

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering & Computer Science  
**6.041/6.431: Probabilistic Systems Analysis**  
(Spring 2006)

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**Tutorial 07 Answers**  
**April 6-7, 2006**

1. This is a simple recurrence relation. Let  $\alpha_k$  be your expected gain after  $k$  rounds. Clearly, we have:

$$\alpha_k = \alpha_{k-1}(2 - 4p + 4p^2) = \alpha_1(2 - 4p + 4p^2)^k$$

and therefore your expected worth after  $n$  rounds is:

$$X \cdot (2 - 4p + 4p^2)^{n+1}$$

2. (a)  $X$  and  $Y$  are not independent.  
(b)  $Y$  and  $Z$  are independent.  
(c) N/A.  
(d) N/A.

3. (a)

$$M_X(s) = \sum_{k=0}^{78} \binom{78}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{78-k} e^{(ks^2/8)+ks}.$$

- (b)

$$M_X(s) = \left(\frac{1}{2} + \frac{1}{2} \left(e^{\frac{s^2}{8}+s}\right)\right)^{78}.$$