

**Massachusetts Institute of Technology**  
**Biological Engineering Division**  
**Department of Mechanical Engineering**  
**Department of Electrical Engineering and Computer Science**

**BEH.410/2.798J/6.524J/10.539J**  
**Molecular, Cellular & Tissue Biomechanics, Spring 2003**

**First quiz**

**Wednesday, April 9, 2003**  
**7:00 – 9:00 pm**

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You will be given two hours to complete the exam.

You may use any of the following materials:

Class notes

Other materials distributed in class

Any of the reference texts

Calculator

If you have any questions, don't hesitate to ask us.

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There are three problems on the exam with the points as indicated.

Please be sure to explain your reasoning as fully as possible.

Write everything in the exam booklet, not on the exam.

Solutions will be available as you leave.

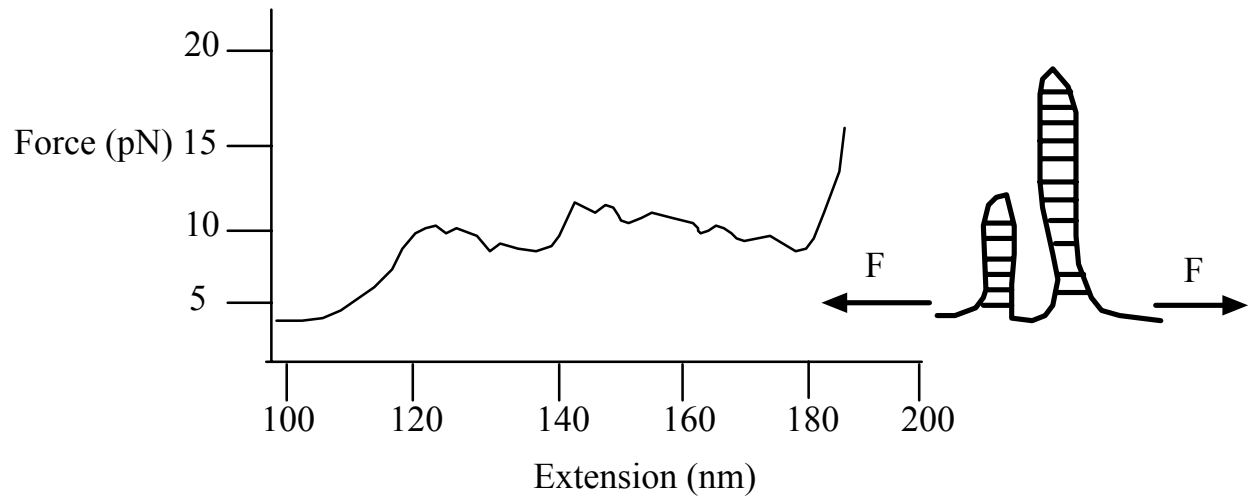
## Problem #1 Biopolymer Mechanics (40 points total)

Recently Maier and Radler (*Phys. Rev. Lett.* **82** 1911, 1999) have adsorbed double stranded DNA onto a lipid bilayer. One use of the system is to enable fundamental studies of polymer dynamics in reduced dimensions. The bilayer is mobile and so we can think about this system as a **flexible polymer undergoing a random walk in two dimensions (2-D)**.

- a) (10pts) Treat the adsorbed DNA molecule as an **ideal** Gaussian chain in 2-D. Derive the force–extension relationship for this model system ( $\langle F \rangle$  for a given end-to-end separation  $R$ ).
- b) (8pts) Does it take more or less force to extend a chain in 2-D versus 3-D to any given extension? **Explain why.**
- c) (7pts) Under what conditions is your answer in a) valid?
- d) (8pts) What will be the *qualitative* change in the average end-to-end distance of the polymer (no applied force) if the solution is heated from 25 to 50° C (for simplicity assume the DNA can be described by the freely-jointed chain mode)? Would you expect the same result for the large protein titin found in sarcomeres which we discussed in class? Recall that titin displayed a ‘saw-tooth’ force extension behavior due to domain unfolding. **Explain why.**
- e) (7pts) Suppose the DNA is denatured such that it is now single stranded. Does it take more or less force to extend a single stranded versus a double stranded DNA to any given extension? You can assume that both the double stranded and single stranded DNA behave as freely jointed chains. **Explain why.**

## Problem #2 RNA Thermodynamics (30 points total)

Optical tweezers are a useful tool to study the thermodynamics of single molecules. Consider the force-extension data achieved when the folded RNA depicted below is stretched. As shown schematically, the RNA has 2 folded hairpin regions which differ in size (small and large).



- (10pts) From the data *estimate* the free energy change (in kJ/mol) in going from completely folded to having just the small hairpin unfolded.
- (10pts) At equilibrium (no applied forces) what is the ratio of probability of having just the small hairpin unfold to the probability of being completely folded?
- (10pts) Now optical tweezers are used to apply a constant force of 5pN to the ends and the extension is monitored in time. For this experiment, what is the ratio of probability of having just the small hairpin unfold to the probability of being completely folded?

**PROBLEM 3: LINEAR ELASTICITY (30 Points)**

A common test used to measure the elastic properties of biological tissues is biaxial extension, in which a uniform normal stress is applied along four edges of a sheet-like tissue specimen (see Fig. 1). The front and back surfaces are stress-free.

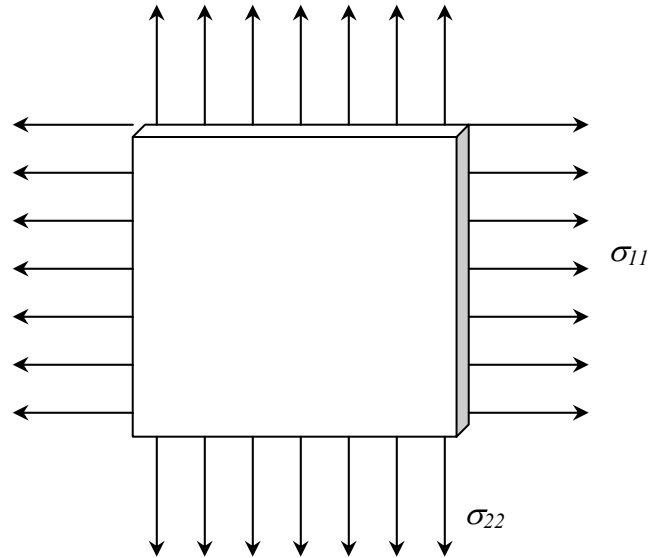


Fig. 1

- a) (10 pts) Given that a stress  $\sigma = \sigma_{11} = \sigma_{22} = \text{constant}$ , and that the material is isotropic, homogeneous, and linearly elastic with a Young's modulus  $E$  and Poisson ratio  $\nu$ , find expressions for the three diagonal elements of the strain tensor,  $\epsilon_{11}$ ,  $\epsilon_{22}$ , and  $\epsilon_{33}$ .

A biaxial extension test is often conducted on tissues that are sheet-like in vivo. For example, oftentimes a segment of an excised artery or vein is sliced open and laid flat prior to biaxial testing. In such cases, it is important to know the state of internal stress experienced by the tissue in the flattened state, prior to the application of stress as in Fig. 1.

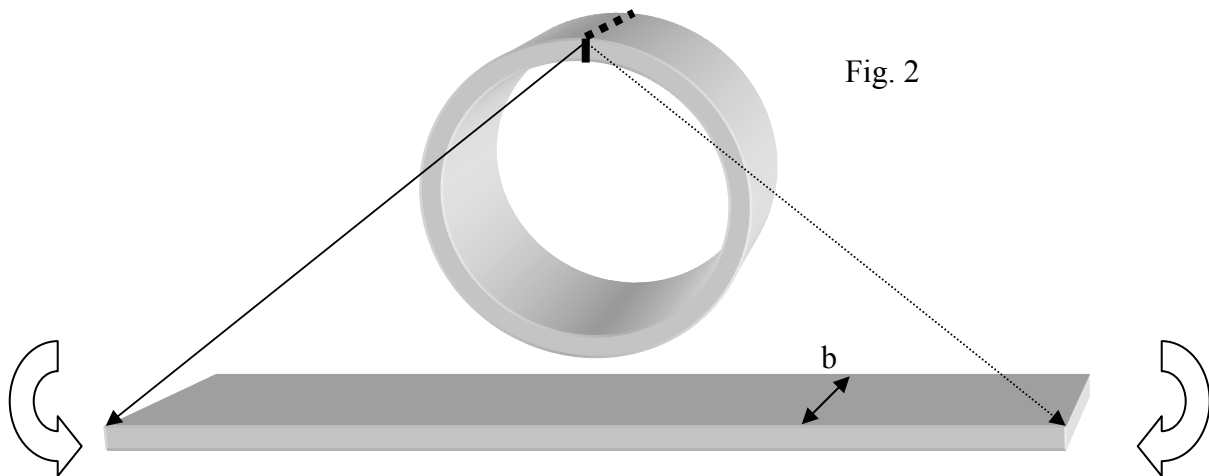


Fig. 2

b) (8 pts) Given that the initial internal radius of the vessel is  $R$ , its axial dimension is  $b$ , and its thickness is  $h$ , what moment must be applied to the specimen in order to lay it flat (Fig. 2)? (For this part of the problem, you may assume that the vessel is initially stress-free in the cylindrical configuration and that all stresses are introduced due to bending moments applied at the edges.)

Now consider the vessel in its normal, inflated state in vivo (Fig. 3a). Experiments have shown that, due to remodeling of the vascular wall, circumferential stresses in this normal, inflated state are nearly uniform through the wall. As you found in your homework, this circumferential stress ( $\sigma_\theta$ ) equals  $PR/h$  where  $P$  is internal pressure (relative to external, ambient pressure),  $h$  is the wall thickness and  $R$  is the radius.

c) (7 pts.) Given that the circumferential stress with an internal pressure  $P_{max}$  (above atmospheric) is uniform, use physical reasoning to obtain a qualitative sketch of circumferential stress ( $\sigma_\theta$ ) as a function of radial position ( $r$ ) between the inner and outer walls when internal pressure is zero (equal to external pressure) (Fig. 3b). We are primarily interested in the *mean value* and whether the stress is *positive* (tensile) or *negative* (compressive). Provide your answer in the form of a qualitative plot of  $\sigma_\theta$  vs.  $r$  as shown in Fig. 3c.

d) (5 pts) When the vessel shown in Fig. 3b is sliced as in Fig. 2 so that no stresses or moments are applied to the cut ends, the ring has a tendency to change shape (either open up more or close down). How will the vessel geometry change, and why?

