MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Solutions to Problem Set 12 Due: 5/8/03

Problem 12.1

a) $P_{rec} = (P_t G_r/4\pi r^2)(\sigma_L/4\pi r^2)A_r = 10\ kTB$
Assume η_A=0.65, the dish area is $\pi D^2/4$, $A_r = 0.65\pi D^2/4$, and $G_t = G_r = 0.65(\pi D/\lambda)^2$.
Therefore $D^4 = 10kTB\lambda^2 r^4 64/P_t\pi\times 0.65^2\sigma_L$
$= 10\times 1.38\times 10^{-23}600B\times 9\times 10^{-4}\times 10^{16}\times 64/10^3\pi\times 0.65^2\times 10^{-4} = 3.6\times 10^{-4}B$, so
$D = 0.14\ B^{0.25}$ meters. (B in Hz)

b) *Longer CW pulses yield better doppler resolution because of reduced B values, but poorer range resolution; larger B (shorter τ) requires larger antenna diameters D.*

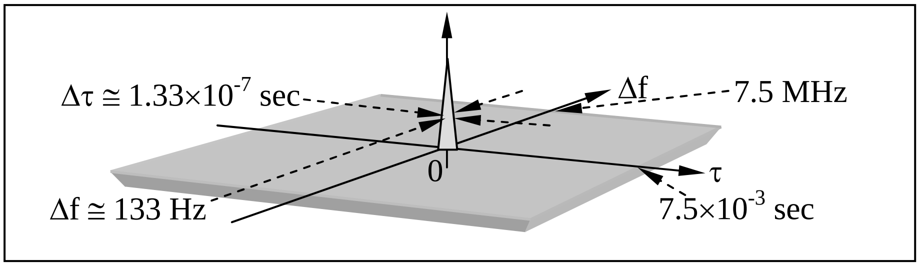
c) *If the pulse spacing T is too short, the delay window for accepting desired pulse echos may also intercept echos from previous pulses reflected from very distant objects. There are no "grandfathered" echoes if there are no reflecting targets beyond $R_{max} = Tc/2$ [m], or if those echoes are so weak as to be negligible; since echo strength declines as r^4, or 12 dB per octave (factor of two), increasing Tc/2 by a factor of 2-10 (12-40 dB) beyond the nominal working range of the radar may suffice.*

c) *The ambiguity function for a pulse τ = 10 m is ~10 meters wide at its half-amplitude points. A factor of 2 compensates the round trip;* $\tau \cong 2\times 10/c$ [s] $\cong 6.7\times 10^{-8}$ [s].

d) *Referring to Fig 5.3-12, we see the ambiguity function width Δf for a doppler shift for $\Delta v = \pm 1\ ms^{-1}$ is $\Delta f \cong 1/2T$ Hz, where $f_o\, 2\Delta v/c = 1/2T$ so*
$T = c/(4\Delta v\, f_o) = 3\times 10^8/4\times 1\times 10^{10} = 7.5\times 10^{-3}$ seconds.

e) *The time ambiguity function is triangular with half-amplitude width = τ seconds, and the round-trip effect means the range ambiguity function width is $c\tau/2$ meters $\cong 2\times 10$, so $\tau \cong 40/c = 1.33\times 10^{-7}$ seconds.*

f) *We can use a large time-bandwidth pulse. To obtain the range accuracy we need a bandwidth $B \cong 1/\tau$, where we use $\tau \cong 1.33\times 10^{-7}$ seconds, so $B \cong 7.5$ MHz. To obtain the doppler accuracy we need a pulse 7.5 milliseconds long. The ambiguity function then is:*



g) *The answer to (a) still applies, where $B \cong 7.5$ MHz, so* $\boxed{D \cong 7.33 \text{ meters.}}$

Problem 12.2

a) $\theta_B \cong \lambda/D$, *and resolution* $L \cong \lambda R/D$, *where R is range. Thus* $L \cong 0.1\times10^4/10 = \boxed{100 \text{ m.}}$

b) *The range ambiguity function width is* $c\tau/2$ *meters = 1 m, so* $\tau \cong 2/c = \boxed{6.67\times10^{-9} \text{ sec.}}$

c) *This is an unfocused SAR, so the resolution* $\boxed{\text{down track} \cong D = 10 \text{ m,}}$ *across track it is the same as for (a):* $\boxed{100 \text{ m.}}$

d) *If D = 10 meters, we must* $\boxed{\text{steer the dish to track the aircraft along a } 10\times100 \text{ m}}$ *trajectory centered overhead, 10 times the original resolution-limited 100-m track. This longer track is then Fourier transformed as before. Note that two airliners following within one km of each other cannot both be mapped because the radar dish can track only one at a time.*

e) *There should be no echos originating below 8 km or above 12 km, so there will be no ambiguity if* $cT > 2\times4000$ *meters;* $RPF = T^{-1} = c/8000 = 3\times10^8/8000 = \boxed{37.5 \text{ kHz.}}$

f) $\boxed{\text{Yes, delay compensation is needed}}$ *because the correlation length of the echo is ~2×1m, and the delay for a 1-km track at 10-km altitude varies from 10 km to* $10^4/\cos[\tan^{-1}(0.5/10)] = 10{,}012.5$ *meters, a difference of 12.5 meters, >> 2 m. The relative delay is* $2\times12.5/c = \boxed{83.3 \text{ ns.}}$

g) *Each separate image has a ratio s/m for intensity ≅ ~0.53, so if we average 10 this is reduced by a factor of* $10^{0.5} = 0.32$, *yielding a ratio of* $\boxed{\sim0.17.}$

Problem 12.3

a) $\theta_{null} = \lambda/D = 0.01/10 = \boxed{10^{-3} \text{ radian.}}$

b) $R_{\underline{E}}(\tau_\lambda)$ *has support over ±D = 20 meters,*
so $\hat{T}_B(\bar{\phi})$ *has its first null at* $\lambda/2D = 0.01/20 = \boxed{5\times10^{-4} \text{ radians}}$

c) *The maximum baseline is 10 m and* $\phi(\tau)$ *has support over a square 20 m on a side, yielding* $\lambda/2D$ *beamwidths =* $\boxed{5\times10^{-4} \text{ radians,}}$ *same as for (b). Both images are limited by the maximum spatial frequency governed by the maximum antenna spacing 10 m.*