6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 5

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- Announcements
  - Turn in problem set 2
  - Pick up problem set 2 solution, problem set 3, lecture notes, slides
- Quantum Harmonic Oscillator
  - Number measurements versus quadrature measurements
  - Coherent states and their measurement statistics

Quantum Harmonic Oscillator: Reprise

- Operator-Valued Dynamics:

$$\hat{a}(t) = \hat{a}_1(t) + j\hat{a}_2(t) = \hat{a}e^{-j\omega t}$$

- Hamiltonian and the Number Operator:

$$\hat{H} = \hbar\omega[\hat{a}_1^2(t) + \hat{a}_2^2(t)] = \hbar\omega[\hat{a}^\dagger\hat{a} + 1/2] = \hbar\omega[\hat{N} + 1/2]$$

- Heisenberg Uncertainty Principle:

$$\langle\Delta\hat{a}_1^2(t)\rangle\langle\Delta\hat{a}_2^2(t)\rangle \geq \frac{1}{16}$$

Quantum Harmonic Oscillator: Reprise

- Number Kets:

$$\hat{N}|n\rangle = n|n\rangle, \quad \text{for } n = 0, 1, 2, \ldots$$

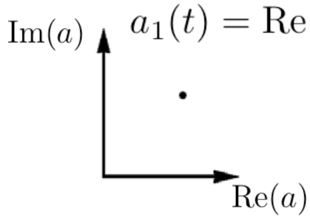
  - Orthonormal: $\langle m|n\rangle = \delta_{nm}$
  - Complete: $\hat{I} = \sum_{n=0}^{\infty} |n\rangle\langle n|$
  - Diagonal representation of number operator: $\hat{N} = \sum_{n=0}^{\infty} n|n\rangle\langle n|$
  - Annihilation and creation operators:

$$\hat{a} = \sum_{n=1}^{\infty} \sqrt{n}|n-1\rangle\langle n|, \quad \hat{a}^\dagger = \sum_{n=0}^{\infty} \sqrt{n+1}|n+1\rangle\langle n|$$

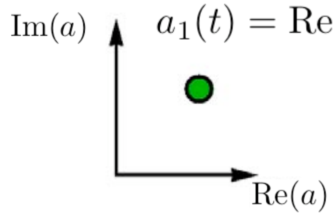
Classical versus Quantum Quadrature Behavior

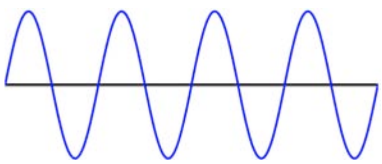
- Classical Oscillator: Noiseless
- Quantum Oscillator: Noisy

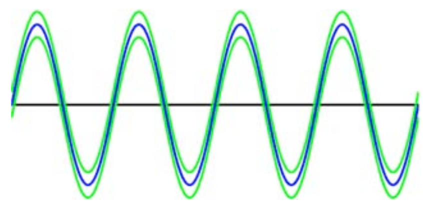
$a_1(t) = \text{Re}[ae^{-j\omega t}]$



$a_1(t) = \text{Re}[ae^{-j\omega t}]$







How can we get to the classical limit?

Quadrature-Statistics of Number Kets

- Quadrature-Measurement Mean Values:

$$\begin{aligned}\langle n|\hat{a}(t)|n\rangle &= \langle n|\hat{a}_1(t)|n\rangle + j\langle n|\hat{a}_2(t)|n\rangle \\ &= \langle n|\hat{a}|n\rangle e^{-j\omega t} = 0\end{aligned}$$

- Quadrature-Measurement Variances:

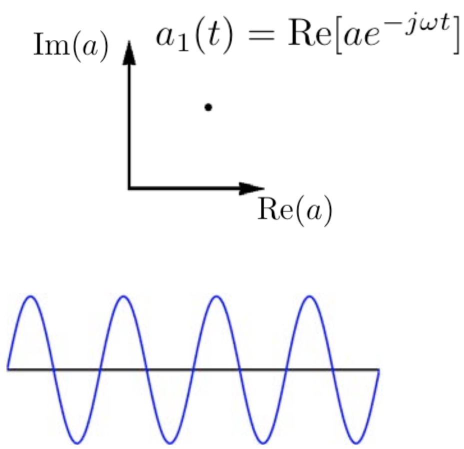
$$\langle n|\Delta\hat{a}_1^2(t)|n\rangle = \langle n|\Delta\hat{a}_2^2(t)|n\rangle = \frac{2n+1}{4}$$

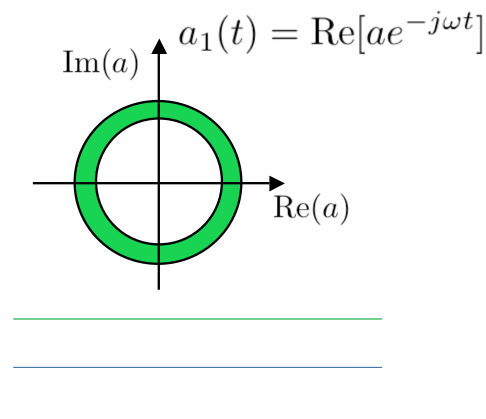
- Non-Minimum Quadrature-Uncertainty Product:

$$\langle n|\Delta\hat{a}_1^2(t)|n\rangle\langle n|\Delta\hat{a}_2^2(t)|n\rangle = \left(\frac{2n+1}{4}\right)^2 > \frac{1}{16}, \text{ for } n \geq 1$$

Classical versus Quantum Quadrature Behavior

- Classical Oscillator: Noiseless
- Quantum Oscillator: $|n\rangle$ State





How can we get to the classical limit?

Coherent States

- Eigenkets of the Annihilation Operator:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \quad \text{for } \alpha = \alpha_1 + j\alpha_2$$

  - Number-ket representation:

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} e^{-|\alpha|^2/2} |n\rangle$$

  - Normalized, non-orthogonal:

$$\langle\alpha|\beta\rangle = \exp(-|\alpha|^2/2 - |\beta|^2/2 + \alpha^*\beta)$$

  - Overcomplete:

$$\hat{I} = \int \frac{d^2\alpha}{\pi} |\alpha\rangle\langle\alpha|$$

Coherent-State Measurement Statistics

- Number-Operator Measurement:

$$\Pr(\,\hat{N}\text{ outcome} = n \mid \text{state is } |\alpha\rangle\,) = |\langle n|\alpha\rangle|^2 = \frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2}$$

- Quadrature-Operator Measurements:

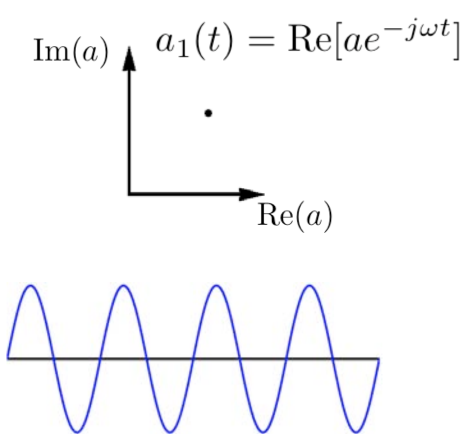
$$\begin{aligned}\langle\alpha|\hat{a}(t)|\alpha\rangle &= \langle\alpha|\hat{a}_1(t)|\alpha\rangle + j\langle\alpha|\hat{a}_2(t)|\alpha\rangle \\ &= \langle\alpha|\hat{a}|\alpha\rangle e^{-j\omega t} = \alpha e^{-j\omega t}\end{aligned}$$

$$\langle\alpha|\Delta\hat{a}_1^2(t)|\alpha\rangle = \langle\alpha|\Delta\hat{a}_2^2(t)|\alpha\rangle = \frac{1}{4}$$

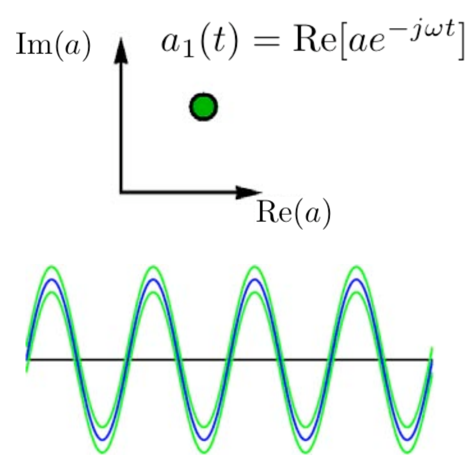
- Minimum Uncertainty-Product with Equal Variances

Classical versus Quantum Quadrature Behavior

- Classical Oscillator: Noiseless



- Quantum Oscillator: $|\alpha\rangle$ State



THIS is how we get to the classical limit!

Coming Attractions: Lectures 6 and 7

- Lecture 6:
  Quantum Harmonic Oscillator
  - Squeezed states and their measurement statistics
  - Probability operator-valued measurement of $\hat{a}$

- Lecture 7:
  Single-Mode Photodetection
  - Direct Detection
  - Homodyne Detection