MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Solutions -- Problem Set No. 4 December 22, 2003

Problem 4.1

a) The preferred amplifier depends on both noise figure and gain. The amplifier with F = 4 is not in the contest. Using $F = F_1 + (F_2-1)/G_1$, it follows that we can choose between:

$F = 1.5 + (1.6 - 1)/10 = 1.56$ and
$F = 0.5 + 1.5/100 = 1.605$

Clearly the first combination has the lower noise figure, so put F = 1.5 first.

b) The equivalent input power $\cong k(FT_o)B = 1.38\times10^{-23}\times1.615\times290\times10^9 = 6.5\times10^{-12}$ W. If the output power is 0.001 watt = GP_{in}, then
$G_{max} \cong 0.001/6.5\times10^{-12} = 1.5\times10^8 =$ 82 dB.

c) We need 80 dB gain (but not over 82), so we can put the 70-dB amplifier in series with the 10 and/or 20-dB amplifiers. The lowest noise figure results when the 20-dB amplifier is followed by the 10 and 70-dB amplifiers, in that order. But then we need an attenuator preceding the final amplifier. If we have all three amplifiers in series, we need to reduce our gain by a factor of 100 - 82 dB = 18 dB $\Rightarrow$ 63. If R is added in series preceding the final amplifier and the Thevenin source resistance is 50 ohms, as is the load, then the power transferred is $0.5\underline{V}\underline{I}^* = 0.5|\underline{V}_{Th}|^2 50/[(R + 50)(R + 100)]$ versus $0.5|\underline{V}_{Th}|^2 50/[(50)(100)]$ when R = 0. This ratio of powers equals 63 when $[(1 + (R/50)][(1 + (R/100)] = 63 = 1 + 3R/100 + R^2/5000$. This happens when $R \cong 485$ ohms. This increases the Johnson noise from the input of the final amplifier in the ratio $[(485 + 50)/50]^{0.5}$, which is small compared to 290K increased by the 30-dB gain that precedes the resistor.

d) Consideration of (a) above suggests the lowest noise system has the amplifiers in the noise-figure sequence 1.6, 1.5, and 4.0 dB, with R just before the final amplifier. Although we could omit the 20-dB amplifier and the resistor R, and still achieve the desired 80-dB gain without non-linearities, the noise figure of this combination would be 1.5 + 4/10 = `1.9, which is decidedly worse. The 20-dB amplifier followed by a smaller R and the 70-dB amplifier would still suffer from the resistor Johnson noise more than happens when an extra 10-dB of gain precedes and effectively diminishes it.

Problem 4.2

a) The general scattering matrix subject to the given constraints is:

$$\underline{\mathbf{S}} = \begin{matrix} a & b & c \\ c & a & b \\ b & c & a \end{matrix}$$

where all entries are complex. Losslessness requires $\underline{\mathbf{S}}^{T*}\underline{\mathbf{S}} = \underline{\mathbf{I}}$, so that:

$$\underline{\mathbf{I}} = \begin{matrix} |a|^2 + |b|^2 + |c|^2 & a^*b + c^*a + b^*c & a^*c + c^*b + b^*a \\ b^*a + a^*c + c^*b & |a|^2 + |b|^2 + |c|^2 & c^*a + b^*c + a^*b \\ c^*a + b^*c + ba^* & c^*b + b^*a + a^*c & |a|^2 + |b|^2 + |c|^2 \end{matrix} = \begin{matrix} \Sigma & \alpha & \beta \\ \alpha^* & \Sigma & \beta^* \\ \beta^* & \beta & \Sigma \end{matrix}$$

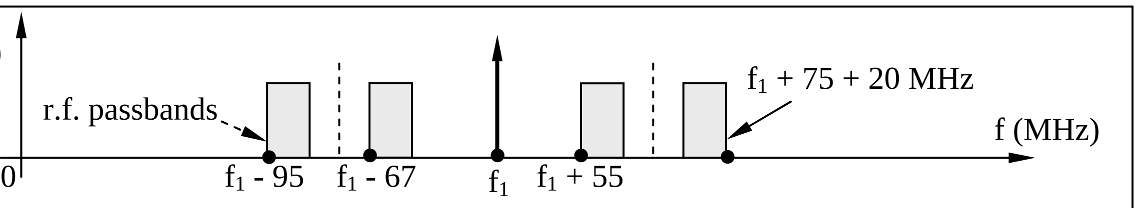
Therefore $\alpha = \beta = 0$ and $\Sigma = 1$. This implies only a, b, or c can be non-zero, and they must have magnitude unity to satisfy $\Sigma = 1$. However, reciprocity requires symmetry so that b = c, and therefore only 'a' can be non-zero.

Thus **the only allowed $\underline{\mathbf{S}}$ is the identity matrix times a phase shift $e^{j\phi}$, where ϕ is arbitrary.**

b) If we relax the requirement for reciprocity (b = c), only three scattering matrices are allowed **(a or b or c = 1), and the phase angle ϕ in each case is again arbitrary.**

Problem 4.3

a)



b) See Equation (2.3.20) in the notes. $F_A = L_C(F_{if} + t_r - 1) = L_{C1}(F_1 + t_{r1} - 1)$

c) The second mixer is like an amplifier of gain G_2L_{C2} and noise figure $F_B' = L_{C2}(F_2 + t_{r2} - 1)$, but with excess noise (still larger F_B) due to the fact that the unused sideband into the second mixer is not at 290K, but at a higher temperature when the first amplifier sees 290K. If we combine this excess noise input to the second mixer into the parameter t_{r2}', where t_{r2} is the manufacturer's specification, then the cascade formula for F yields:

$$F = F_A + (F_B - 1)/G_A = L_{c1}(F_{if1} + t_{r1} - 1) + L_{c1}[L_{c2}(F_{if2} + t_{r2}' - 1) - 1]/G_1$$

By definition $t_r = N_{if2}/kT_o$ = (shot etc. + kT_o)/kT_o. (see p 2-44 in text).

Here N_{if2}' = shot etc. + $(kT_o/2)(1 + [G_1/L_{c1}])$ where the G_1/L_{c1} term accounts for the excess thermal noise ($G_1/L_{c1} = 1$ if there is no first amplifier). Thus, $t_{r2}' = N_{if2}/kT_o = t_{r2} + ([G_1/L_{c1}] - 1)/2$ and

$$F = L_{c1}(F_{if1} + t_{r1} - 1) + L_{c1}\{L_{c2}(F_{if2} - 1 + t_{r2} + [([G_1/L_{c1}] - 1)/2]) - 1\}/G_1$$

12/22/03

Problem 4.4

a) Nothing. The local-oscillator noise is identical on both reversed diodes and still cancels, while the signal terms are still out of phase and pass through the transformer to the i.f. amplifier.

b) The local oscillator noise at i.f. frequencies results when the local oscillator signal at the diode mixes with its own noise sidebands, both of which are subject to the same phase changes at the junctions of the multiport. Therefore their product is the same in both arms of the 4-port device regardless of whether both diodes are reversed or remain the same, and thus both noises cancel in the transformer. The products of the signal and local oscillator signals remain out of phase in the two diodes, provided they are both up or both reversed, and therefore the signal passes through the transformer to the i.f. amplifier. Therefore nothing happens and no other changes should be made.