6.453 Quantum Optical Communication
Spring 2009

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6.453 *Quantum Optical Communication*
Lecture 1

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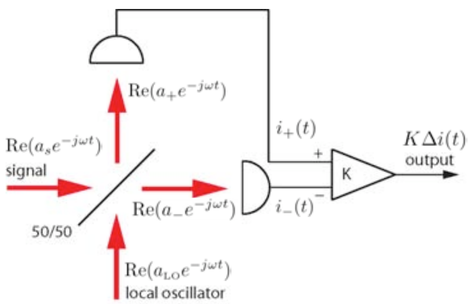
6.453 *Quantum Optical Communication* — Lecture 1

- Handouts
  - Syllabus, schedule/policy, probability chapter, lecture notes, slides, problem set 1
  - Sign-up on class list
- Introductory Remarks
  - Subject organization
  - Subject outline
- Technical Overview
  - Optical eavesdropping tap — quadrature-noise squeezing
  - Action at a distance — polarization entanglement
  - Long-distance quantum state transmission — qubit teleportation

Optical Homodyne Detection — Semiclassical

Balanced Homodyne Receiver



- Signal is weak, LO is strong
- Energy conservation

$$a_\pm \equiv \frac{a_s \pm a_{\mathrm{LO}}}{\sqrt{2}}$$

- Detectors are noisy square laws

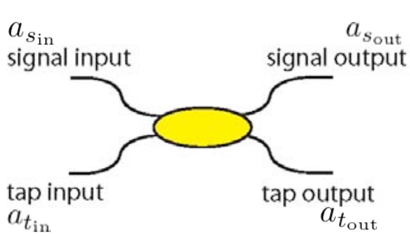
$$i_\pm(t) \text{ Poisson distributed}$$
$$\text{mean} = |a_\pm|^2$$

- Output mean and variance

$$\begin{aligned} \langle K\Delta i(t)\rangle &= 2K\mathrm{Re}(a_s a_{\mathrm{LO}}^*) \\ \mathrm{var}(K\Delta i(t)) &= K^2|a_{\mathrm{LO}}|^2 \end{aligned}$$

Optical Waveguide Tap — Semiclassical

Fused Fiber Coupler



- Coupler is a beam splitter

$$\begin{aligned} a_{s_{\mathrm{out}}} &= \sqrt{T}a_{s_{\mathrm{in}}} + \sqrt{1-T}a_{t_{\mathrm{in}}} \\ a_{t_{\mathrm{out}}} &= \sqrt{1-T}a_{s_{\mathrm{in}}} - \sqrt{T}a_{t_{\mathrm{in}}} \end{aligned}$$

- Tap input is zero
- Homodyne SNR at signal input

$$\mathrm{SNR}_{\mathrm{in}} = 4|a_{s_{\mathrm{in}}}|^2$$

- Homodyne SNR at signal output

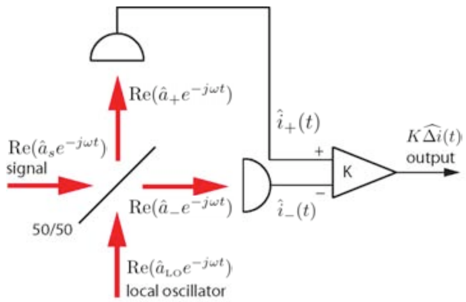
$$\mathrm{SNR}_{\mathrm{out}} = 4T|a_{s_{\mathrm{in}}}|^2$$

- Homodyne SNR at tap output

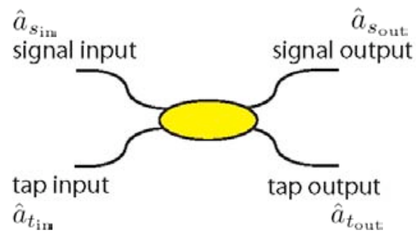
$$\mathrm{SNR}_{\mathrm{tap}} = 4(1-T)|a_{s_{\mathrm{in}}}|^2$$

Quantum Homodyne Detection and Waveguide Tap

Balanced Homodyne Receiver



Fused Fiber Coupler



Homodyne SNR at signal output

$$\mathrm{SNR}_{\mathrm{out}} \approx 4|a_{s_{\mathrm{in}}}|^2$$

Homodyne SNR at tap output

$$\mathrm{SNR}_{\mathrm{tap}} \approx 4|a_{s_{\mathrm{in}}}|^2$$

Billiard-Ball Photons and the Poincaré Sphere

- Polarization of $+z$-going photon:

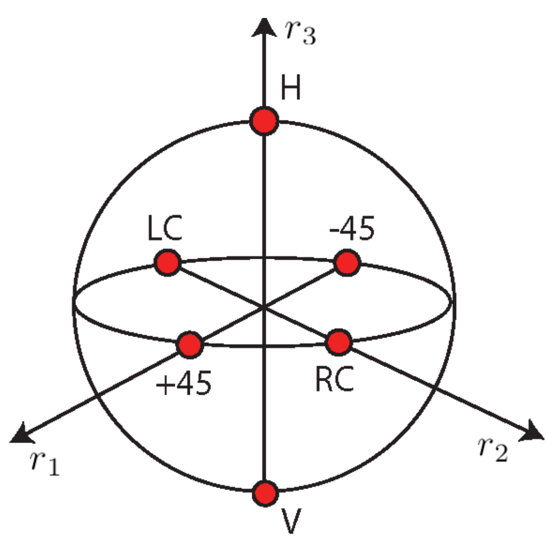
$$\mathbf{i} \equiv \begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix}, \quad \mathbf{i}^\dagger \mathbf{i} = 1$$

- Poincaré sphere representation

$$\mathbf{r} \equiv \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 2\mathrm{Re}(\alpha_x^* \alpha_y) \\ 2\mathrm{Im}(\alpha_x^* \alpha_y) \\ |\alpha_x|^2 - |\alpha_y|^2 \end{bmatrix}$$

- $\pm\mathbf{r}_m$ polarization measurement

$$\Pr(\text{polarized } \pm\mathbf{r}_{\mathrm{m}}) = \frac{1 \pm \mathbf{r}_{\mathrm{m}}^{\mathrm{T}}\mathbf{r}}{2}$$



Classical Correlation vs. Quantum Entanglement

- Classical-Correlated, Randomly-Polarized Photons
  - Source produces $\pm\mathbf{r}$ photon pair with $\mathbf{r}$ completely random

    $$\Pr(\text{photon } 1 \ = \pm\mathbf{r}_m) = \Pr(\text{photon } 2 = \mp\mathbf{r}_m) = 1/2$$

  - Conditional probability given photon 1 is $\mathbf{r}_m$ instead of $-\mathbf{r}_m$

    $$\Pr(\text{ photon } 2 \ = -\mathbf{r}_m \mid \text{photon } 1 \ = \mathbf{r}_m\,) = 2/3$$

- Maximally-Entangled Photons
  - Source produces $\pm\mathbf{r}$ photon pair with $\mathbf{r}$ completely random

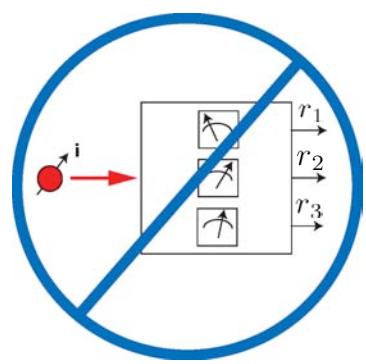
    $$\Pr(\text{photon } 1 \ = \pm\mathbf{r}_m) = \Pr(\text{photon } 2 = \mp\mathbf{r}_m) = 1/2$$

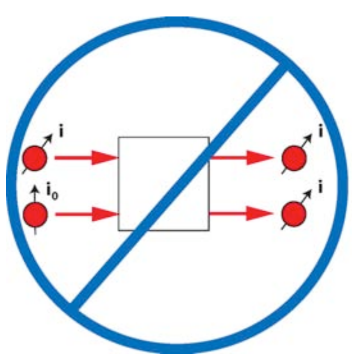
  - Conditional probability given photon 1 is $\mathbf{r}_m$ instead of $-\mathbf{r}_m$

    $$\Pr(\text{ photon } 2 \ = -\mathbf{r}_m \mid \text{photon } 1 \ = \mathbf{r}_m\,) = 1$$

Properties of Single-Photon Polarization States

- Polarization cannot be perfectly measured →



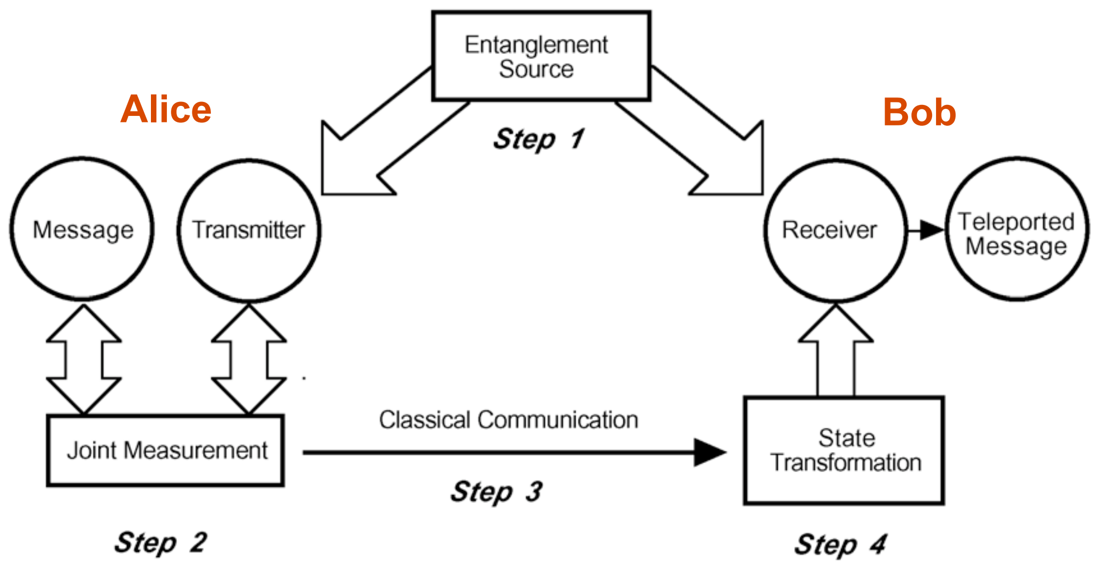


- ← Polarization cannot be perfectly cloned

- Photons can be lost in propagation:

  $$\Pr(\text{photon loss in 50 km of low-loss fiber}) = 0.9$$

Photon Polarization States Can Be Teleported



The Road Ahead: Problem Set 1, Lectures 2 and 3

- Problem Set 1
  - Reviews of essential probability theory and linear algebra
- Lectures 2 and 3:
  Fundamentals of Dirac-Notation Quantum Mechanics
  - Quantum systems
  - States as ket vectors
  - State evolution via Schrödinger's equation
  - Quantum measurements — observables
  - Schrödinger picture versus Heisenberg picture