6.453 Quantum Optical Communication
Spring 2009

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6.453 *Quantum Optical Communication* Lecture 14

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Optical and Quantum Communications Group

- Announcements
  - Turn in problem set 7
  - Pick up problem set 7 solutions, problem set 8, lecture notes, slides

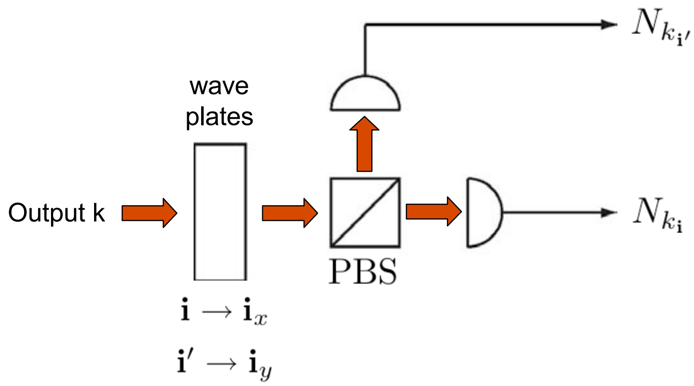
- Teleportation
  - Polarization entanglement and qubit teleportation
  - Quadrature entanglement and continuous-variable teleportation

Polarization-Entangled Photon Pairs

- Post-Selected Bi-Photon State from Dual-Paramp Source

$$|\psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|x\rangle_1|y\rangle_2 - |y\rangle_1|x\rangle_2)$$

- Measure Outputs 1 and 2 in Conjugate Polarizations



$$|\mathbf{i}\rangle_k = \alpha|x\rangle_k + \beta|y\rangle_k \text{ and } |\mathbf{i}'\rangle_k = \beta^*|x\rangle_k - \alpha^*|y\rangle_k, \text{ for } k = 1,2$$

Polarization-Entangled Photon Pairs

- Classical Theory: correlated, randomly-polarized photons

$$\Pr(N_{1_\mathbf{i}} = 1, N_{2_{\mathbf{i}'}} = 1) = \left\langle \frac{1+\mathbf{r}^T\mathbf{r}_1}{2}\frac{1+\mathbf{r}'^T\mathbf{r}_2}{2} \right\rangle = 1/3$$

$$\Pr(N_{1_{\mathbf{i}'}} = 1, N_{2_\mathbf{i}} = 1) = \left\langle \frac{1+\mathbf{r}'^T\mathbf{r}_1}{2}\frac{1+\mathbf{r}^T\mathbf{r}_2}{2} \right\rangle = 1/3$$

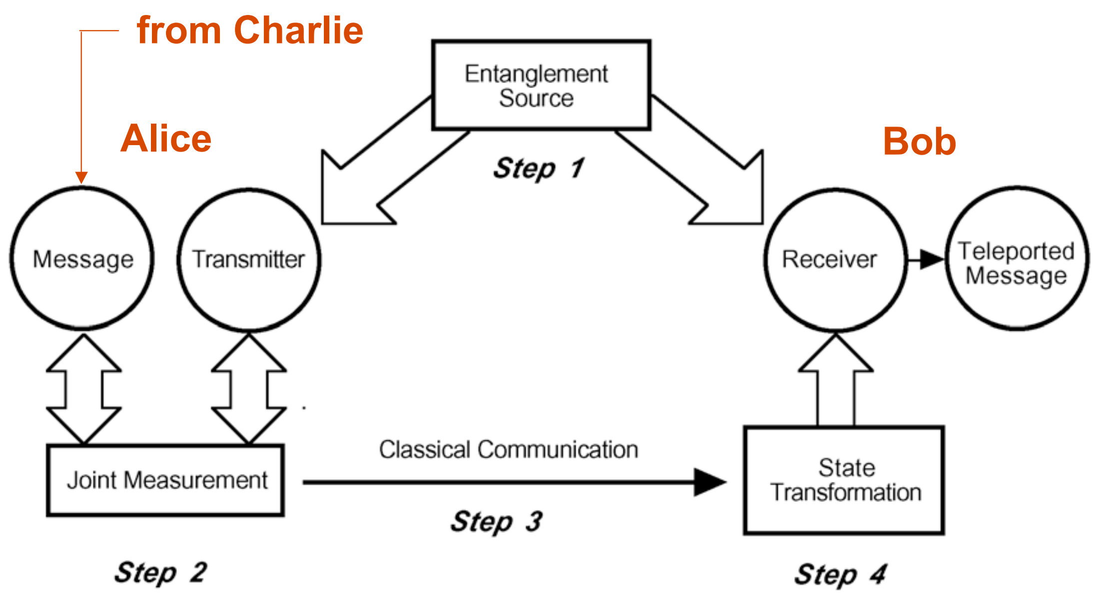
where $\mathbf{r} \leftrightarrow \mathbf{i}, \mathbf{r}' = -\mathbf{r} \leftrightarrow \mathbf{i}', \mathbf{r}_2 = -\mathbf{r}_1 = \text{ random}$

- Quantum Theory: polarization-entangled photons

$$\Pr(N_{1_\mathbf{i}} = 1, N_{2_{\mathbf{i}'}} = 1) = |{}_1\langle \mathbf{i}|_2\langle \mathbf{i}'|\psi^-\rangle_{12}|^2 = 1/2$$

$$\Pr(N_{1_{\mathbf{i}'}} = 1, N_{2_\mathbf{i}} = 1) = |{}_1\langle \mathbf{i}'|_2\langle \mathbf{i}|\psi^-\rangle_{12}|^2 = 1/2$$

The Four Steps of Qubit Teleportation



What's Under the Teleportation Hood

- Step 1: Alice and Bob share and store an entangled state
  - Bob's state intimately tied to result of Alice's measurement
- Step 2: Alice measures her state ⊗ message state
  - she obtains two bits of classical information
  - she learns nothing about her state or the message
- Step 3: Alice sends her measurement bits to Bob…
  - using classical communication: nothing moves faster than light speed
- Step 4: Bob applies polarization transformation…
  - chosen in accordance with Alice's measurement bits
  - entanglement guarantees that Bob has recovered the message

And Now the Details...

- Step 1: Alice and Bob share and store an entangled state

$$|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|x\rangle_A|y\rangle_B - |y\rangle_A|x\rangle_B)$$

- Step 2: Alice measures the Bell observable

$$\hat{B}_{AC} = \sum_{n=0}^{3} n|B_n\rangle_{AC\,AC}\langle B_n|$$

$$|B_0\rangle_{AC} = \frac{|x\rangle_A|y\rangle_C - |y\rangle_A|x\rangle_C}{\sqrt{2}}, \quad |B_1\rangle_{AC} = \frac{|x\rangle_A|y\rangle_C + |y\rangle_A|x\rangle_C}{\sqrt{2}}$$

$$|B_2\rangle_{AC} = \frac{|x\rangle_A|x\rangle_C - |y\rangle_A|y\rangle_C}{\sqrt{2}}, \quad |B_3\rangle_{AC} = \frac{|x\rangle_A|x\rangle_C + |y\rangle_A|y\rangle_C}{\sqrt{2}}$$

Here's the Magic of Entanglement...

- Alice's Measurement Result Determines Bob's State

$$|\psi\rangle_C|\psi^-\rangle_{AB} = (\alpha|x\rangle_C + \beta|y\rangle_C)|\psi^-\rangle_{AB}$$

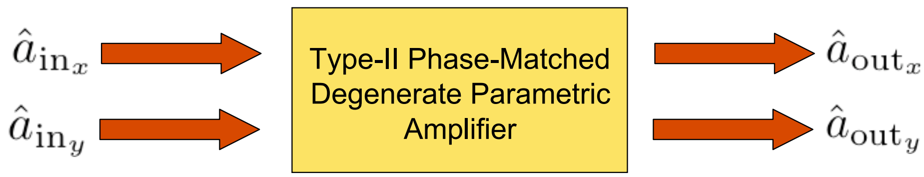
$$= \frac{1}{2}\Big[|B_0\rangle_{AC} \otimes \underbrace{(\alpha|x\rangle_B + \beta|y\rangle_B)}_{\text{Bob's state if } \hat{B}_{AC}=0} - |B_1\rangle_{AC} \otimes \underbrace{(\alpha|x\rangle_B - \beta|y\rangle_B)}_{\text{Bob's state if } \hat{B}_{AC}=1}$$

$$+ |B_2\rangle_{AC} \otimes \underbrace{(\alpha|y\rangle_B + \beta|x\rangle_B)}_{\text{Bob's state if } \hat{B}_{AC}=2} + |B_3\rangle_{AC} \otimes \underbrace{(\alpha|y\rangle_B - \beta|x\rangle_B)}_{\text{Bob's state if } \hat{B}_{AC}=3}\Big]$$

- Step 3: Alice sends her two measurement bits to Bob

- Step 4: Bob makes the appropriate state transformation

Quadrature Entanglement: Vacuum-Input Paramp



- Quadrature Variances:

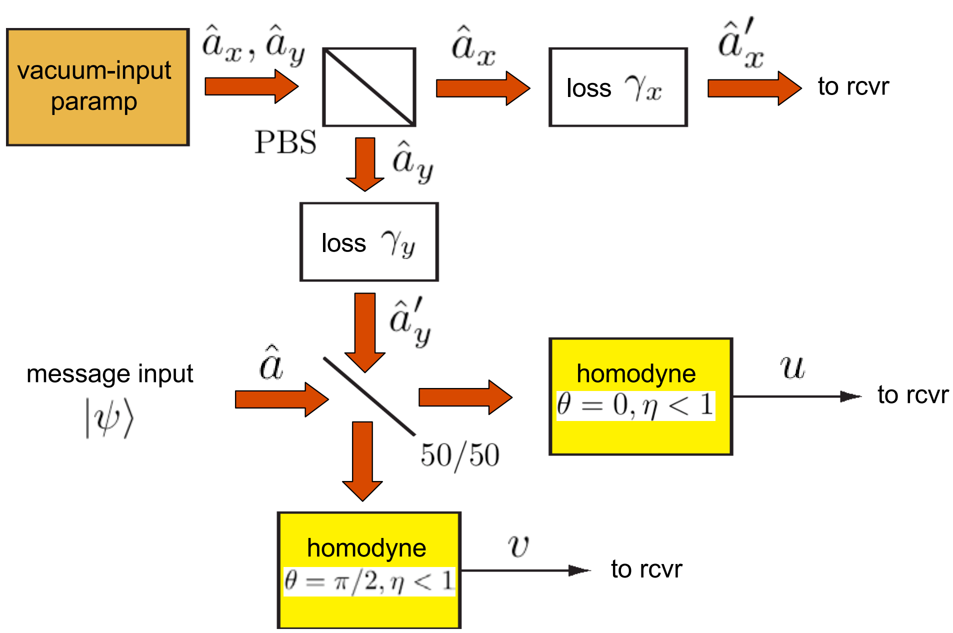
$$\langle \Delta \hat{a}^2_{\text{out}_{x_k}} \rangle = \langle \Delta \hat{a}^2_{\text{out}_{y_k}} \rangle = \frac{2G-1}{4} > \frac{1}{4}, \ \text{for } k = 1, 2$$

- Quadrature-Difference Variances:

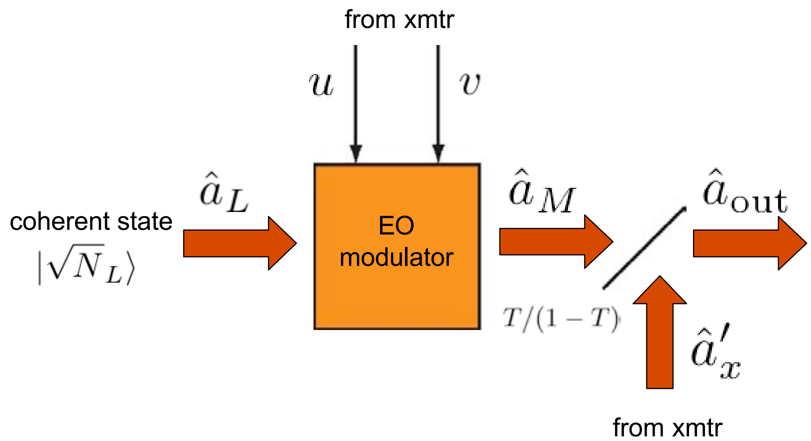
$$\left\langle \left(\frac{\Delta \hat{a}_{\text{out}_{x_1}} - \Delta \hat{a}_{\text{out}_{y_1}}}{\sqrt{2}} \right)^2 \right\rangle = \left\langle \left(\frac{\Delta \hat{a}_{\text{out}_{x_2}} + \Delta \hat{a}_{\text{out}_{y_2}}}{\sqrt{2}} \right)^2 \right\rangle$$

$$= \frac{(\sqrt{G} - \sqrt{G-1})^2}{4} \approx \frac{1}{16G} \ll \frac{1}{4}, \ \text{for } G \gg 1$$

Continuous-Variable Teleportation: the Xmtr



Continuous-Variable Teleportation: the Rcvr



Coming Attractions: Lectures 15 and 16

- Lecture 15:
  Quadrature Teleportation
  - Fidelity analysis for coherent-state inputs
- Lecture 16:
  Quantum Cryptography
  - Bennett-Brassard protocol
  - Ekert protocol