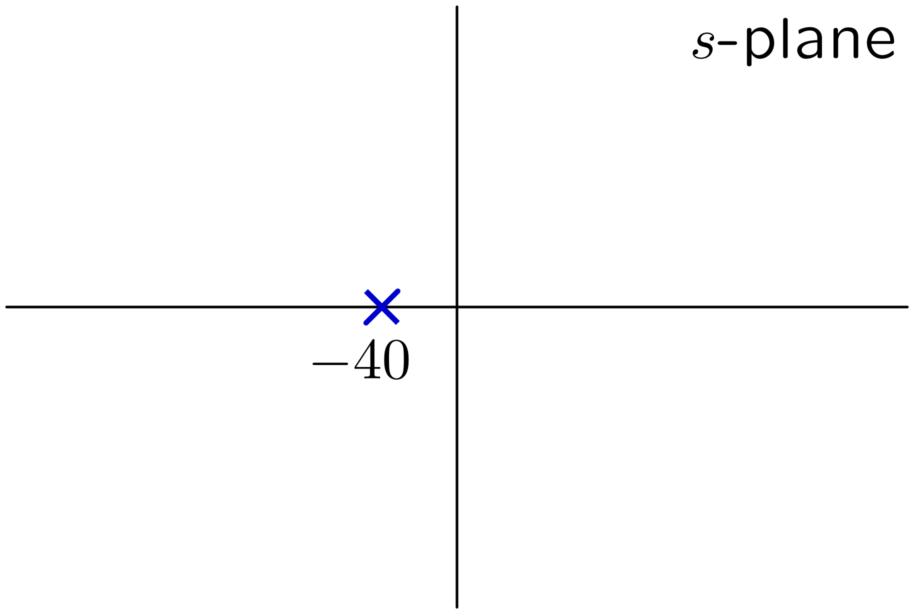
6.003 model



$$\frac{V_o}{V_i} = \frac{K(s)}{1 + \beta K(s)}$$

Dominant Pole

Op-amps are designed to have a dominant pole at low frequencies:
$\longrightarrow$ simplifies the application of feedback.

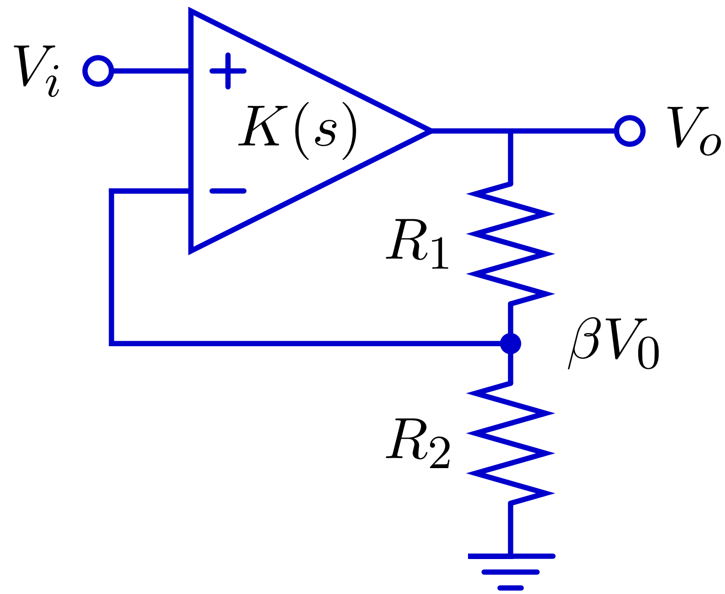


$$\alpha = 40\,\text{rad/s} = \frac{40\,\text{rad/s}}{2\pi\,\text{rad/cycle}} \approx 6.4\,\text{Hz}$$

Improving Performance

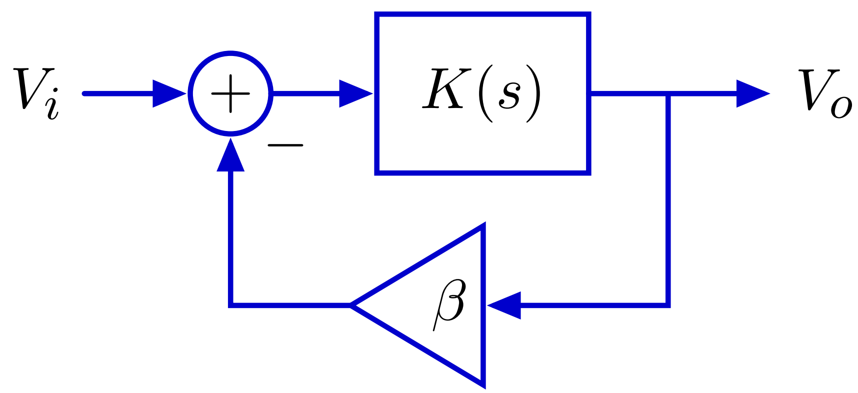
Using feedback to improve performance parameters.

circuit



$$V_- = \beta V_o = \left(\frac{R_2}{R_1 + R_2}\right) V_o$$

6.003 model



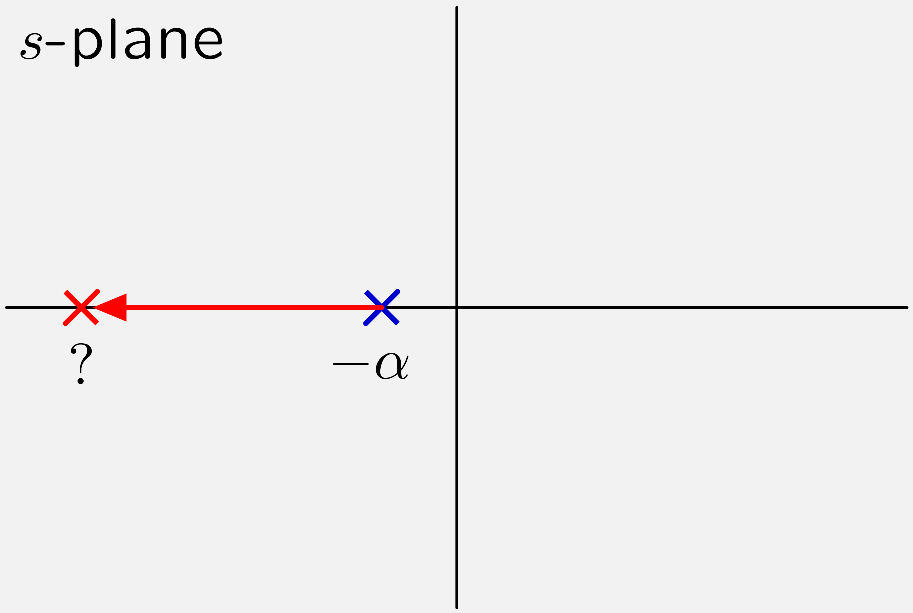
$$\frac{V_o}{V_i} = \frac{K(s)}{1 + \beta K(s)}$$

$$= \frac{\frac{\alpha K_0}{s+\alpha}}{1 + \beta \frac{\alpha K_0}{s+\alpha}}$$

$$= \frac{\alpha K_0}{s + \alpha + \alpha \beta K_0}$$

Check Yourself

What is the most negative value of the closed-loop pole that can be achieved with feedback?

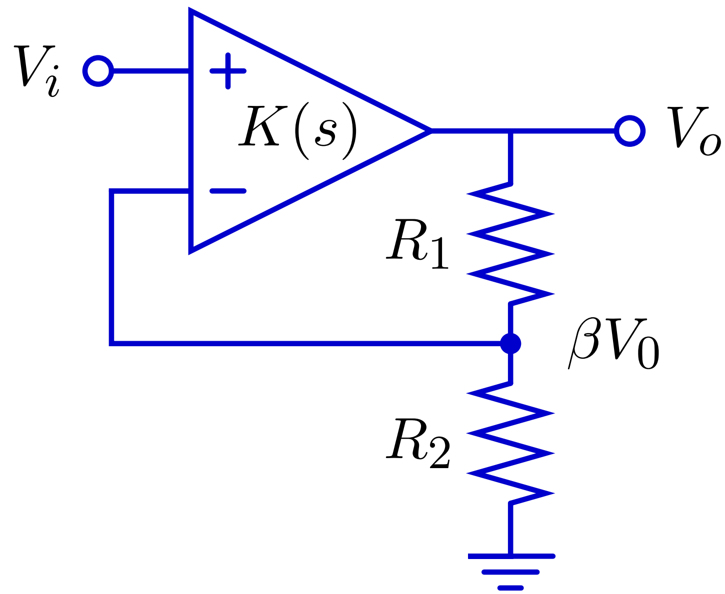


1. $-\alpha(1+\beta)$
2. $-\alpha(1+\beta K_0)$
3. $-\alpha(1+K_0)$
4. $-\infty$
5. none of the above

Improving Performance

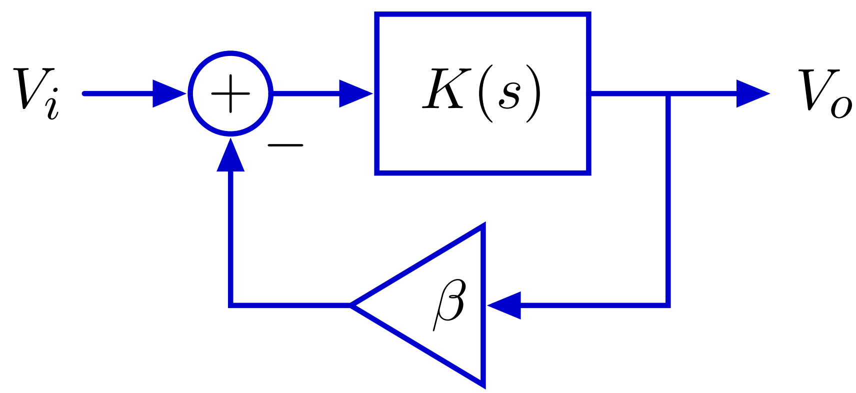
Using feedback to improve performance parameters.

circuit



$$V_- = \beta V_o = \left(\frac{R_2}{R_1+R_2}\right) V_o$$

6.003 model



$$\frac{V_o}{V_i} = \frac{K(s)}{1+\beta K(s)}$$

$$= \frac{\frac{\alpha K_0}{s+\alpha}}{1+\beta\frac{\alpha K_0}{s+\alpha}}$$

$$= \frac{\alpha K_0}{s+\alpha+\alpha\beta K_0}$$

Check Yourself

What is the most negative value of the closed-loop pole that can be achieved with feedback?

Open loop system function: $\dfrac{\alpha K_0}{s+\alpha}$

$\rightarrow$ pole: $s=-\alpha$.

Closed-loop system function: $\dfrac{\alpha K_0}{s+\alpha+\alpha\beta K_0}$

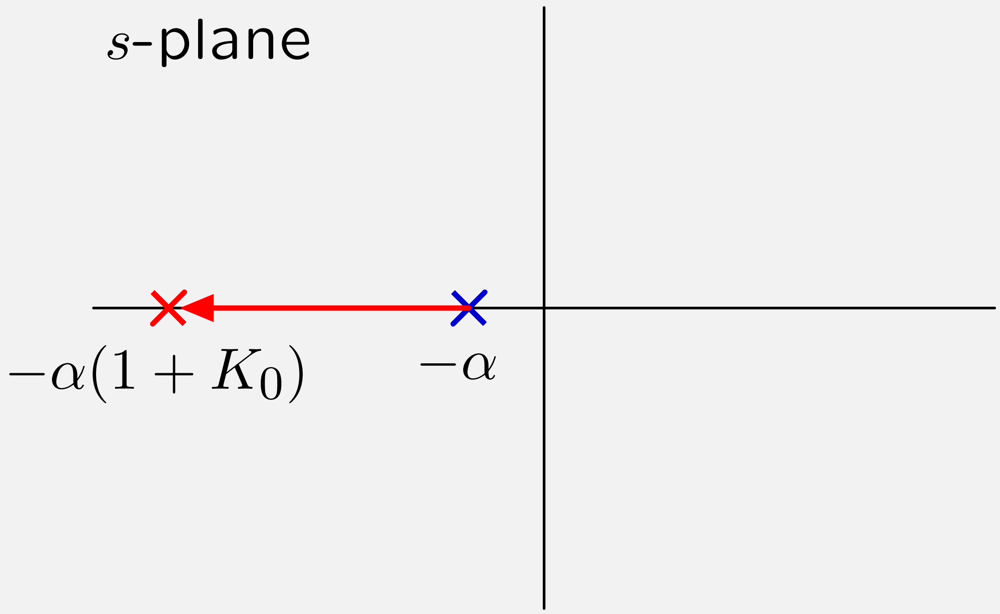
$\rightarrow$ pole: $s=-\alpha(1+\beta K_0)$.

The feedback constant is $0 \le \beta \le 1$.

$\rightarrow$ most negative value of the closed-loop pole is $s=-\alpha(1+K_0)$.

Check Yourself

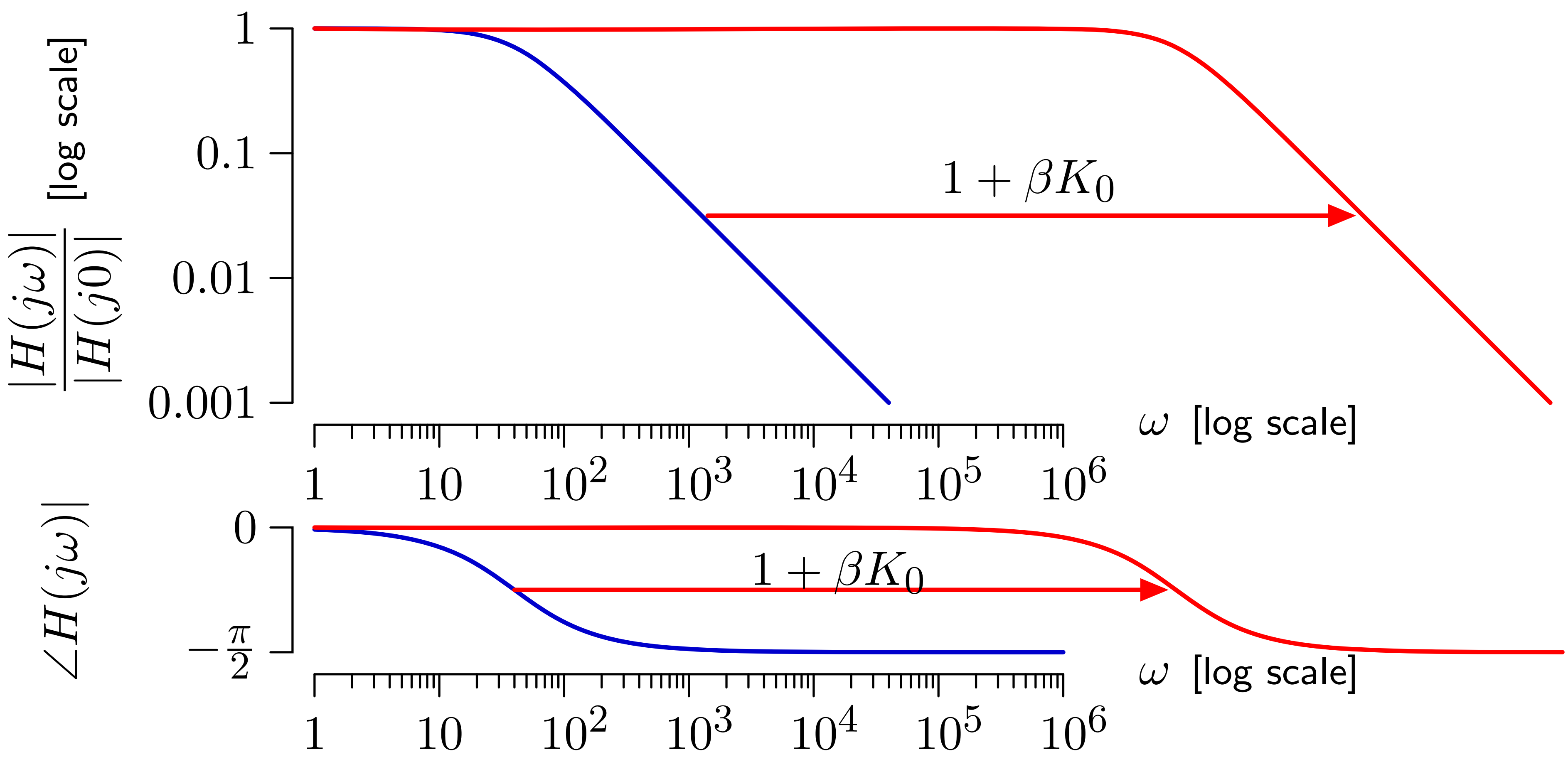
What is the most negative value of the closed-loop pole that can be achieved with feedback? 3



1. $-\alpha(1+\beta)$
2. $-\alpha(1+\beta K_0)$
3. $-\alpha(1+K_0)$
4. $-\infty$
5. none of the above

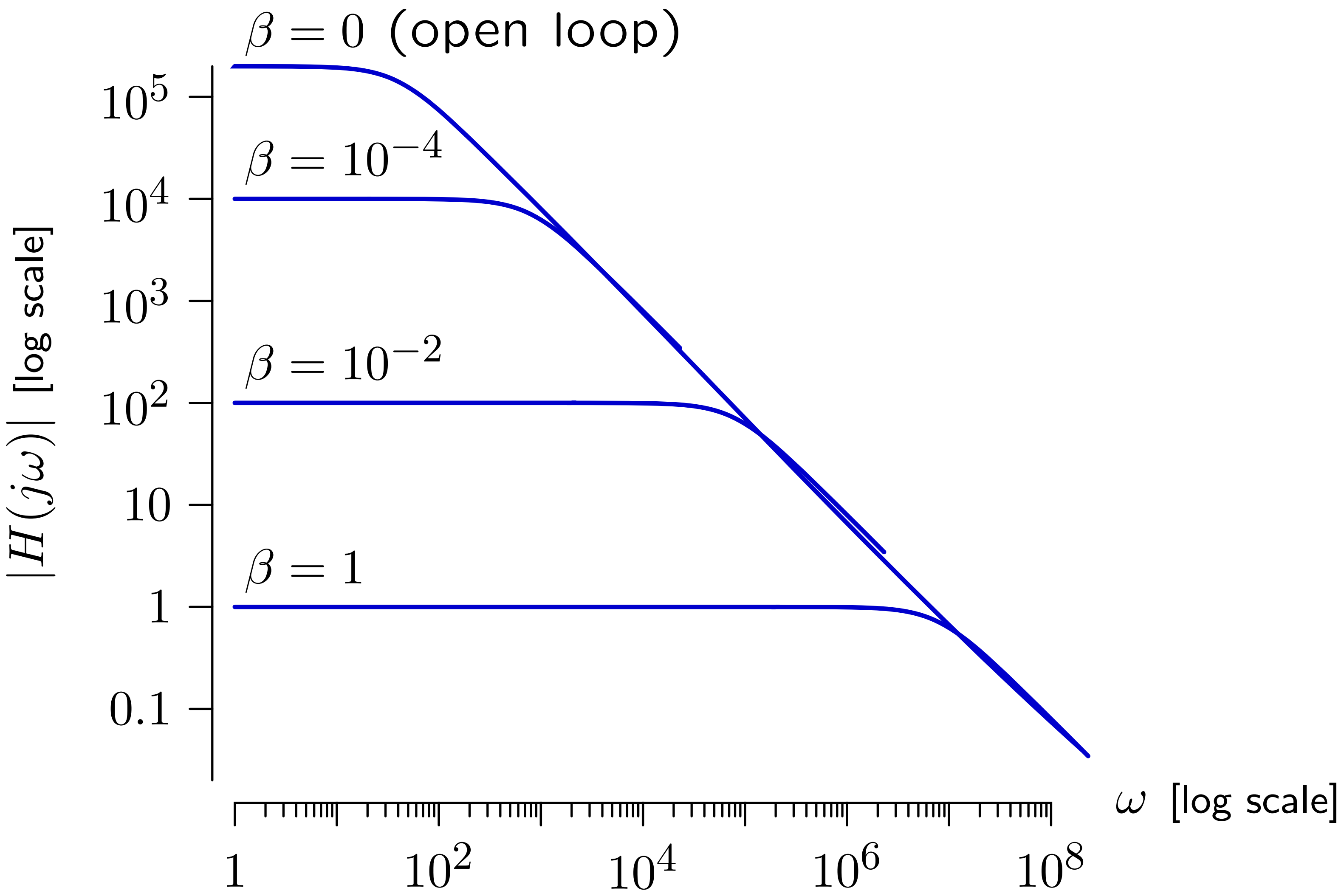
Improving Performance

Feedback extends frequency response by a factor of $1+\beta K_0$ ($K_0 = 2\times 10^5$).



Improving Performance

Feedback produces higher bandwidths by **reducing** the gain at low frequencies. It trades gain for bandwidth.

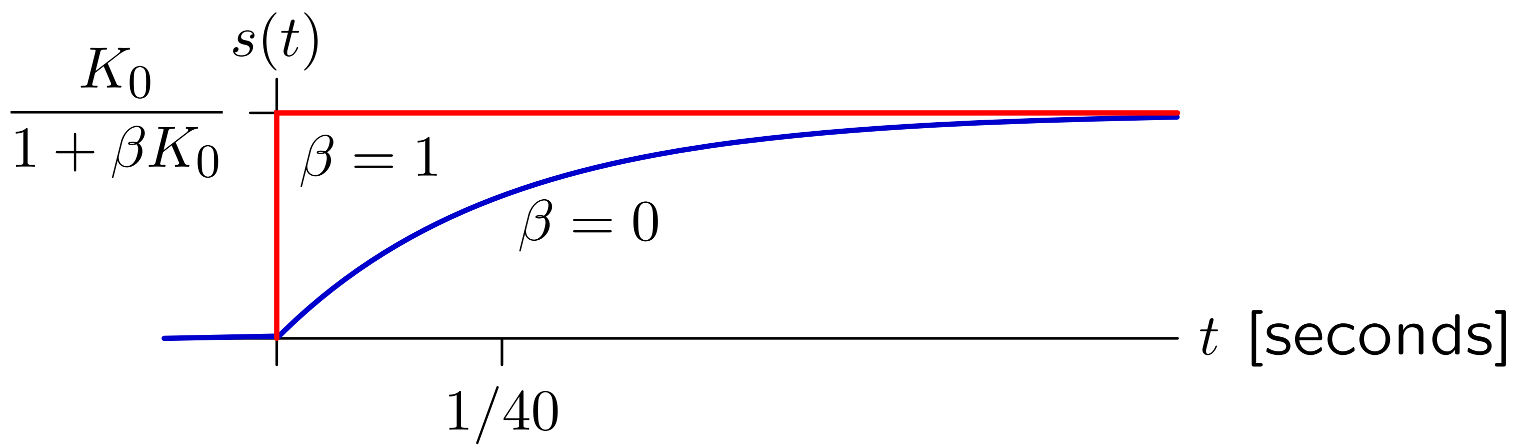


Improving Performance

Feedback makes the time response faster by a factor of $1 + \beta K_0$ ($K_0 = 2 \times 10^5$).

Step response

$$s(t) = \frac{K_0}{1 + \beta K_0}(1 - e^{-\alpha(1+\beta K_0)t})u(t)$$

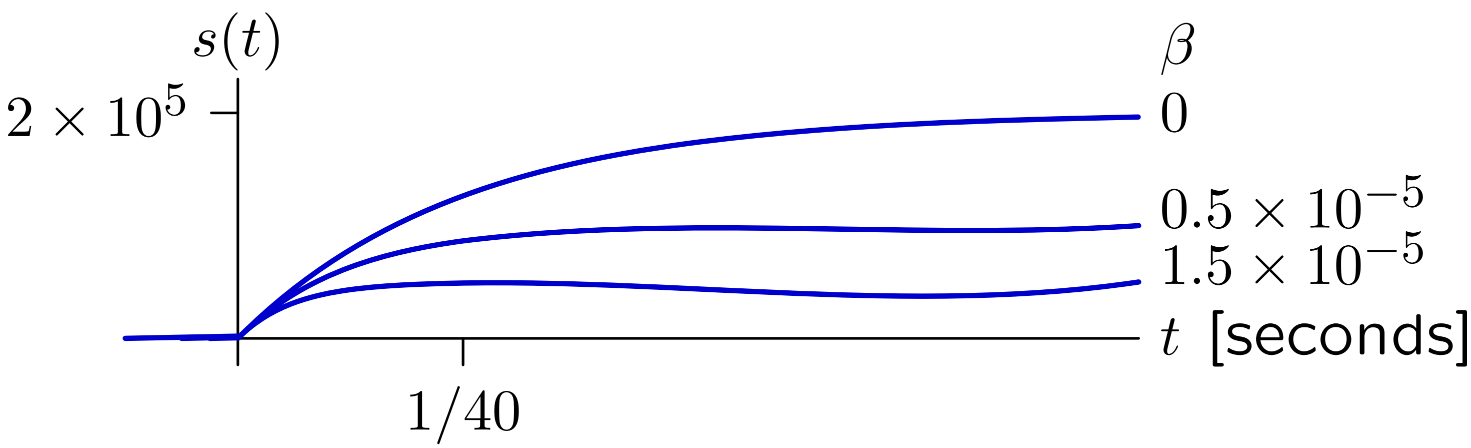


Improving Performance

Feedback produces faster responses by **reducing** the final value of the step response. It trades gain for speed.

Step response

$$s(t) = \frac{K_0}{1+\beta K_0}(1 - e^{-\alpha(1+\beta K_0)t})u(t)$$



The maximum rate of voltage change $\left.\frac{ds(t)}{dt}\right|_{t=0+}$ is not increased.

Improving Performance

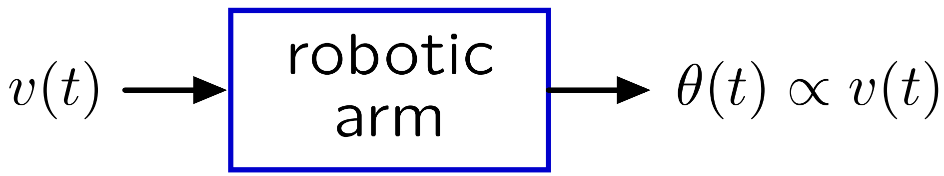
Feedback improves performance parameters of op-amp circuits.

- can extend frequency response
- can increase speed

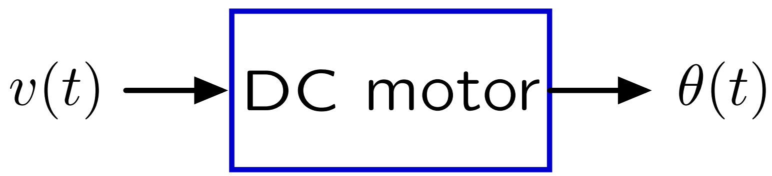
Performance enhancements are achieved through a reduction of gain.

Motor Controller

We wish to build a robot arm (actually its elbow). The input should be voltage $v(t)$, and the output should be the elbow angle $\theta(t)$.



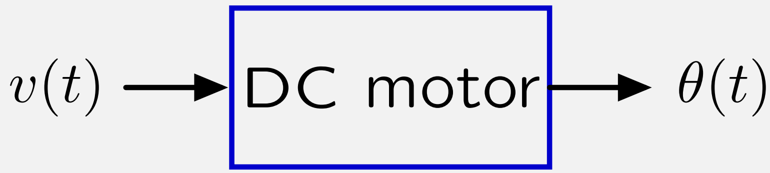
We wish to build the robot arm with a DC motor.



This problem is similar to the head-turning servo in 6.01 !

Check Yourself

What is the relation between $v(t)$ and $\theta(t)$ for a DC motor?

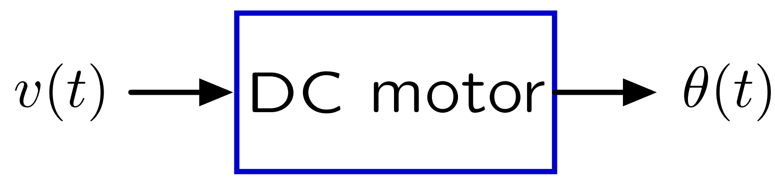


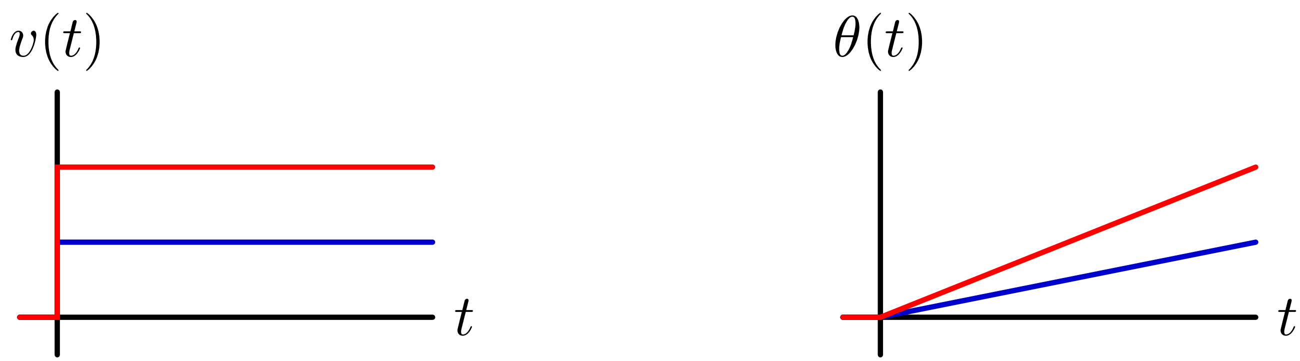
1. $\theta(t) \propto v(t)$
2. $\cos\theta(t) \propto v(t)$
3. $\theta(t) \propto \dot{v}(t)$
4. $\cos\theta(t) \propto \dot{v}(t)$
5. none of the above

Check Yourself

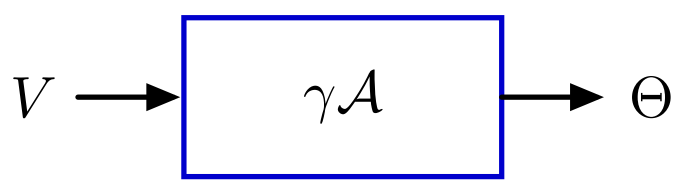
What is the relation between $v(t)$ and $\theta(t)$ for a DC motor?

To first order, the rotational speed $\dot{\theta}(t)$ of a DC motor is proportional to the input voltage $v(t)$.



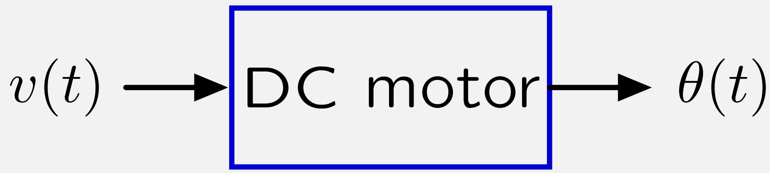


First-order model: integrator



Check Yourself

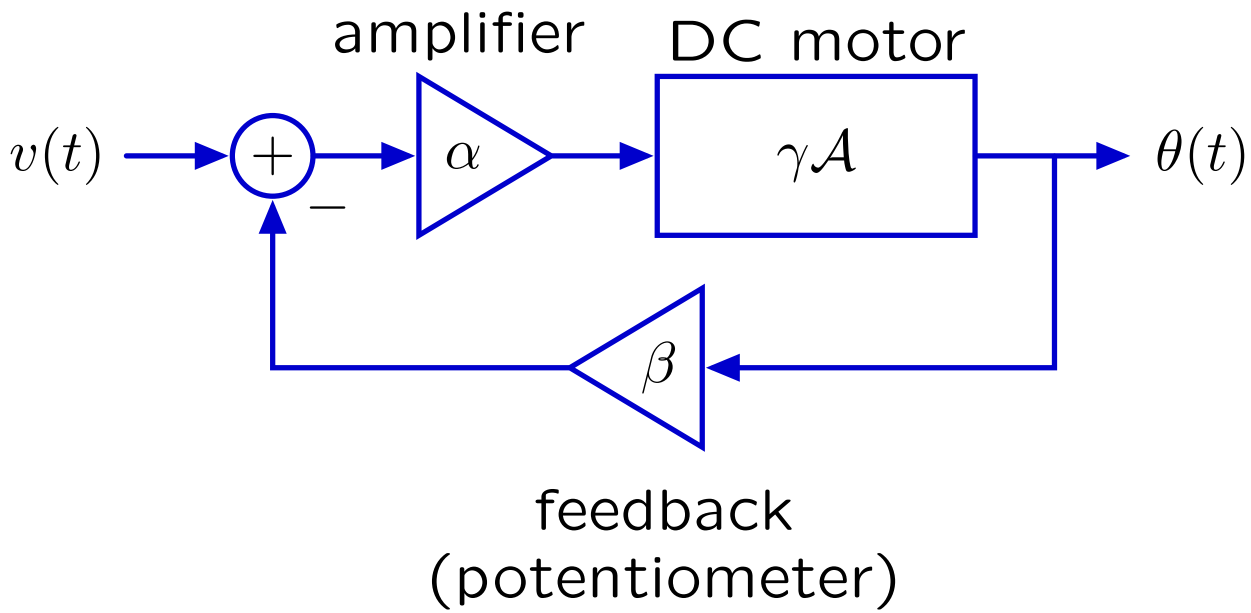
What is the relation between $v(t)$ and $\theta(t)$ for a DC motor?



1. $\theta(t) \propto v(t)$
2. $\cos\theta(t) \propto v(t)$
3. $\theta(t) \propto \dot{v}(t)$
4. $\cos\theta(t) \propto \dot{v}(t)$
5. none of the above

Motor Controller

Use proportional feedback to control the angle of the motor's shaft.

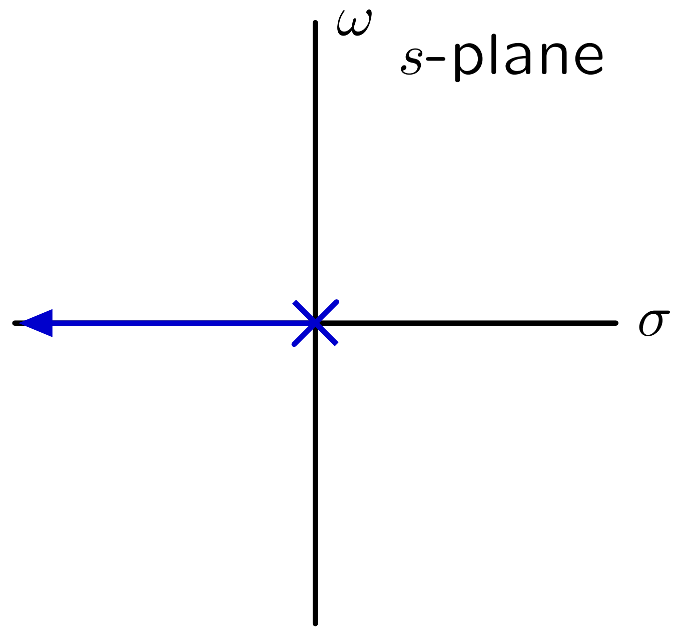


$$\frac{\Theta}{V} = \frac{\alpha\gamma\mathcal{A}}{1+\alpha\beta\gamma\mathcal{A}} = \frac{\alpha\gamma\frac{1}{s}}{1+\alpha\beta\gamma\frac{1}{s}} = \frac{\alpha\gamma}{s+\alpha\beta\gamma}$$

Motor Controller

The closed loop system has a single pole at $s = -\alpha\beta\gamma$.

$$\frac{\Theta}{V} = \frac{\alpha\gamma}{s + \alpha\beta\gamma}$$



As α increases, the closed-loop pole becomes increasingly negative.

Motor Controller

Find the impulse and step response.

The system function is

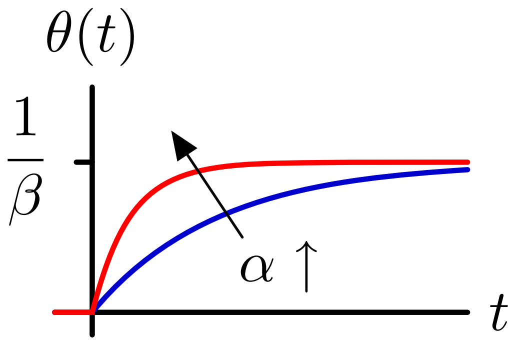
$$\frac{\Theta}{V} = \frac{\alpha\gamma}{s + \alpha\beta\gamma}\,.$$

The impulse response is

$$h(t) = \alpha\gamma e^{-\alpha\beta\gamma t} u(t)$$

and the step response is therefore

$$s(t) = \frac{1}{\beta}\left(1 - e^{-\alpha\beta\gamma t}\right) u(t)\,.$$



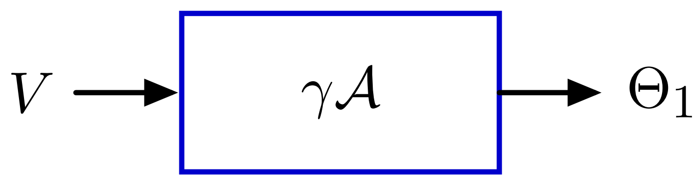
The response is faster for larger values of α.

Try it: Demo.

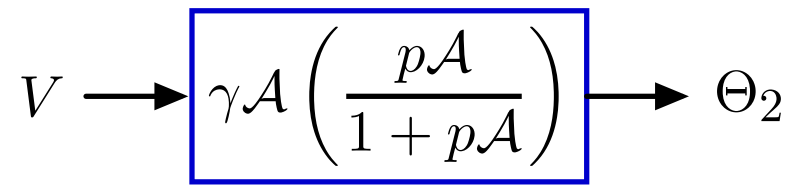
Motor Controller

The speed of a DC motor does not change instantly if the voltage is stepped. There is lag due to rotational inertia.

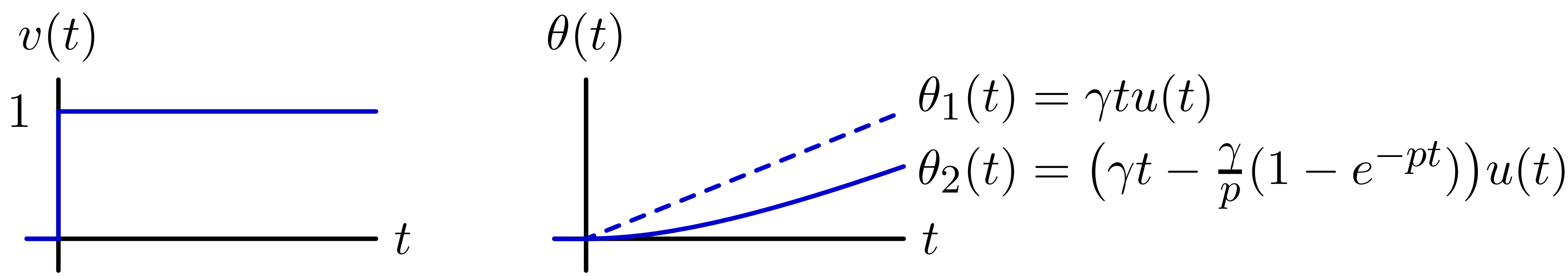
First-order model
integrator



Second-order model
integrator with lag

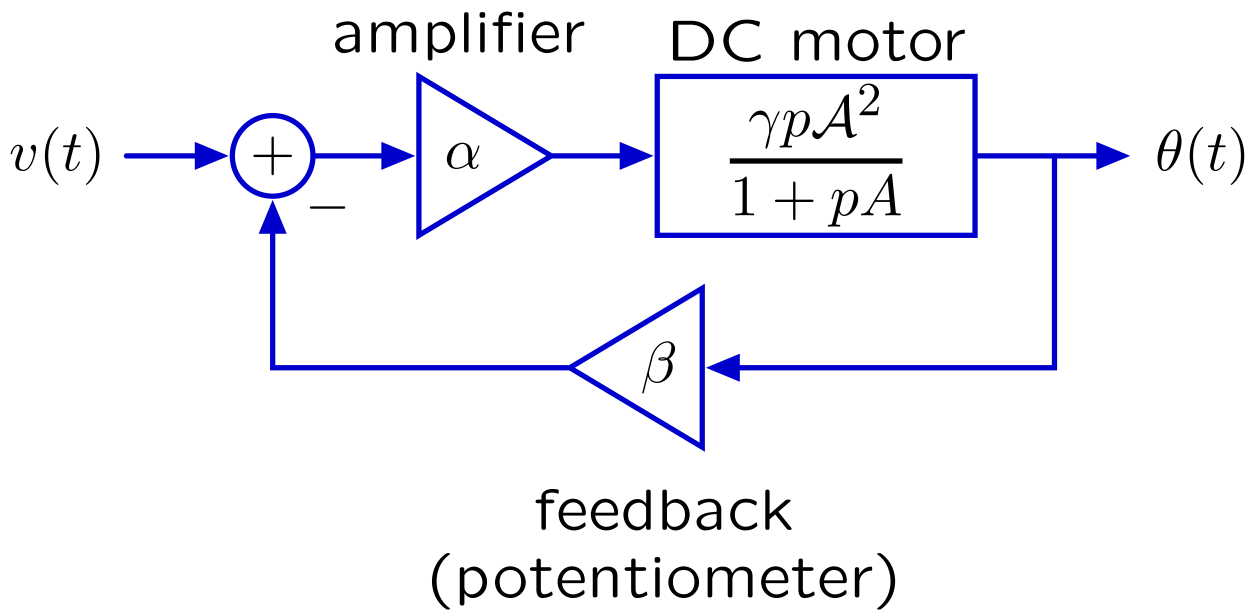


Step response of the models:



Motor Controller

Analyze second-order model.

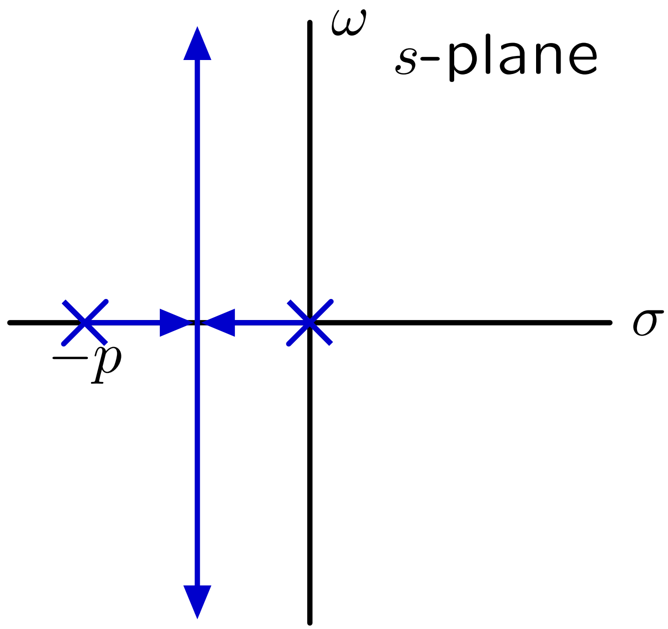


$$\frac{\Theta}{V} = \frac{\frac{\alpha\gamma p \mathcal{A}^2}{1+pA}}{1+\frac{\alpha\beta\gamma p \mathcal{A}^2}{1+p\mathcal{A}}} = \frac{\alpha\gamma p \mathcal{A}^2}{1+pA+\alpha\beta\gamma p \mathcal{A}^2} = \frac{\alpha\gamma p}{s^2+ps+\alpha\beta\gamma p}$$

$$s = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - \alpha\beta\gamma p}$$

Motor Controller

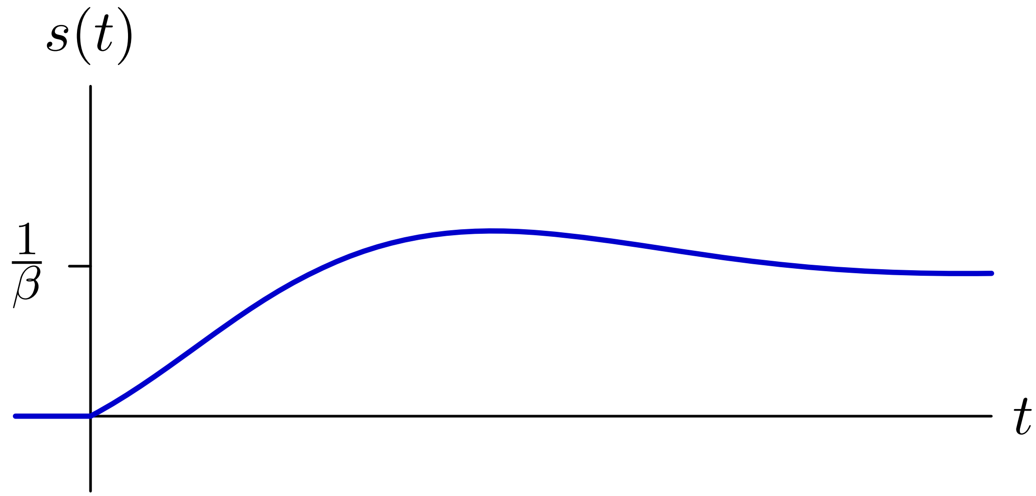
For second-order model, increasing α causes the poles at 0 and $-p$ to approach each other, collide at $s = -p/2$, then split into two poles with imaginary parts.



Increasing the gain α does not increase speed of convergence.

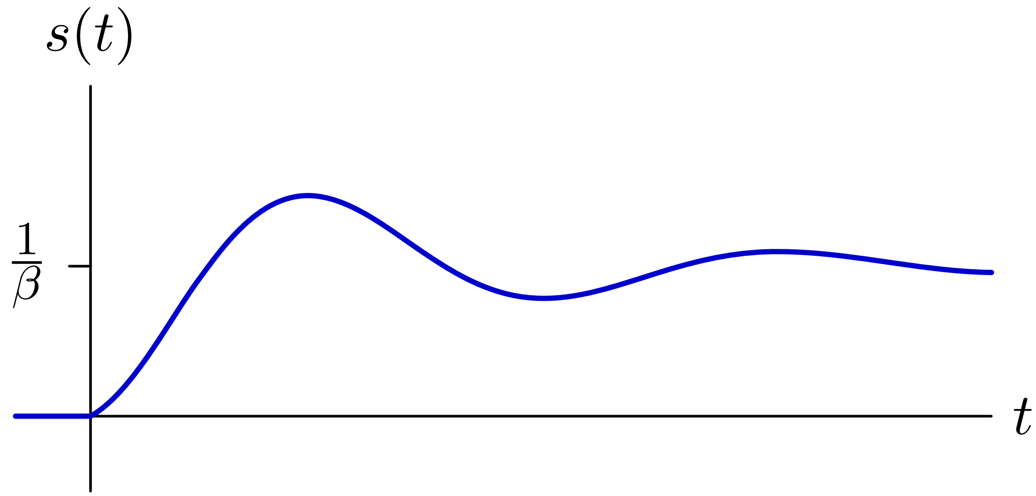
Motor Controller

Step response.



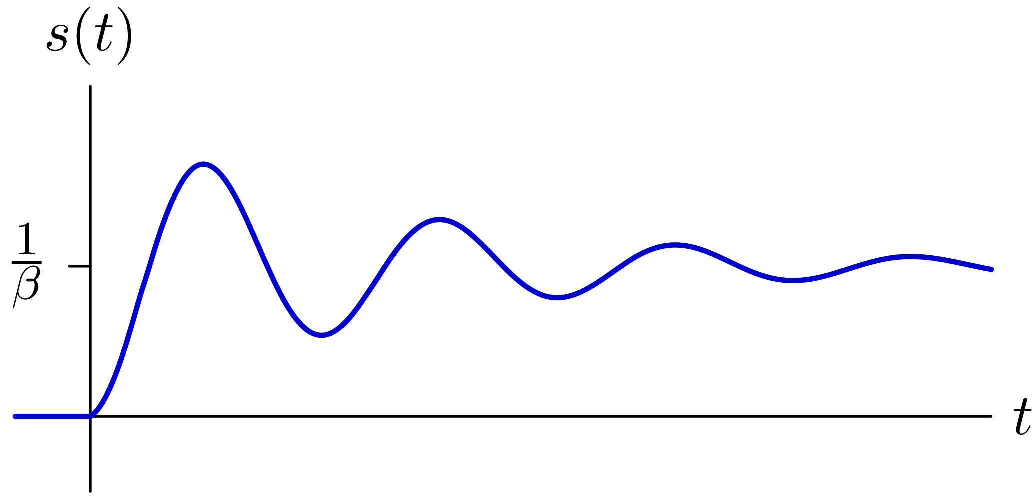
Motor Controller

Step response.



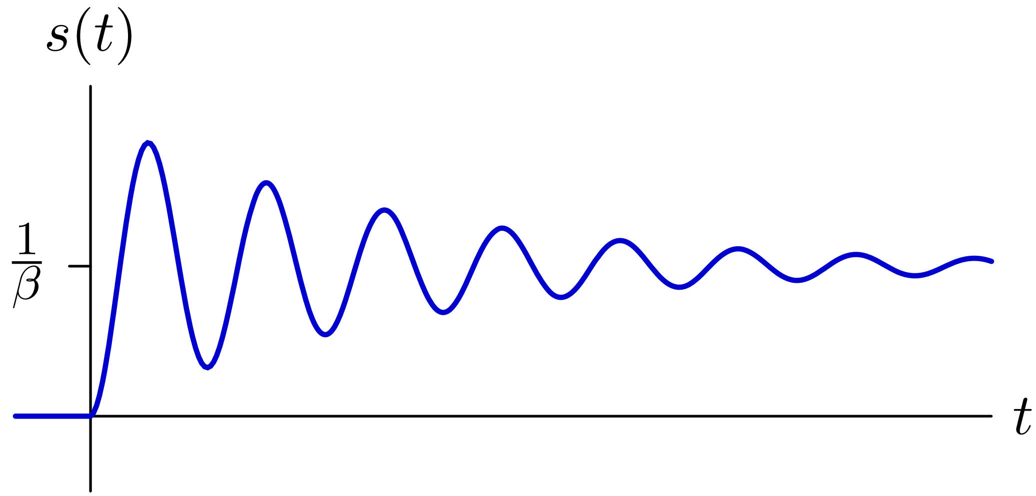
Motor Controller

Step response.



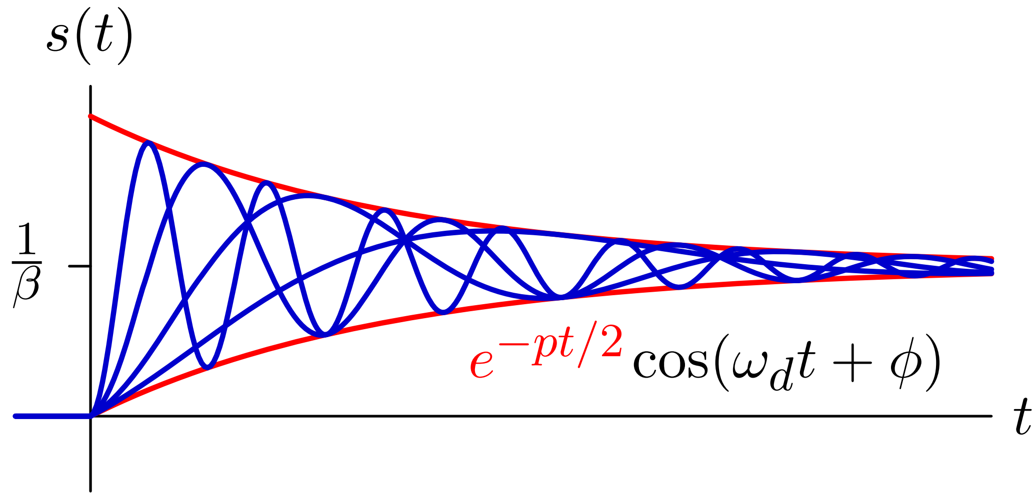
Motor Controller

Step response.



Motor Controller

Step response.



Feedback and Control: Summary

CT feedback is useful for many reasons. Today we saw two:

- increasing speed and bandwidth
- controlling position instead of speed

Next time we will look at several others:

- reduce sensitivity to parameter variation
- reduce distortion
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum