MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Solutions to Problem Set 11 Due: 5/1/03

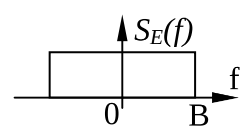
Problem 11.1

a) $\boxed{\langle (a+b)^2 \rangle = \langle a^2 \rangle + \langle b^2 \rangle + 2\langle ab \rangle}$ *where the DC terms* $\langle a^2 \rangle + \langle b^2 \rangle$ *can be subtracted, leaving the time average of the product of the two signals a(t) and b(t). Q.E.D.*

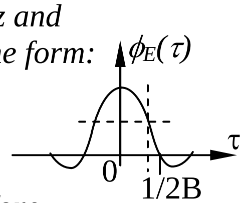
b) $\langle (a+b)^2 \rangle + \langle (a+b)^3 \rangle = \langle a^2 \rangle + \langle b^2 \rangle + 2\langle ab \rangle + c[\langle a^3 \rangle + 2\langle a^2 b \rangle + 2\langle ab^2 \rangle + \langle b^3 \rangle]$. *All the cubed terms average to zero, so a non-ideal diode with odd terms in the diode characteristic* **still functions as an ideal multiplier.** *The higher-order even powers are of greater potential interest, but the ability of the square term plus all higher order odd terms to approximate the local i-v characteristic of a diode leaves little room for producing much DC energy due to other terms. Only in highly sensitive systems is this a typical concern.*

Problem 11.2

Referring to Equation (5.2.5)and Figure 5.2-2 in the text, we see that $\langle ab \rangle$ *is the fringe modulation envelope, which is proportional to the field correlation function* $\phi_E(d/c)$, *where d is the differential distance traveled by the two rays, analogous to L* $\sin\phi_x$ *in the figure and c is the velocity of light. Note that the field E(t) to which* $\phi_E(\tau)$ *refers is the slowly varying envelope of the sine wave of interest, and so its bandwidth corresponds to the bandwidth B of the optical signal:* $f_{max} - f_{min} = c/\lambda_{min} - c/\lambda_{max} = 3\times10^8/5\times10^{-7} - 3\times10^8/6\times10^{-7} = 10^{14}$ *Hz and its power density spectrum* $S_E(f)$ *has the form:*



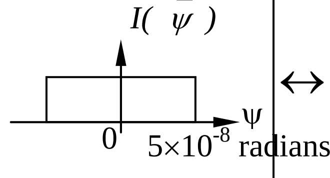
Since $\leftrightarrow \phi_E(\tau)$, *we have:*

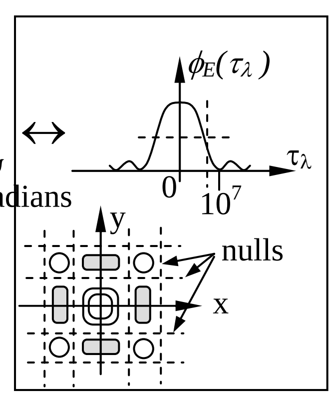


and the half-power point on $\phi_E(\tau)$ *is approximately* $\tau = 1/3B = d/c$. *Therefore* $d \cong c/3B = 3\times10^8/3\times10^{14} =$ **one micron.**

Problem 11.3

This Hanbury-Brown-Twiss interferometer deserves $\left|\phi_{\overline{E}}(\overline{\tau}_\lambda)\right|^2$





Referring to (5.2.15) we see that $\phi_E(\tau_\lambda) \leftrightarrow |E(\overline{\psi})|^2 \propto I(\overline{\psi})$, *as illustrated. It follows that the first null in the 2-D sinc function* $\phi_E(\tau_\lambda)$ *falls at* 10^7 *wavelengths, or* $10^7\times0.5\times10^{-9} =$ **5 mm.**