

Recitation 5 Solutions
February 28, 2006

1. Problem 2.22, page 123 in the text. See online solutions.
2. Total expectation follows easily from total probability. This could be a good time to point out that the Total Probability Theorem and Total Expectation Theorem each have versions phrased with (a) conditioning on events forming a partition; and (b) conditioning on a discrete random variable. These are equivalent because the collection of events $\{Y = y\}$ over all y is a partition. You could also point out that technically, when we write

$$\mathbf{E}[X] = \sum_y p_Y(y) \mathbf{E}[X | Y = y]$$

we better only include in the summation y such that $\mathbf{P}(Y = y) > 0$.

3. The result follows by rewriting the expectation summation in the following manner:

$$\begin{aligned} \mathbf{E}[X] &= \sum_{k=0}^{\infty} k p_X(k) = \sum_{k=1}^{\infty} \left(\sum_{\ell=1}^k 1 \right) p_X(k) = \sum_{\ell=1}^{\infty} \sum_{k=\ell}^{\infty} p_X(k) \\ &= \sum_{\ell=1}^{\infty} \mathbf{P}(X > \ell - 1) = \sum_{n=0}^{\infty} \mathbf{P}(X > n). \end{aligned}$$

The manipulations could look unmotivated, but if you sketch the k - ℓ plane, then the interchange of summations is clear.