Quiz III Review
Signals and Systems
6.003

Massachusetts Institute of Technology

April 26th, 2010

Quiz 3 Details

- *Date:* Wednesday April 28th, 2010
- *Time:* 7.30pm–9.30pm
- *Content:* (boundaries inclusive)
  - Lectures 1–20
  - Recitations 1–20
  - Homeworks 1–11

Review Outline

- CT Fourier Series
- CT Fourier Transforms
- DT Frequency Response
- DT Fourier Series
- DT Fourier Transforms
- Fourier Relations
- The Impulse Train and Periodic Extension
- Filters

CT Fourier Series

- Periodic signals can be represented by a sum of harmonics
- The integral over one period of a harmonic is equal to zero, except for k=0.

$$\int_T e^{jk\omega_0 t} dt = T\delta[k]$$

- The "analysis" equation gives us the Fourier coefficients

$$a_k = \frac{1}{T}\int_T x(t)e^{-j\frac{2\pi}{T}kt}dt$$

- The "synthesis" equation reconstructs the periodic signal

$$x(t) = x(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T}kt}$$

CT Fourier Transform

- The aperiodic extension: *The Fourier series can be generalized to an aperiodic signal by viewing the signal as a periodic signal with an infinite period.*
- The "analysis" equation gives us the Fourier Transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

- The "synthesis" equation reconstructs the periodic signal

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

Fourier and Laplace Transform Properties

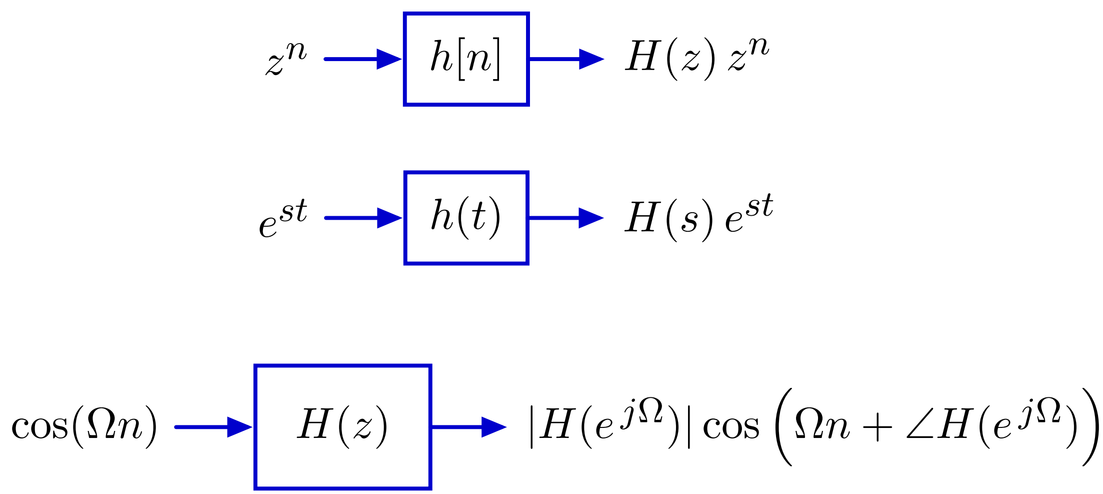
$$X(j\omega) = X(s)|_{s=j\omega}$$

Property	$x(t)$	$X(s)$	$X(j\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0)$	$e^{-st_0}X(s)$	$e^{-j\omega t_0}X(j\omega)$
Time scale	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
Differentiation	$\frac{dx(t)}{dt}$	$sX(s)$	$j\omega X(j\omega)$
Multiply by t	$tx(t)$	$-\frac{d}{ds}X(s)$	$-\frac{1}{j}\frac{d}{d\omega}X(j\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \times X_2(s)$	$X_1(j\omega) \times X_2(j\omega)$

* The form of the Fourier transform and its inverse are very similar. We can therefore use duality to find new transform pairs.

DT Frequency Response

- Similar to the continuous time case, *complex geometrics* are eigenfunctions of DT LTI systems



$$H(e^{j\Omega}) = H(z)\big|_{z=e^{j\Omega}}$$

- The DT frequency response is equivalent to the z-transform evaluated along the unit circle. It is **periodic** with period 2π.

$$H(e^{j(\Omega+2\pi k)}) = H(e^{j\Omega})$$

- The "highest" DT frequency is $\Omega = \pi$

DT Fourier Series

- Discrete time periodic signals can also be represented by a sum of harmonics
- The "analysis" equation gives us the Fourier coefficients

$$a_k = \frac{1}{N} \sum_{<N>} x[n] e^{-j\frac{2\pi}{N}kn}$$

- The "synthesis" equation reconstructs the periodic signal

$$x[n] = x[n+N] = \sum_{<N>} a_k e^{j\frac{2\pi}{N}kn}$$

- In the discrete Fourier series there are a **finite** number of periodic harmonics

DT Fourier Transform

The aperiodic extension of the discrete Fourier series.

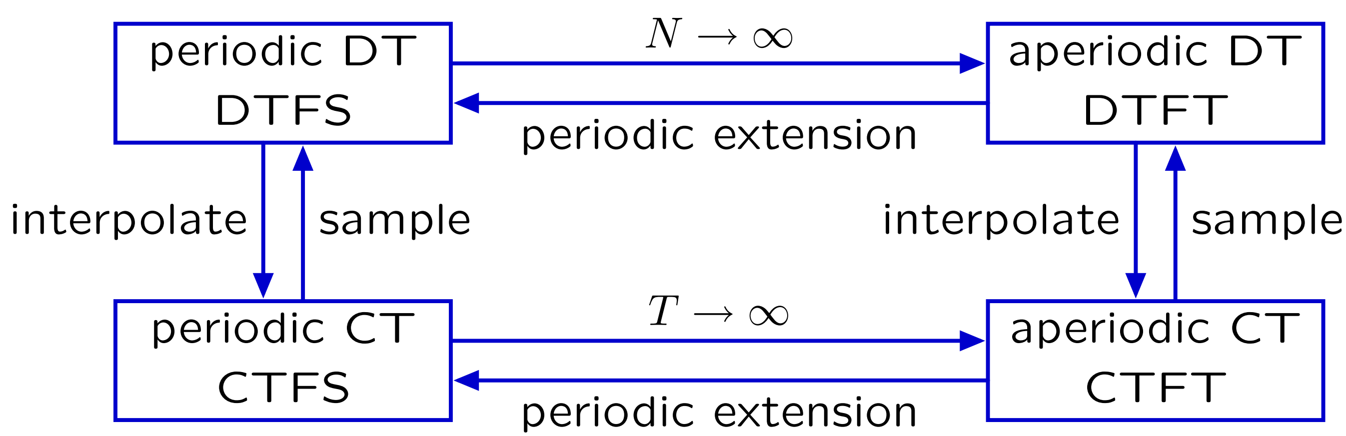
$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$$

Property	$x(t)$	$X(s)$	$X(j\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0)$	$e^{-st_0}X(s)$	$e^{-j\omega t_0}X(j\omega)$
Time scale	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
Differentiation	$\frac{dx(t)}{dt}$	$sX(s)$	$j\omega X(j\omega)$
Multiply by t	$tx(t)$	$-\frac{d}{ds}X(s)$	$-\frac{1}{j}\frac{d}{d\omega}X(j\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \times X_2(s)$	$X_1(j\omega) \times X_2(j\omega)$

Fourier Relations

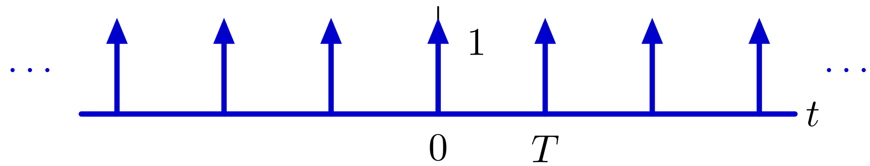
DTFS (discrete-time Fourier series):	periodic DT
DTFT (discrete-time Fourier transform):	aperiodic DT
CTFS (continuous-time Fourier series):	periodic CT
CTFT (continuous-time Fourier transform):	aperiodic CT



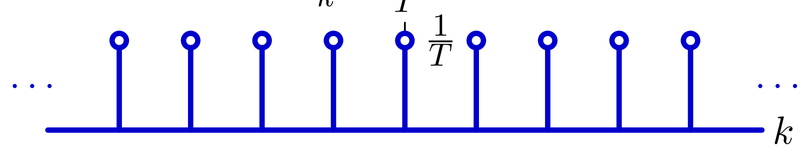
Impulse Trains and Periodic Extension

- Periodic extension can be accomplished by convolving a signal with an impulse train. This is equivalent to multiplying by an impulse train in frequency.

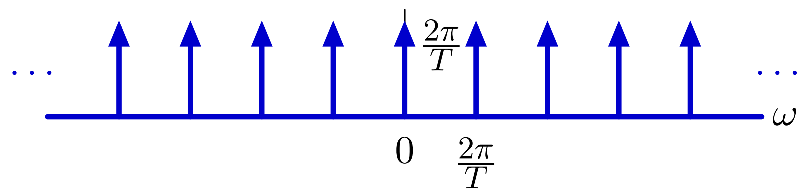
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$



$$a_k = \frac{1}{T} \quad \forall \; k$$



$$X(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\delta(\omega - k\frac{2\pi}{T})$$



Filters

- Fourier representations allow us to think of systems as **filters**
- LTI systems cannot create new frequencies
- LTI systems scale the magnitude and shift the phase of existing frequency components.

End of Review

Good luck on Wednesday! :-)

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