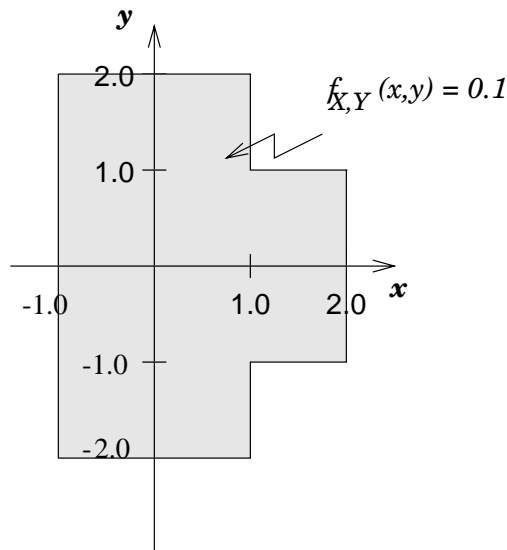


**Recitation 08**  
**March 09, 2006**

1. Random variables  $X$  and  $Y$  have the joint PDF shown below:



- (a) Prepare neat, fully labeled sketches of  $f_X(x)$ ,  $f_Y(y)$ ,  $f_{Y|X}(y|x)$  and  $f_{X|Y}(x|y)$ .
  - (b) Are  $X$  and  $Y$  independent?
  - (c) Find  $f_{X,Y|A}(x,y)$ , where the event  $A$  corresponds to points  $(x,y)$  within the unit circle centered at the origin.
  - (d) Find  $\mathbf{E}[X|Y = y]$  and  $\text{var}(X|Y = y)$ .
2. Alexei is vacationing in Monte Carlo. The amount  $X$  (in dollars) he takes to the casino each evening is a random variable with a PDF of the form

$$f_X(x) = \begin{cases} ax & \text{if } 0 \leq x \leq 40 \\ 0 & \text{otherwise} \end{cases}$$

At the end of each night, the amount  $Y$  that he has when leaving the casino is uniformly distributed between zero and twice the amount that he came with.

- (a) Determine the joint PDF  $f_{X,Y}(x,y)$
- (b) What is the probability that on a given night Alexei makes a positive profit at the casino?
- (c) Find the PDF of Alexei's profit  $Y - X$  on a particular night, and also determine its expected value.