6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 21

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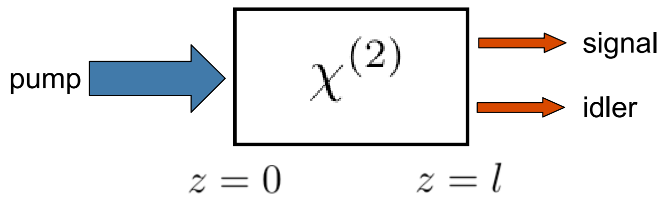
6.453 *Quantum Optical Communication* - Lecture 21

- Announcements
  - Pick up lecture notes, slides
- Nonlinear Optics of $\chi^{(2)}$ Interactions
  - Coupled-mode equations for parametric downconversion
  - Phase-matching for efficient interactions
  - Classical and quantum solutions
  - Gaussian-state characterization

Second-Order Nonlinear Optics

- Spontaneous Parametric Downconversion



  - Strong pump at frequency $\omega_P = \omega_S + \omega_I$
  - No input at signal frequency ω_S
  - No input at idler frequency ω_I
  - Nonlinear mixing in $\chi^{(2)}$ crystal produces signal and idler outputs

Coupled Equations for Plane-Wave Modes

- Monochromatic Pump, Signal, and Idler Electric Fields:

$$\begin{aligned}\vec{E}(z,t) &= (A_S(z)e^{-j(\omega_S t - k_S z)} + \text{cc})\vec{i}_S/2 \\ &+ (A_I(z)e^{-j(\omega_I t - k_I z)} + \text{cc})\vec{i}_I/2 \\ &+ (A_P e^{-j(\omega_P t - k_P z)} + \text{cc})\vec{i}_P/2\end{aligned}$$

  - Non-depleting pump
  - Slowly-varying signal and idler complex amplitudes

- Photon-Units Coupled-Mode Equations:

$$\begin{aligned}\frac{\partial A_S(z)}{\partial z} &= j\kappa A_I^*(z)e^{j\Delta k z} \\ \frac{\partial A_I(z)}{\partial z} &= j\kappa A_S^*(z)e^{j\Delta k z}\end{aligned}$$

Type-II Phase Matched Operation at Degeneracy

- Phase Matching for Efficient Coupling: $\Delta k = 0$
  - Conservation of photon momentum: $k_P = k_S + k_I$
  - Type-II system: $\vec{i}_S = \vec{i}_x,\ \vec{i}_I = \vec{i}_y$
- Operation at Frequency Degeneracy: $\omega_S = \omega_I = \omega_P/2$
- Classical Input-Output Relation:

$$
\begin{aligned}
A_S(l) &= \cosh(|\kappa|l)A_S(0) + j\frac{\kappa}{|\kappa|}\sinh(|\kappa|l)A_I^*(0) \\
A_I(l) &= \cosh(|\kappa|l)A_I(0) + j\frac{\kappa}{|\kappa|}\sinh(|\kappa|l)A_S^*(0)
\end{aligned}
$$

Quantum Coupled-Mode Equations

- Strong, Monochromatic, Coherent-State Pump
- Positive-Frequency Signal and Idler Field Operators:

$$
\begin{aligned}
\hat{E}_S^{(+)}(z,t) &= \int \frac{\mathrm{d}\omega}{2\pi}\,\hat{A}_S(z,\omega)e^{-j[(\omega_P/2+\omega)t - k_S(\omega_P/2+\omega)z]} \\
\hat{E}_I^{(+)}(z,t) &= \int \frac{\mathrm{d}\omega}{2\pi}\,\hat{A}_I(z,\omega)e^{-j[(\omega_P/2-\omega)t - k_I(\omega_P/2-\omega)z]}
\end{aligned}
$$

- Quantum Coupled-Mode Equations:

$$
\begin{aligned}
\frac{\partial \hat{A}_S(z,\omega)}{\partial z} &= j\kappa \hat{A}_I^\dagger(z,\omega)e^{j\omega\Delta k' z} \\
\frac{\partial \hat{A}_I(z,\omega)}{\partial z} &= j\kappa \hat{A}_S^\dagger(z,\omega)e^{j\omega\Delta k' z}
\end{aligned}
$$

Quantum Input-Output Relation

- Two-Mode Bogoliubov Relation

$$\hat{A}_S(l,\omega) = \mu(\omega)\hat{A}_S(0,\omega) + \nu(\omega)\hat{A}_I^\dagger(0,\omega)$$
$$\hat{A}_I(l,\omega) = \mu(\omega)\hat{A}_I(0,\omega) + \nu(\omega)\hat{A}_S^\dagger(0,\omega)$$

where

$$\mu(\omega) = \left(\cosh(pl) - \frac{j\omega\Delta k' l}{2}\frac{\sinh(pl)}{pl}\right)e^{j\omega\Delta k' l/2}$$
$$\nu(\omega) = j\kappa l\frac{\sinh(pl)}{pl}e^{j\omega\Delta k' l/2},$$
$$p = \sqrt{|\kappa|^2 - (\omega\Delta k'/2)^2}$$

Gaussian-State Characterization

- Signal and Idler at $z = 0$ are in Vacuum States
- Signal and Idler at $z = l$ are in Zero-Mean Gaussian States
- Baseband Signal and Idler Field Operators:

$$\hat{E}_m^{(+)}(l,t) = \hat{E}_m(t)e^{-j(\omega_P t/2 - k_m(\omega_P/2)l)}, \quad \text{for } m = S, I$$

- Non-Zero Covariance Functions:

$$K_{SS}^{(n)}(\tau) = K_{II}^{(n)}(-\tau) = \int \frac{\mathrm{d}\omega}{2\pi}\,|\nu(\omega)|^2 e^{j\omega\tau}$$
$$K_{SI}^{(p)}(\tau) = \int \frac{\mathrm{d}\omega}{2\pi}\,\mu(-\omega)\nu(-\omega)e^{j\omega(\tau - \Delta k' l)}$$

Operation in the Low-Gain Regime

- Low-Gain Regime: $|\kappa|l \ll 1$

- Approximate Bogoliubov Parameters:

$$\mu(\omega) \approx 1 \quad \text{and} \quad \nu(\omega) \approx j\kappa l \frac{\sin(\omega \Delta k' l/2)}{\omega \Delta k' l/2} e^{j\omega \Delta k' l/2}$$

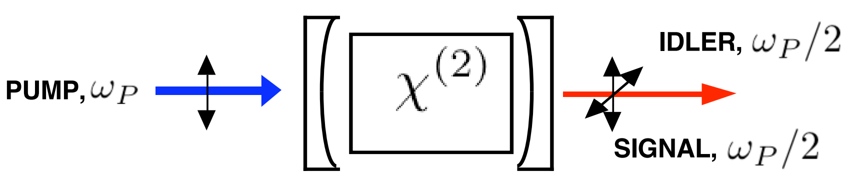
- Normally-Ordered and Phase-Sensitive Spectra:

$$\mathcal{S}_{SS}^{(n)}(\omega) = \mathcal{S}_{II}^{(n)}(\omega) \approx |\kappa|^2 l^2 \left(\frac{\sin(\omega \Delta k' l/2)}{\omega \Delta k' l/2} \right)^2$$

$$\mathcal{S}_{SI}^{(p)}(\omega) \approx j\kappa l \frac{\sin(\omega \Delta k' l/2)}{\omega \Delta k' l/2} e^{j\omega \Delta k' l/2}$$

Type-II Optical Parametric Amplifier

- Doubly-Resonant Operation at Frequency Degeneracy



- Normally-Ordered and Phase-Sensitive Covariances:

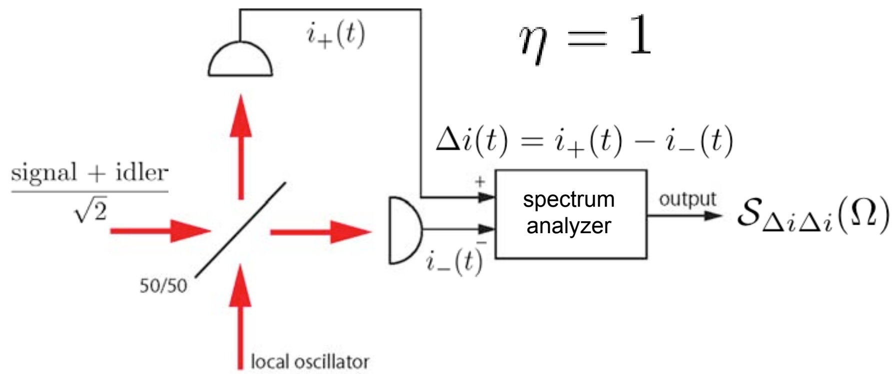
$$K^{(n)}(\tau) = \frac{G\Gamma}{2} \left[\frac{e^{-(1-G)\Gamma|\tau|}}{1-G} - \frac{e^{-(1+G)|\tau|}}{1+G} \right]$$

$$K_{SI}^{(p)}(\tau) = \frac{G\Gamma}{2} \left[\frac{e^{-(1-G)\Gamma|\tau|}}{1-G} + \frac{e^{-(1+G)|\tau|}}{1+G} \right]$$

Quadrature Noise Squeezing

- Homodyne Detection of 45° Polarization (Signal + Idler)

$\eta = 1$



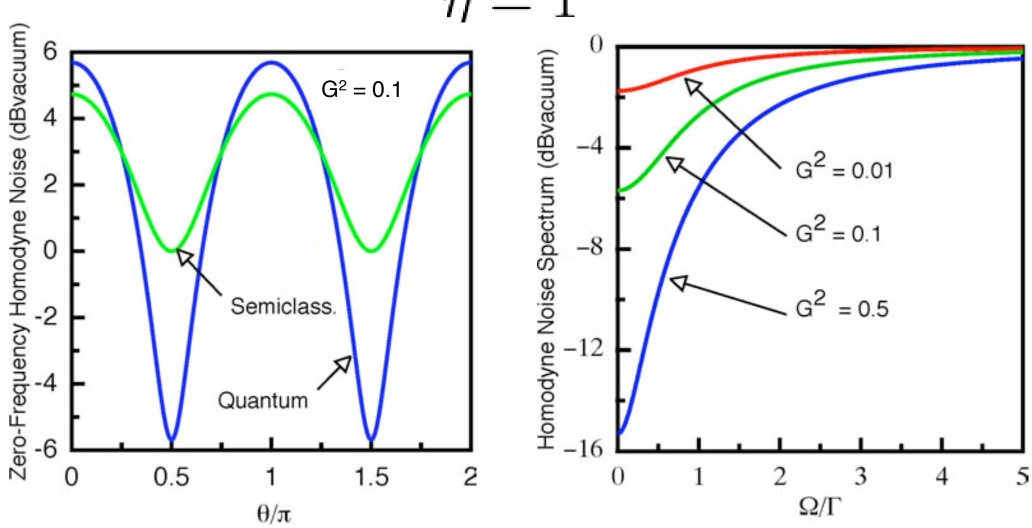
$$\frac{\mathcal{S}_{\Delta i \Delta i}(\Omega)}{\mathcal{S}_{\Delta i \Delta i}(\Omega)|_{\text{CS}}} = |\mu(\Omega) + \nu(\Omega) e^{-2j\theta}|^2$$

$$\mu(\Omega) \equiv \frac{1 + G^2 + \Omega^2/\Gamma^2}{1 - G^2 - \Omega^2/\Gamma^2 - 2j\Omega/\Gamma} \text{ and } \nu(\Omega) \equiv \frac{2G}{1 - G^2 - \Omega^2/\Gamma^2 - 2j\Omega/\Gamma}$$

Quadrature Noise Squeezing: Quantum Efficiency 1

- Homodyne Detection of 45° Polarization (Signal + Idler)

$\eta = 1$



Coming Attractions: Lecture 22

- Lecture 22:
  Quantum Signatures from Parametric Interactions
  - Hong-Ou-Mandel dip produced by parametric downconversion
  - Polarization entanglement produced by parametric downconversion
  - Photon twins from parametric amplifiers