6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 18

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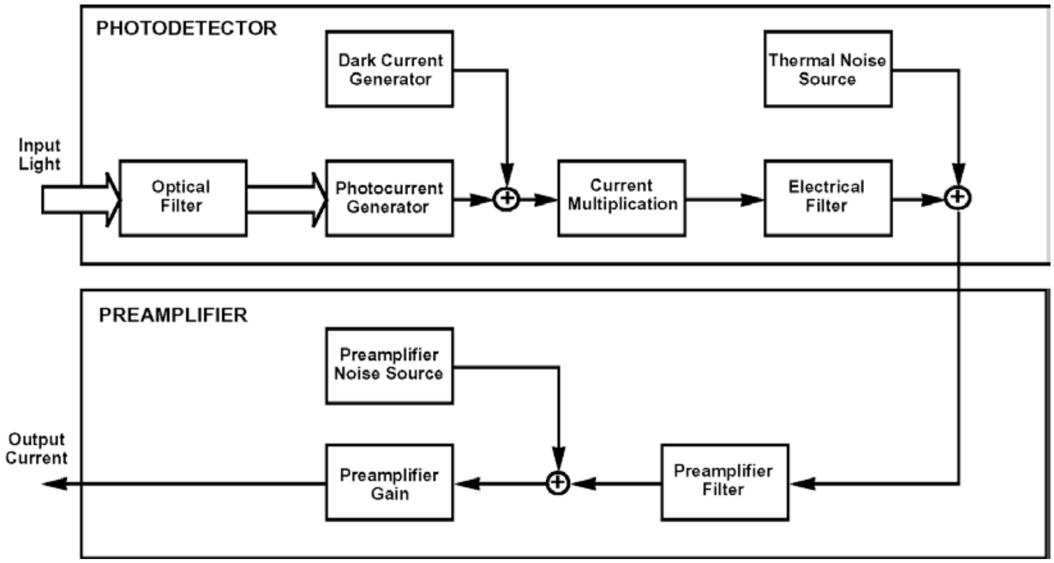
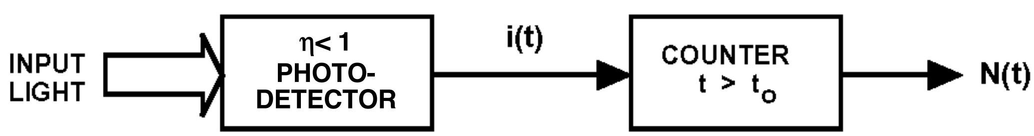
6.453 *Quantum Optical Communication* - Lecture 18

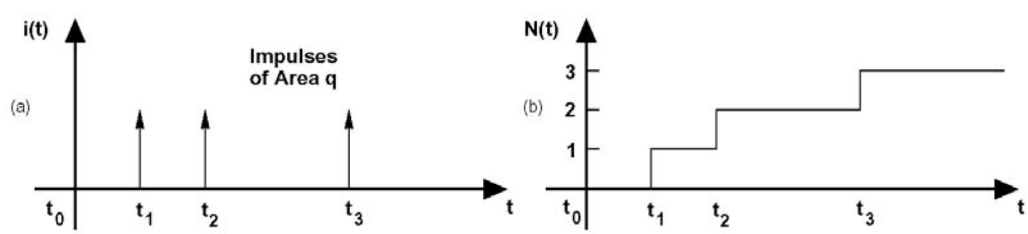
- Announcements
  - Pick up random processes notes, lecture notes, slides
- Continuous-Time Photodetection
  - Semiclassical theory — Poisson shot noise
  - Quantum theory — photon-flux operator measurement
  - Direct-detection signatures of non-classical light

Real Photodetection Systems



"Ideal" Photodetection System: Efficiency < 1





Classical Field versus Quantum Field Operator

- For Semiclassical Photodetection of Narrowband Light
  - Illumination is a classical photon-units positive-frequency field:

$$E(t)e^{-j\omega_o t}$$

  - Short-Time Average Power on Detector:

$$P(t) = \hbar\omega_o |E(t)|^2$$

- For Quantum Photodetection of Narrowband Light
  - Illumination is a photon-units positive-frequency field operator:

$$\hat{E}(t)e^{-j\omega_o t}$$

  - Only non-vacuum frequency components are within $\Delta\omega \ll \omega_o$

Semiclassical versus Quantum Photodetection

- Semiclassical Theory: Given $\{ P(t) : 0 \le t \le T \}$
  - $\{ N(t) : 0 \le t \le T \}$ is an inhomogeneous Poisson Counting Process
  - Rate function $\lambda(t) \equiv \eta P(t)/\hbar\omega_o$
- Quantum Theory:

$$\hat{E}'(t) \equiv \sqrt{\eta}\,\hat{E}(t) + \sqrt{1-\eta}\,\hat{E}_\eta(t)$$

$$i(t) \leftrightarrow \hat{i}(t) \equiv q\hat{E}'^\dagger(t)\hat{E}'(t)$$

$$N(t) \leftrightarrow \frac{1}{q}\int_0^t \mathrm{d}\tau\, \hat{i}(\tau), \quad \text{for } 0 \le t \le T$$

Inhomogeneous Poisson Counting Process (IPCP)

- Definition: $\{ N(t) : t \geq 0 \}$ is an IPCP with rate $\lambda(t)$
  - Starts counting at zero: $N(0) = 0$
  - Has statistically independent increments
  - Increments are Poisson distributed

$$\Pr(N(t) - N(u) = n) = \frac{\left(\int_u^t \mathrm{d}\tau\, \lambda(\tau)\right)^n \exp\left(-\int_u^t \mathrm{d}\tau\, \lambda(\tau)\right)}{n!}$$

Mean and Covariance: Deterministic Illumination

- Semiclassical Photocount and Photocurrent Mean Functions:

$$\langle N(t) \rangle = \int_0^t \mathrm{d}\tau\, \frac{\eta P(\tau)}{\hbar\omega_o} \quad \text{and} \quad \langle i(t) \rangle = q\frac{\eta P(t)}{\hbar\omega_o}$$

- Semiclassical Covariance Functions:

$$\begin{aligned} \langle \Delta N(t) \Delta N(u) \rangle &= \langle N(\min(t,u)) \rangle \\ \langle \Delta i(t) \Delta i(u) \rangle &= q \langle i(t) \rangle \delta(t-u) \end{aligned}$$

Mean and Covariance: Random Illumination

- Semiclassical Photocount and Photocurrent Mean Functions:

$$\langle N(t)\rangle = \int_0^t \mathrm{d}\tau\, \frac{\eta\langle P(\tau)\rangle}{\hbar\omega_o}$$

$$\langle i(t)\rangle = q\frac{\eta\langle P(\tau)\rangle}{\hbar\omega_o}$$

- Semiclassical Covariance Functions:

$$\langle \Delta N(t)\Delta N(u)\rangle = \underbrace{\langle N(\min(t,u))\rangle}_{\text{shot noise}} + \underbrace{\int_0^t \mathrm{d}\tau \int_0^u \mathrm{d}\tau'\, \frac{\eta^2\langle \Delta P(\tau)\Delta P(\tau')\rangle}{(\hbar\omega_o)^2}}_{\text{excess noise}}$$

$$\langle \Delta i(t)\Delta i(u)\rangle = \underbrace{q\langle i(t)\rangle\delta(t-u)}_{\text{shot noise}} + \underbrace{q^2\frac{\eta^2\langle \Delta P(t)\Delta P(u)\rangle}{(\hbar\omega_o)^2}}_{\text{excess noise}}$$

Mean and Covariance Functions: Quantum Case

- Quantum Photocount and Photocurrent Mean Functions:

$$\langle N(t)\rangle = \int_0^t \mathrm{d}\tau\, \eta\langle \hat{E}^\dagger(\tau)\hat{E}(\tau)\rangle \quad \text{and} \quad \langle i(t)\rangle = q\eta\langle \hat{E}^\dagger(t)\hat{E}(t)\rangle$$

- Quantum Covariance Functions:

$$\langle \Delta N(t)\Delta N(u)\rangle = \langle N(\min(t,u))\rangle$$
$$+ \int_0^t \mathrm{d}\tau \int_0^u \mathrm{d}\tau'\, \eta^2 \left[\langle \hat{E}^\dagger(\tau)\hat{E}^\dagger(\tau')\hat{E}(\tau)\hat{E}(\tau')\rangle - \langle \hat{E}^\dagger(\tau)\hat{E}(\tau)\rangle\langle \hat{E}^\dagger(\tau')\hat{E}(\tau')\rangle\right]$$

$$\langle \Delta i(t)\Delta i(u)\rangle = q\langle i(t)\rangle\delta(t-u)$$
$$+ q^2\eta^2 \left[\langle \hat{E}^\dagger(t)\hat{E}^\dagger(u)\hat{E}(t)\hat{E}(u)\rangle - \langle \hat{E}^\dagger(t)\hat{E}(t)\rangle\langle \hat{E}^\dagger(u)\hat{E}(u)\rangle\right]$$

Direct-Detection Signatures of Non-Classical Light

- Semiclassical Theory is Quantitatively Correct:
  - For coherent-state inputs $|E(t)\rangle$
  - For inputs that are classically-random mixtures of coherent states
- Sub-Poissonian Photon Counting:
  - Semiclassical theory:

$$\langle \Delta N^2(t)\rangle \geq \langle N(t)\rangle$$

  - Quantum theory:

$$\langle \Delta N^2(t)\rangle \geq 0$$

  - Non-classical signature:

$$\langle \Delta N^2(t)\rangle < \langle N(t)\rangle$$

Direct-Detection Signatures of Non-Classical Light

- Photocurrent Noise Spectral Density for CW Sources:

$$\mathcal{S}_{ii}(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}\tau\, \langle \Delta i(t+\tau)\Delta i(t)\rangle e^{-j\omega\tau}$$

- Semiclassical Theory:

$$\mathcal{S}_{ii}(\omega) = q\langle i\rangle + q^2 \frac{\eta^2 \mathcal{S}_{PP}(\omega)}{(\hbar\omega_o)^2} \geq q\langle i\rangle$$

- Quantum Theory:

$$\mathcal{S}_{ii}(\omega) \geq 0$$

- Sub-Shot-Noise Non-Classical Signature:

$$\mathcal{S}_{ii}(\omega) < q\langle i\rangle$$

Coming Attractions: Lecture 19

- Lecture 19:
  Continuous-Time Photodetection
  - Noise spectral densities in direct detection
  - Semiclassical theory of coherent detection
  - Quantum theory of coherent detection
  - Coherent-detection signatures of non-classical light