Problem 1. [Cooperation over social network]
Consider a social network, in which agents are matched pairwise at each date according to a matrix of probabilities P (where all entries of P are strictly positive). Once matched, each pair plays the prisoner's dilemma game with the following payoffs.

prisoner 1 / prisoner 2	Cooperate	Defect
Cooperate	1, 1	-1, 2
Defect	2, -1	0, 0

Table 1: Prisoner's Dilemma

1. Consider trigger strategies in which each player chooses its actions against a specific opponent only according to the history of their own past interactions. Show that for any discount factor $\delta < 1$, there exist matching probabilities such that players will not be able to support cooperation with more than one other player.

2. Suppose now that strategies are function of the specific opponents entire history (against each of his past opponents). In this case, show that cooperation against all players can be supported for $\delta \geq 1/2$.

Problem 2. [Network effects]
Consider the following dynamic game among N players. At each date, all players simultaneously choose whether to use a new technology. If in the previous state, at least one **other** person used the technology, the payoff from using the technology is 1. Otherwise it is $-\gamma$ (this includes the payoff of using it in the initial date, since the technology was not used by anybody prior to the beginning of the game). The payoff from not using the technology is always equal to 0. Each player discounts the future at the rate δ.

1. Show that if in the initial date a single player, say player i, has used the new technology, then all other players will use the new technology at all future dates. What about player i?

2. Now consider a situation in which no individual has used the technology in the initial period. Show that if δ is sufficiently close to 1, an individual can use the technology, induce everybody else to use this technology and obtain higher payoff than the situation in which the technology is not used.

3. Finally, show that there exists a symmetric mixed strategy equilibrium in which each player uses the technology with probability p at each date if in the previous state the technology was not used. Write down the expression that gives p.

Problem 3. [Bertrand competition under imperfect information] Consider the Bertrand game between two firms. There is one unit of demand with reservation price R ($R \geq 1$) and each firm has marginal cost of production $c_i, i = 1, 2$ drawn uniformly at random from $[0, 1]$. Find the symmetric Bayesian-Nash equilibrium of this game.

Problem 4. [Herding with crazy types] Consider the basic herding model, where a sequence of agents are choosing between Chinese and Indian restaurants. Assume that the Chinese restaurant is the better one with probability 1/2. Each agent is one of two types. Regular types have probability α and crazy types that have probability $1 - \alpha$. Regular types receive a binary signal indicating whether the Chinese or Indian restaurant is higher quality that is correct with probability $p > 1/2$. Crazy types randomly choose one of the two restaurants regardless of history of past choices. Show that if the first n players choose the Chinese restaurant there will be a herd in which all regular types, regardless of their signal, choose the Chinese restaurant. Characterize n as a function of α and p (It suffices to obtain the inequality that n has to satisfy).

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