Harvard-MIT Division of Health Sciences and Technology
HST.951J: Medical Decision Support, Fall 2005
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6.873/HST.951 Medical Decision Support
Fall 2005

Decision Analysis

(part 1 of 2)

Lucila Ohno-Machado

Outline

- Review Bayes rule
- Example of a decision problem: Knee injury
- Elements of a decision tree
- Conditional probabilities in a decision tree
- Expected value
- Value of information (value of tests)
- Sensitivity analysis
- Utilities
- Risk attitudes

Bayes Rule

Conditional Probabilities

- probability of **PPD-** given that patient has **TB** is 0.2
- This patient has **PPD-**
- What is the probability that he has **TB**?

2 x 2 table
(contingency table)

	PPD+	PPD-	
TB	**8**	**2**	**10**
no TB	**3**	**87**	**90**
	11	**89**	**100**

Probability of TB given PPD- = 2/89

Bayes rule

- Definition of conditional probability:
- $P(A|B) = P(AB)/P(B)$

$$P(B|A) = P(BA)/P(A)$$

$$P(AB) = P(BA)$$

$$P(A|B)P(B) = P(B|A)P(A)$$

$$\mathbf{P(A|B) = P(B|A)P(A)/P(B)}$$

Simple Bayes

Probability of PPD- given TB = P(PPD-|TB) = 0.2
Probability of TB = P(TB) = 0.1
Probability of PPD- = P(PPD-) = 0.89

$$P(TB|PPD-) = \frac{P(PPD-|TB)\ P(TB)}{P(PPD-)}$$

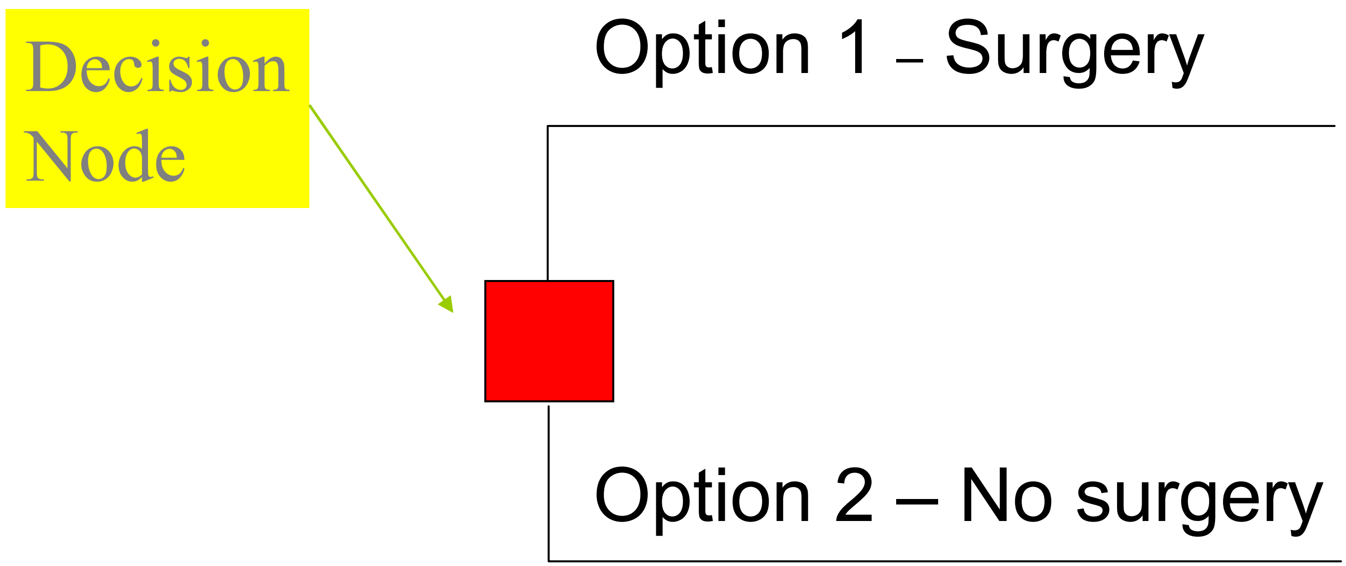
$$P(TB|PPD-) = \frac{(.2)\ (.1)}{(.89)}$$

	PPD+	PPD-
TB	8	2
no TB	3	87

Example of a Decision Problem

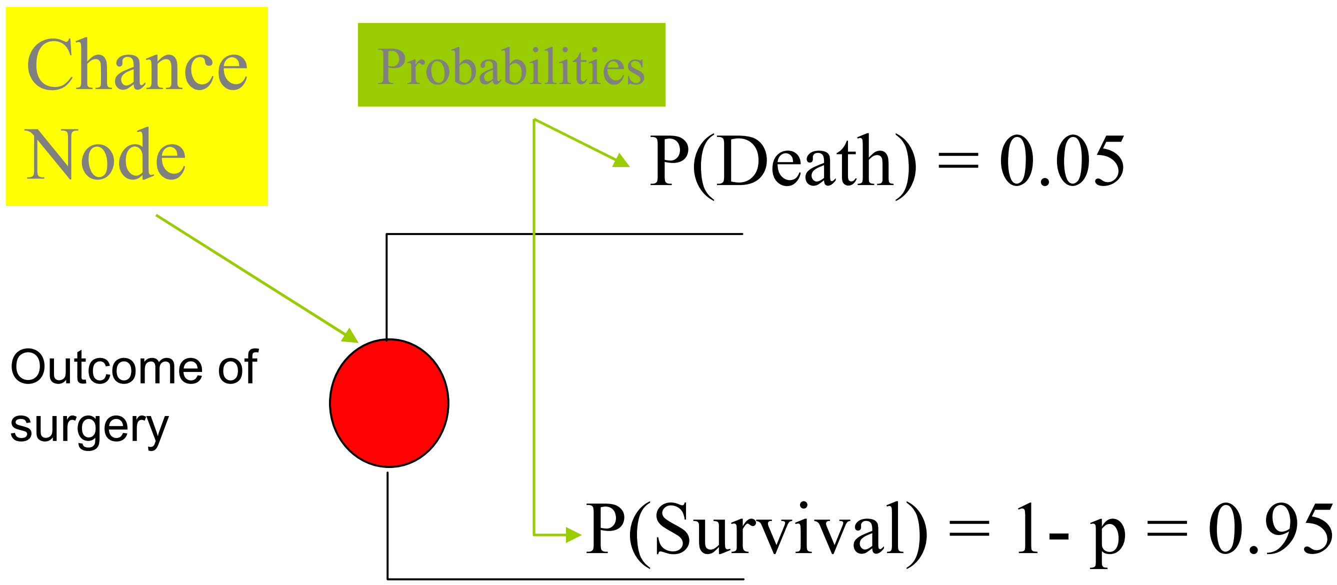
- College athlete considering knee surgery
- Uncertainties:
  - success in recovering perfect mobility
  - infection in surgery (if so, needs another surgery and may loose more mobility)
  - survive surgery

Decision Nodes (squares)



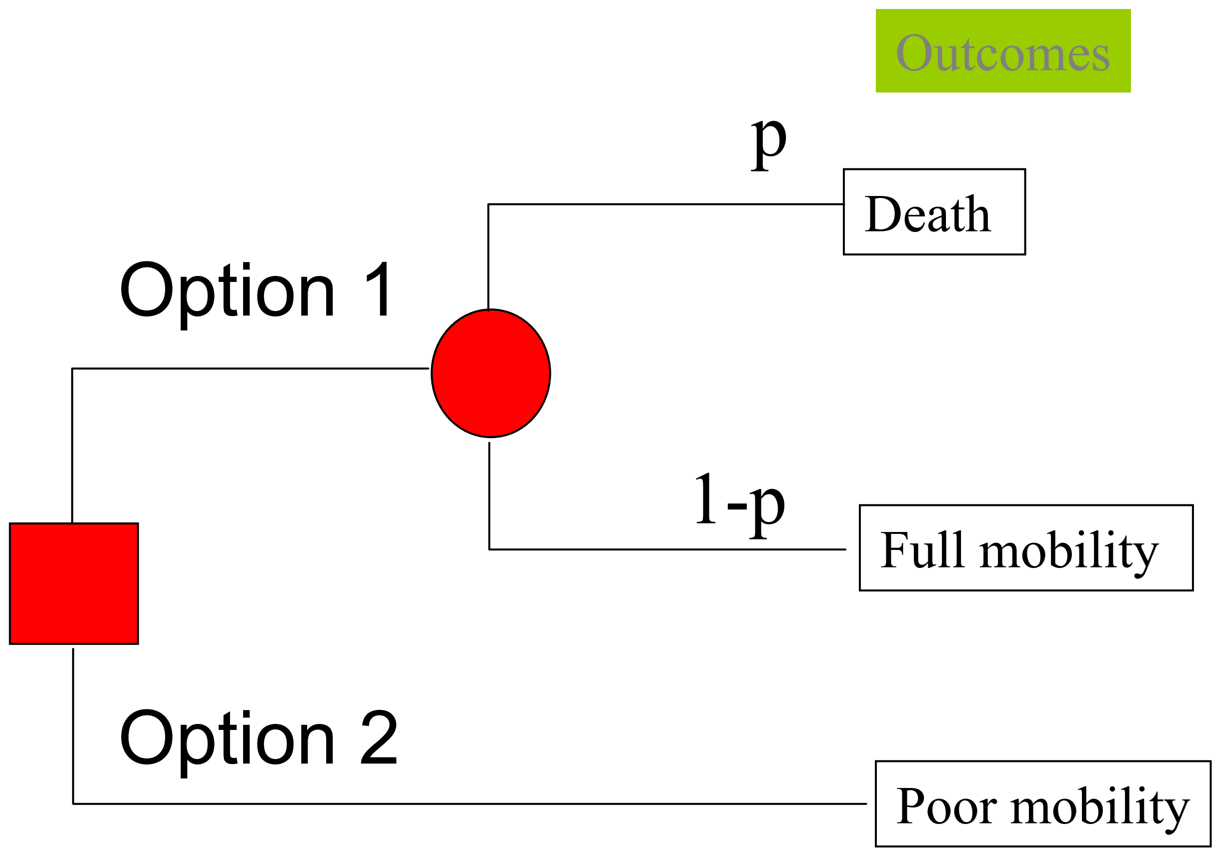
- Choices

Chance Nodes (circles)

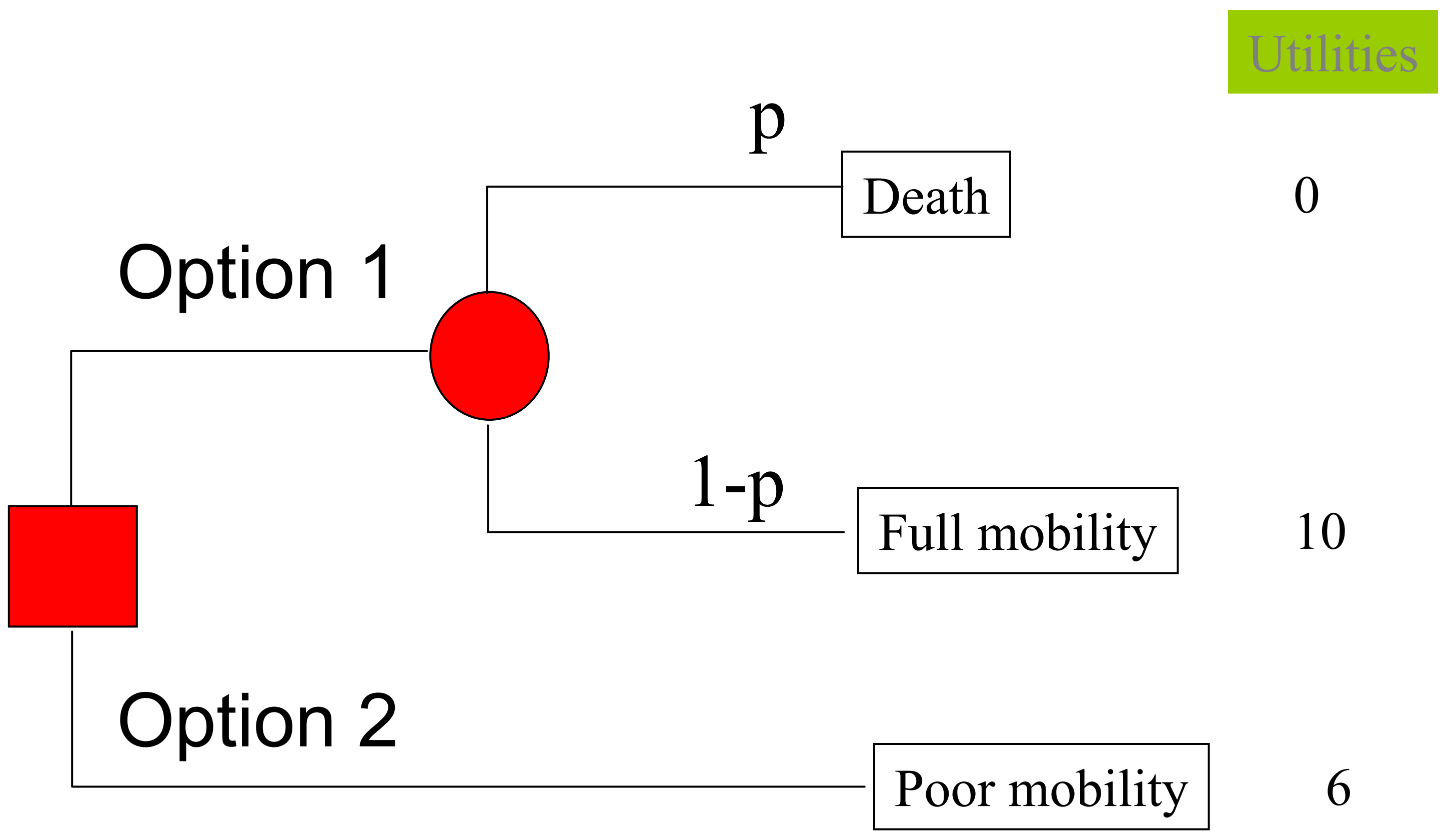


- Uncertain events
- Determined by complementary probabilities
- Mutually exclusive
- Collectively exhaustive

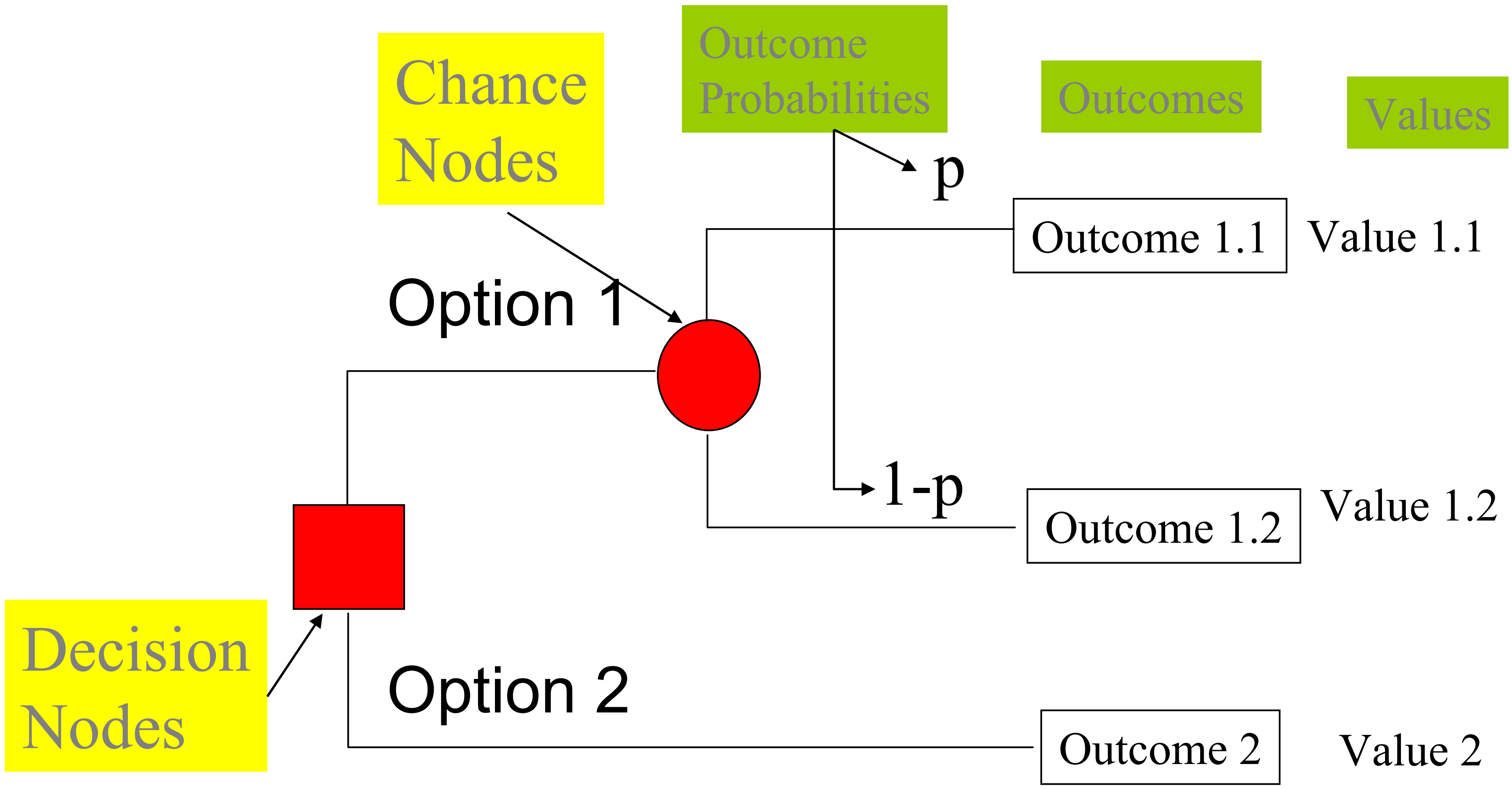
Outcomes


Outcomes
p
Death
Option 1
1-p
Full mobility
Option 2
Poor mobility

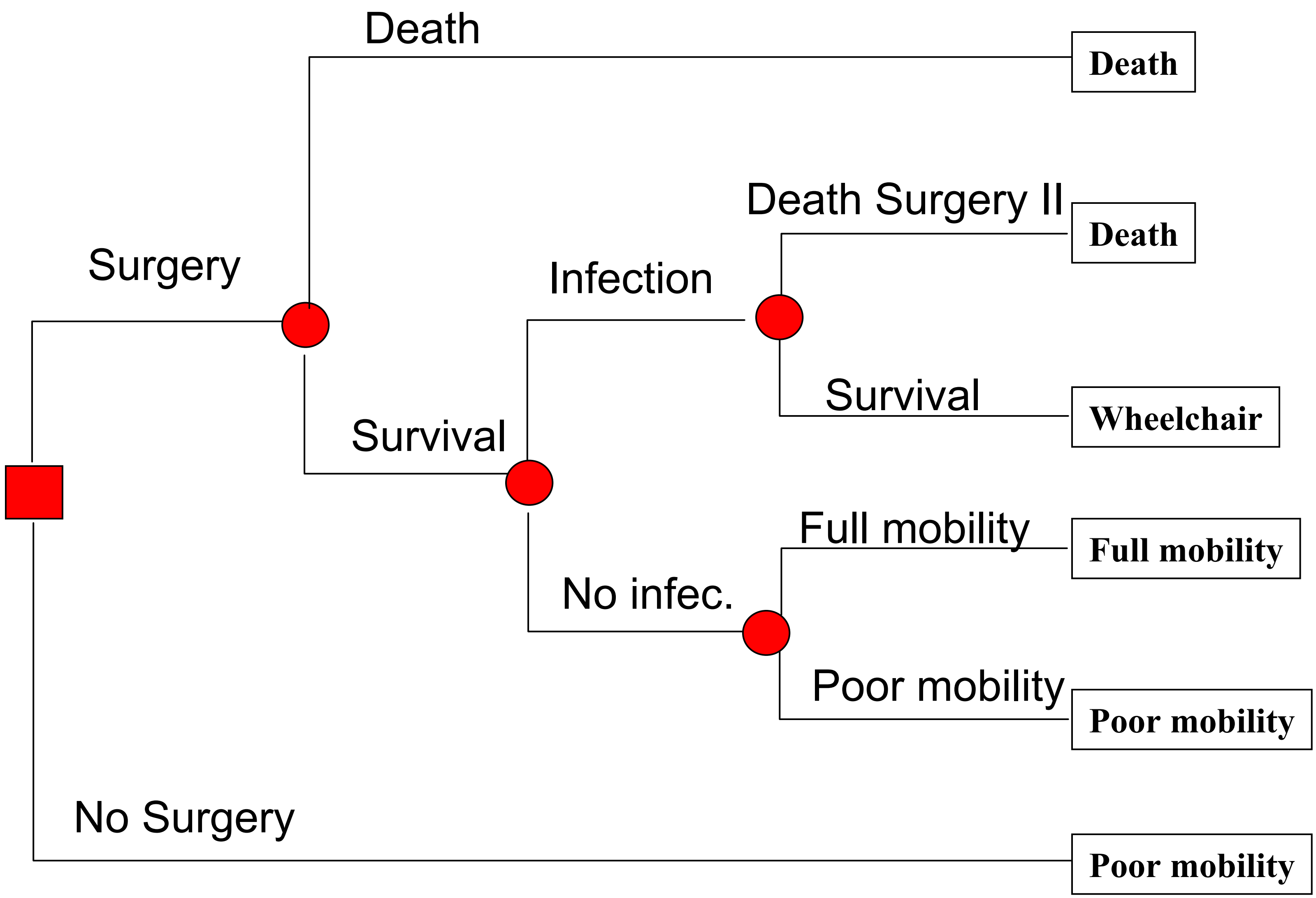
Values or Utilities (or Costs)



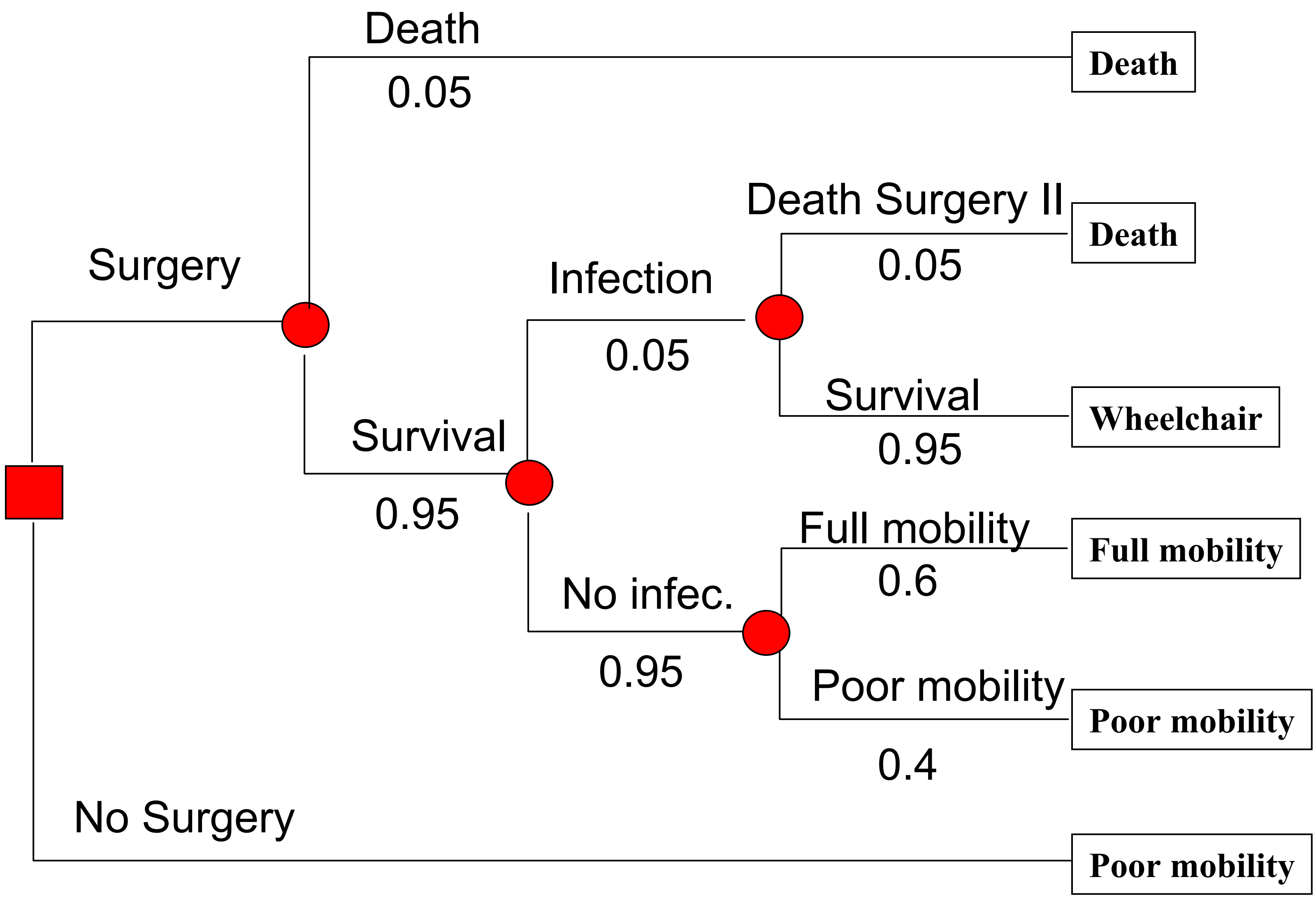
Elements of Decision Trees

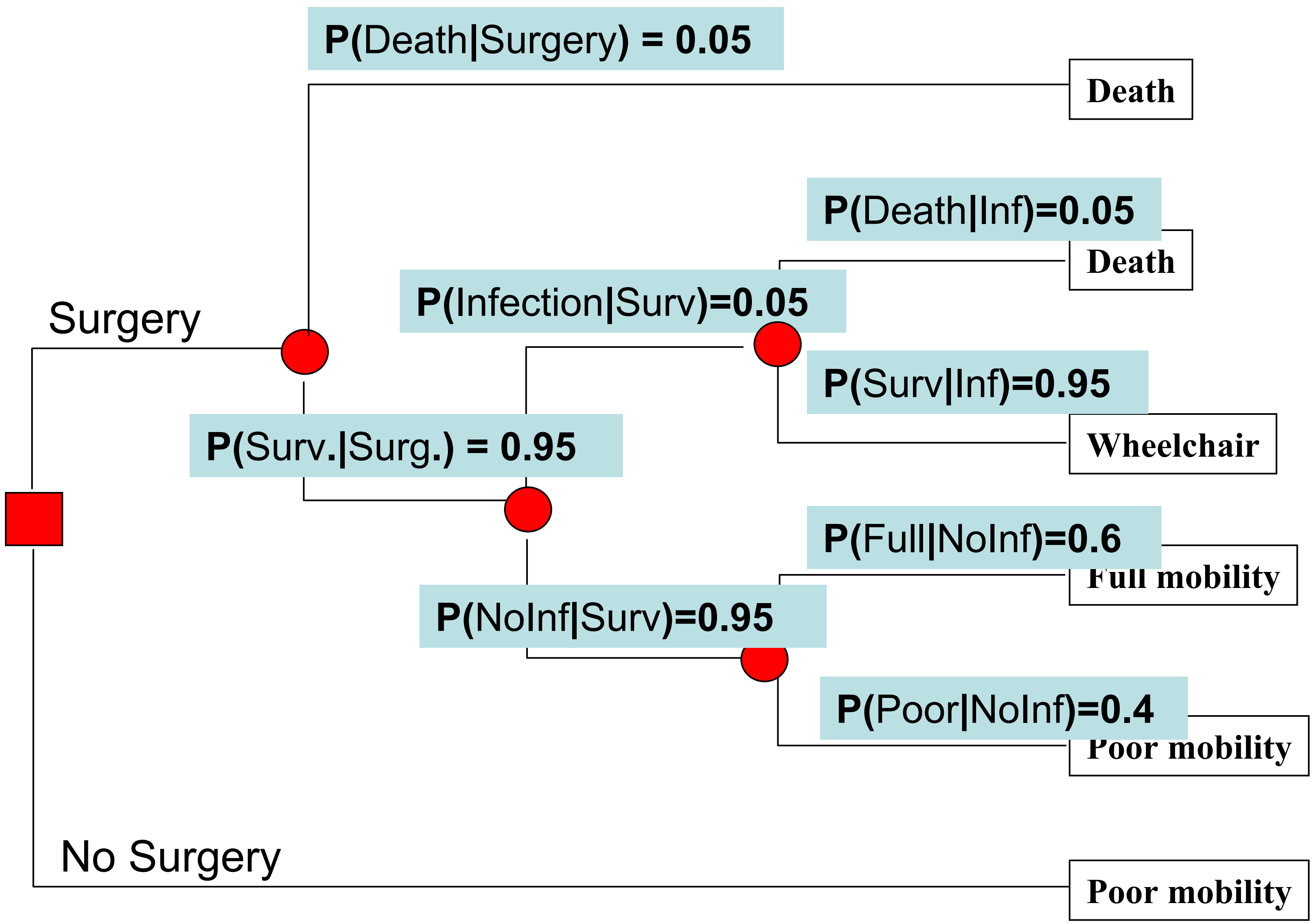


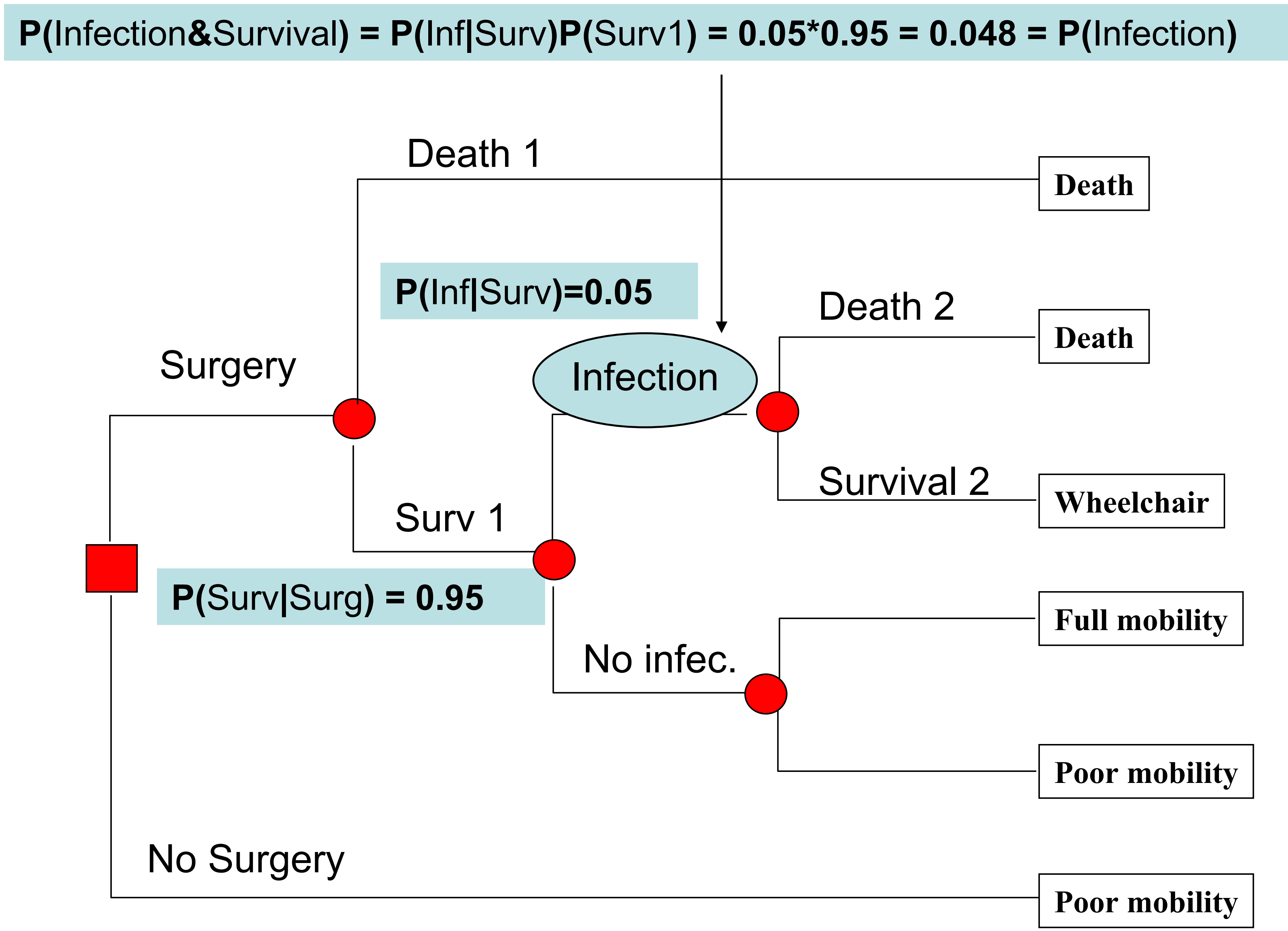
Knee Surgery Example



Assigning Probabilities

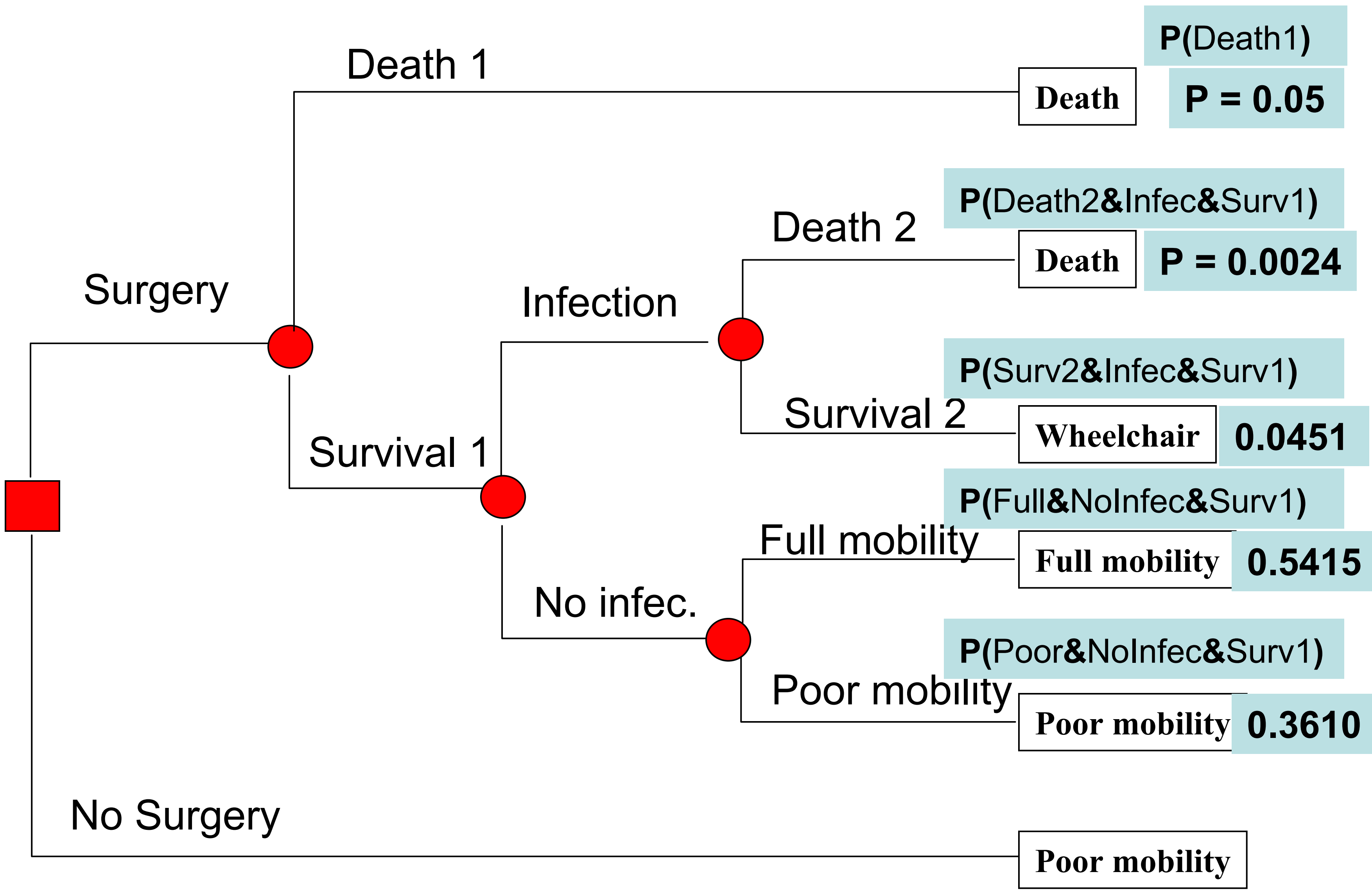



P(Death|Surgery) = 0.05
Death
P(Death|Inf)=0.05
Death
Surgery
P(Infection|Surv)=0.05
P(Surv|Inf)=0.95
P(Surv.|Surg.) = 0.95
Wheelchair
P(Full|NoInf)=0.6
Full mobility
P(NoInf|Surv)=0.95
P(Poor|NoInf)=0.4
Poor mobility
No Surgery
Poor mobility

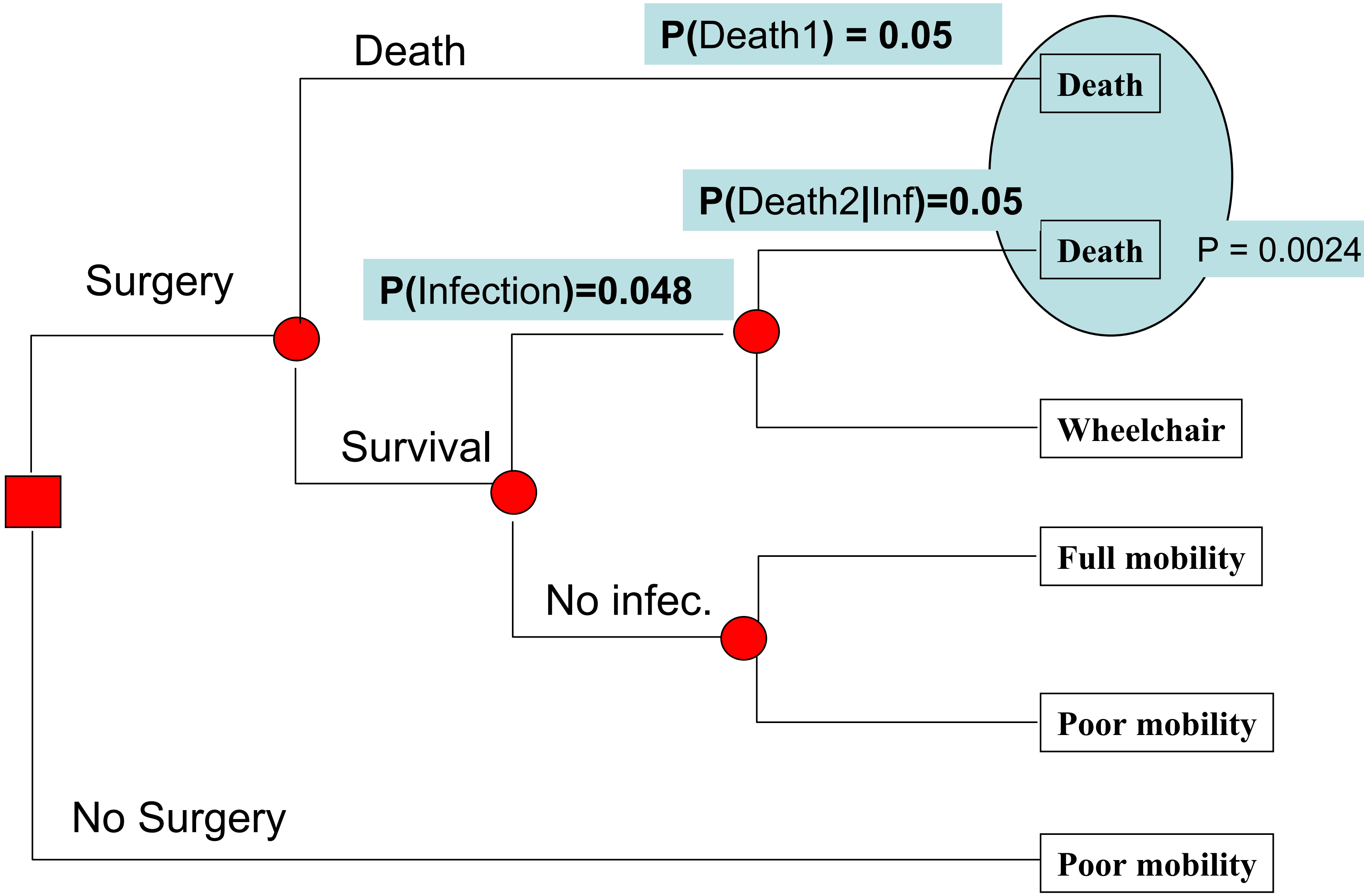


P(Infection&Survival) = P(Inf|Surv)P(Surv1) = 0.05*0.95 = 0.048 = P(Infection)
Death 1
Death
P(Inf|Surv)=0.05
Death 2
Death
Surgery
Infection
Survival 2
Wheelchair
Surv 1
P(Surv|Surg) = 0.95
Full mobility
No infec.
Poor mobility
No Surgery
Poor mobility

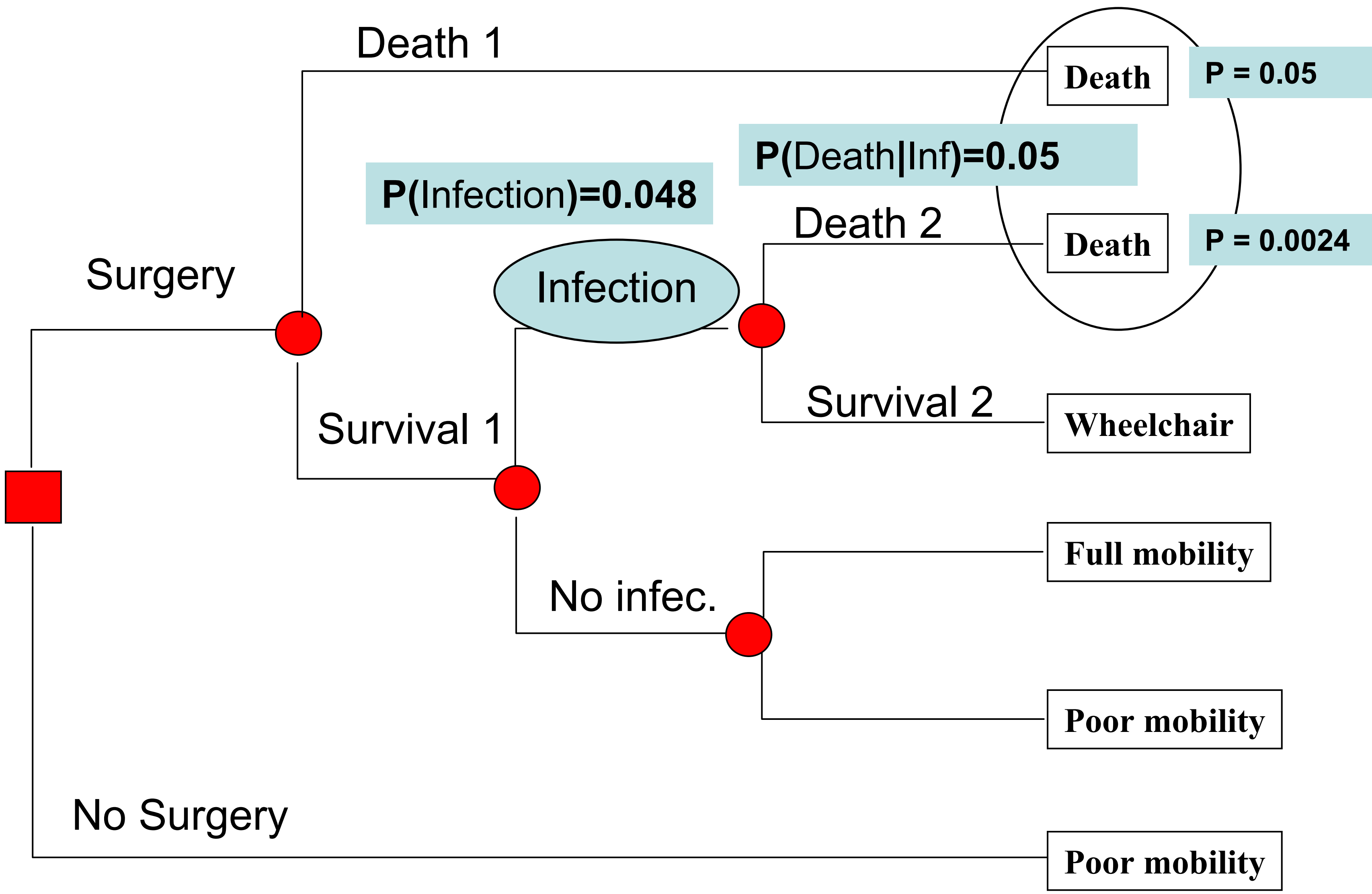
Joint Probabilities



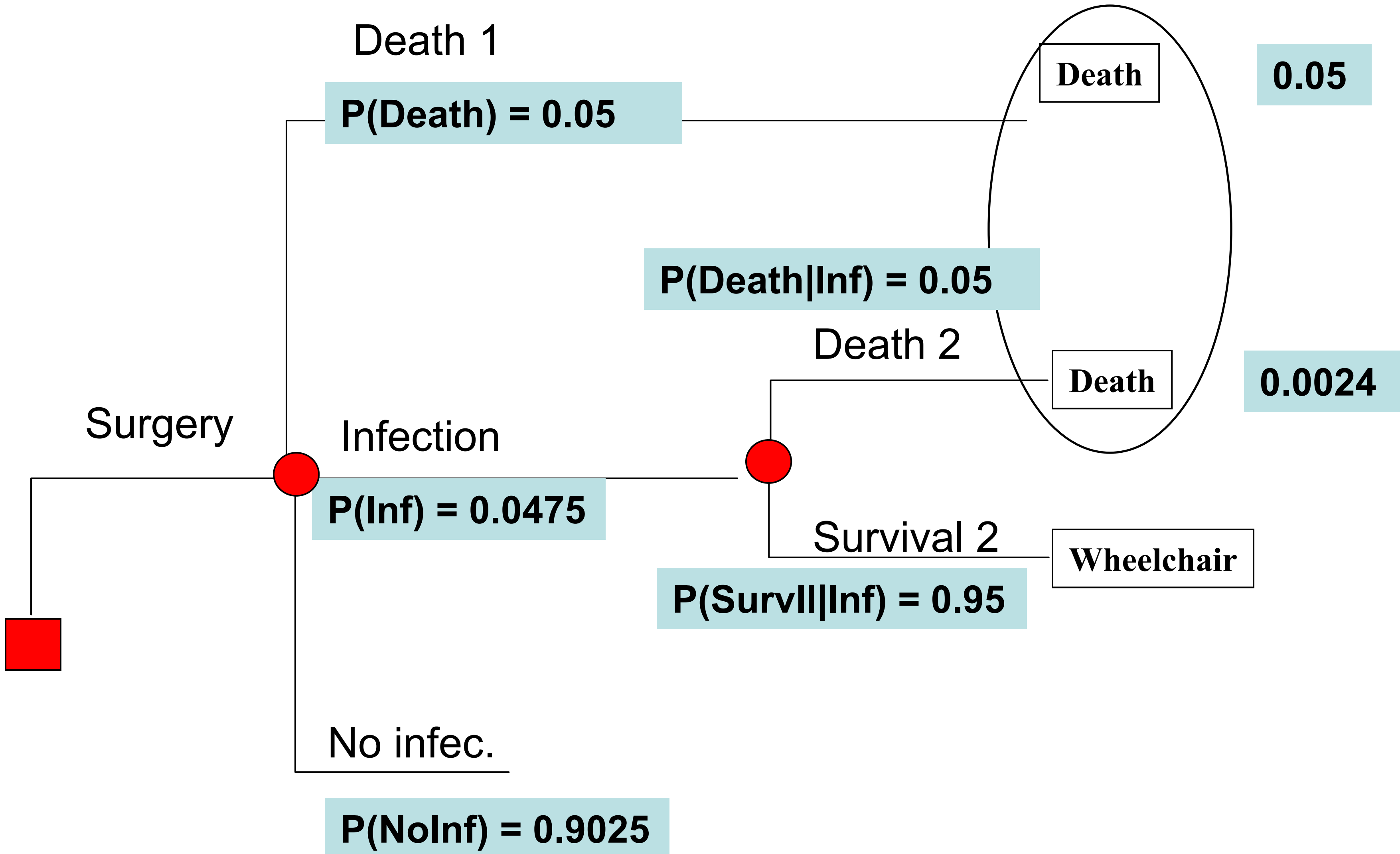
$$P(\text{Death}) = P(\text{Death1}) + P(\text{Death2}|\text{Inf})P(\text{Infection}) = 0.05+0.05*0.048 = 0.05+0.0024 = 0.0524$$



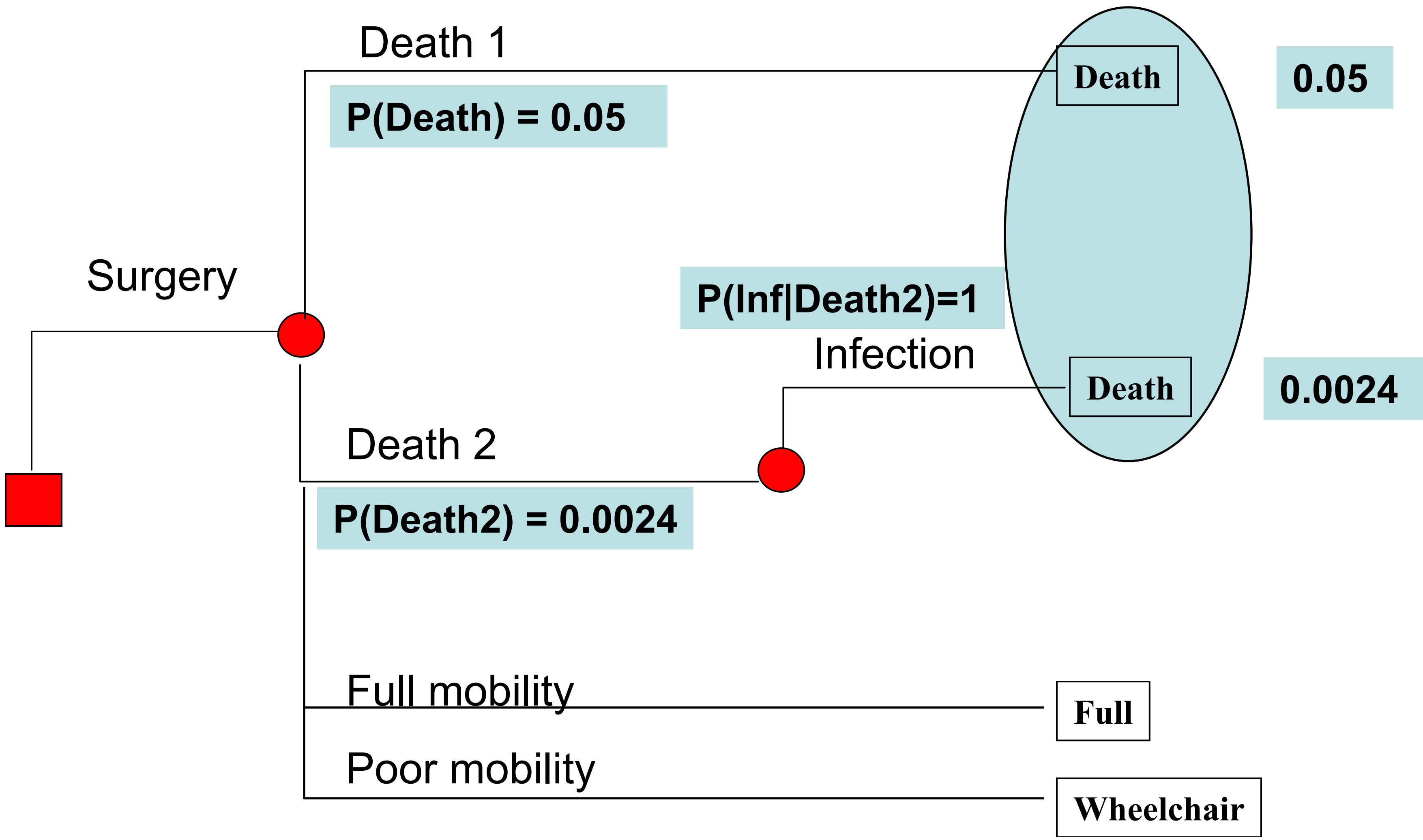
P(Infection|Death) **= P**(Death|Infection)***** **P**(Infection)**/P**(Death) **=**
= 0.05*0.048/0.0524 = 0.0024/0.0524 = 0.045



Simplifying the tree

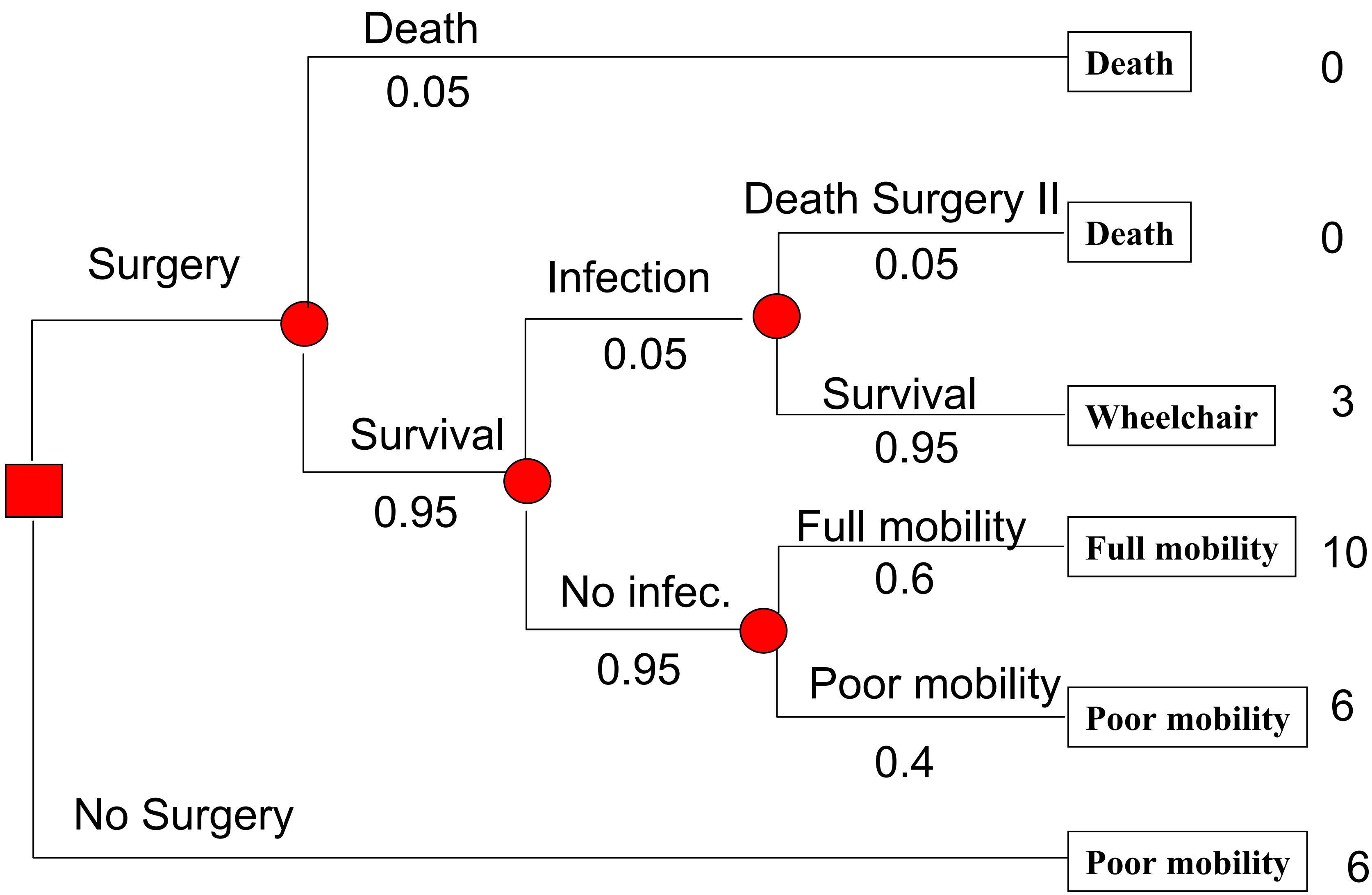


Alternative tree



Utilities - QALYs

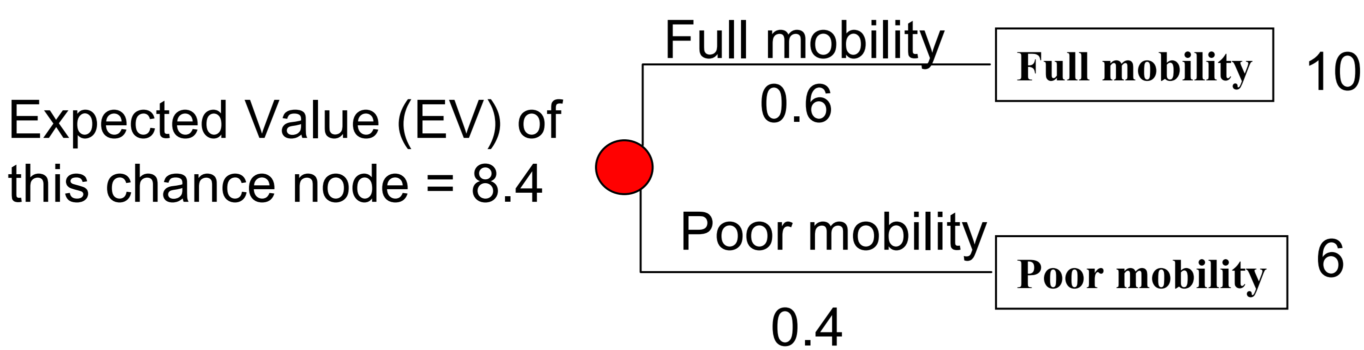
- Quality Adjusted Life Years
- How many years with problem are equivalent to years without problem
- E.g.:
  - x years with poor mobility are equivalent to y years with full mobility
  - x years wheelchair-bound are equivalent to y years of full mobility
- These are judgement calls that can represent an individual preference or a collective (societal) preference

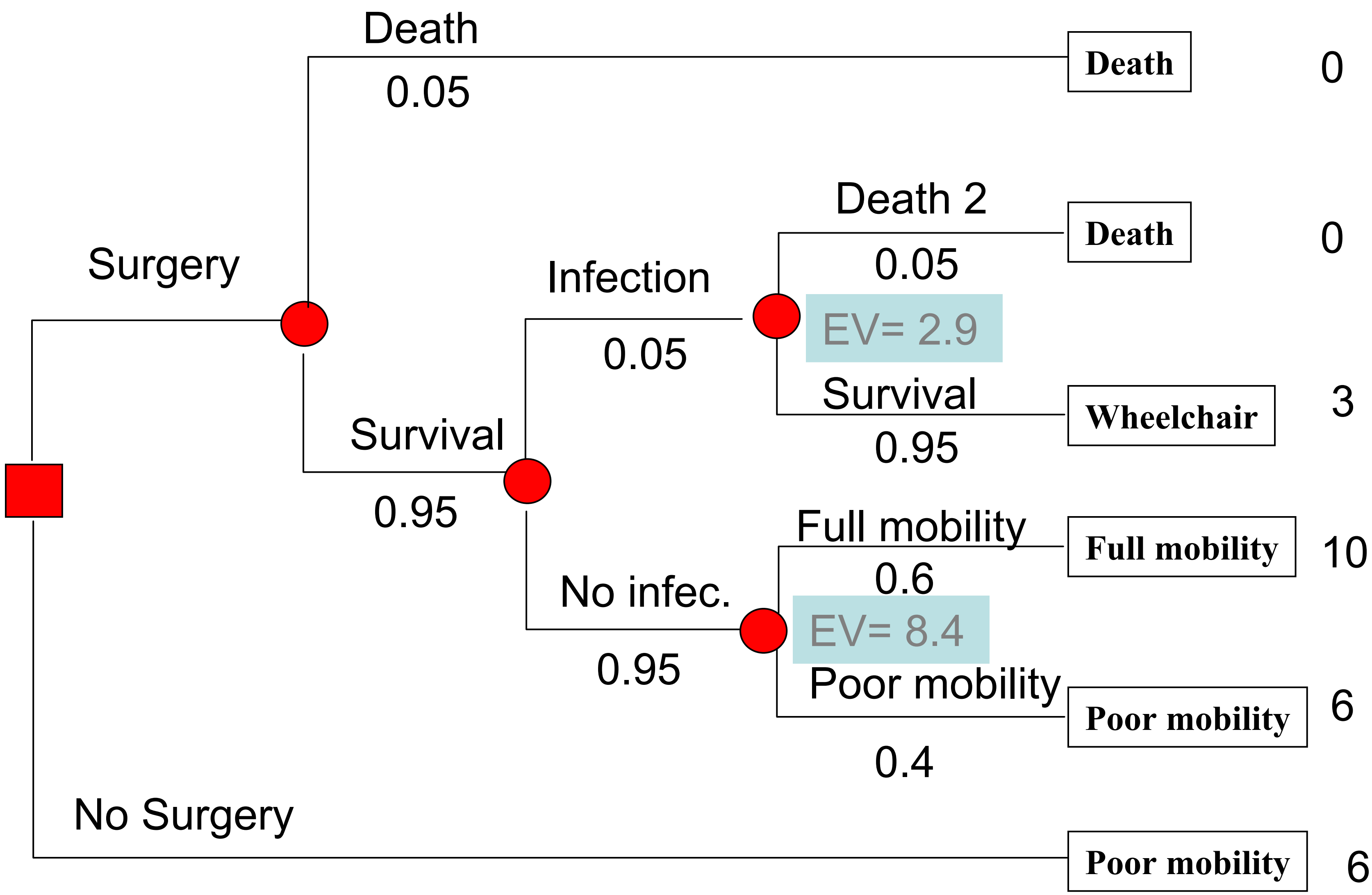

Death
0.05
Death
0
Death Surgery II
Death
0
Surgery
0.05
Infection
0.05
Survival
Wheelchair
3
Survival
0.95
0.95
Full mobility
Full mobility
10
0.6
No infec.
0.95
Poor mobility
Poor mobility
6
0.4
No Surgery
Poor mobility
6

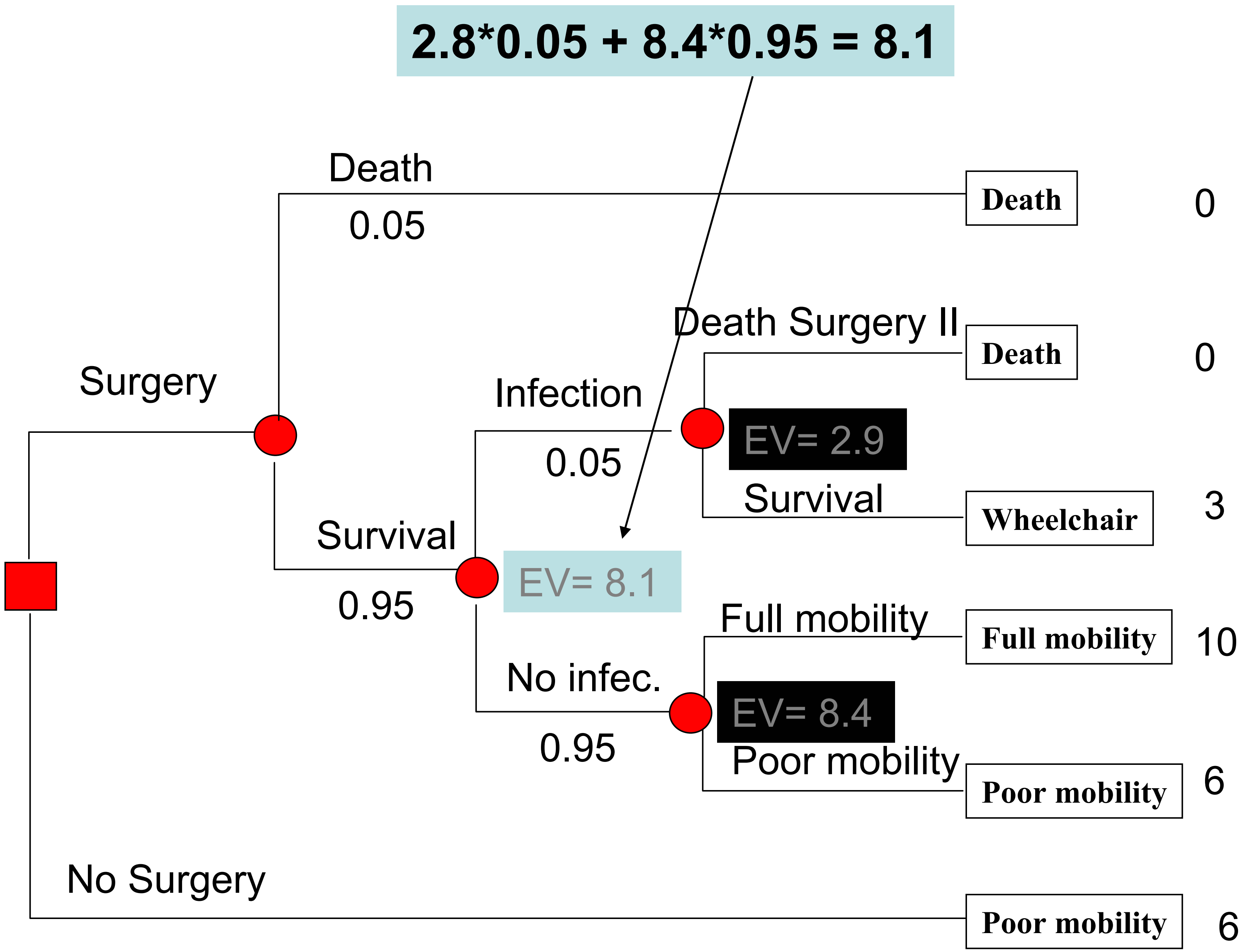
Expected Values

- Value of outcomes, weighed by the respective probability that they happen

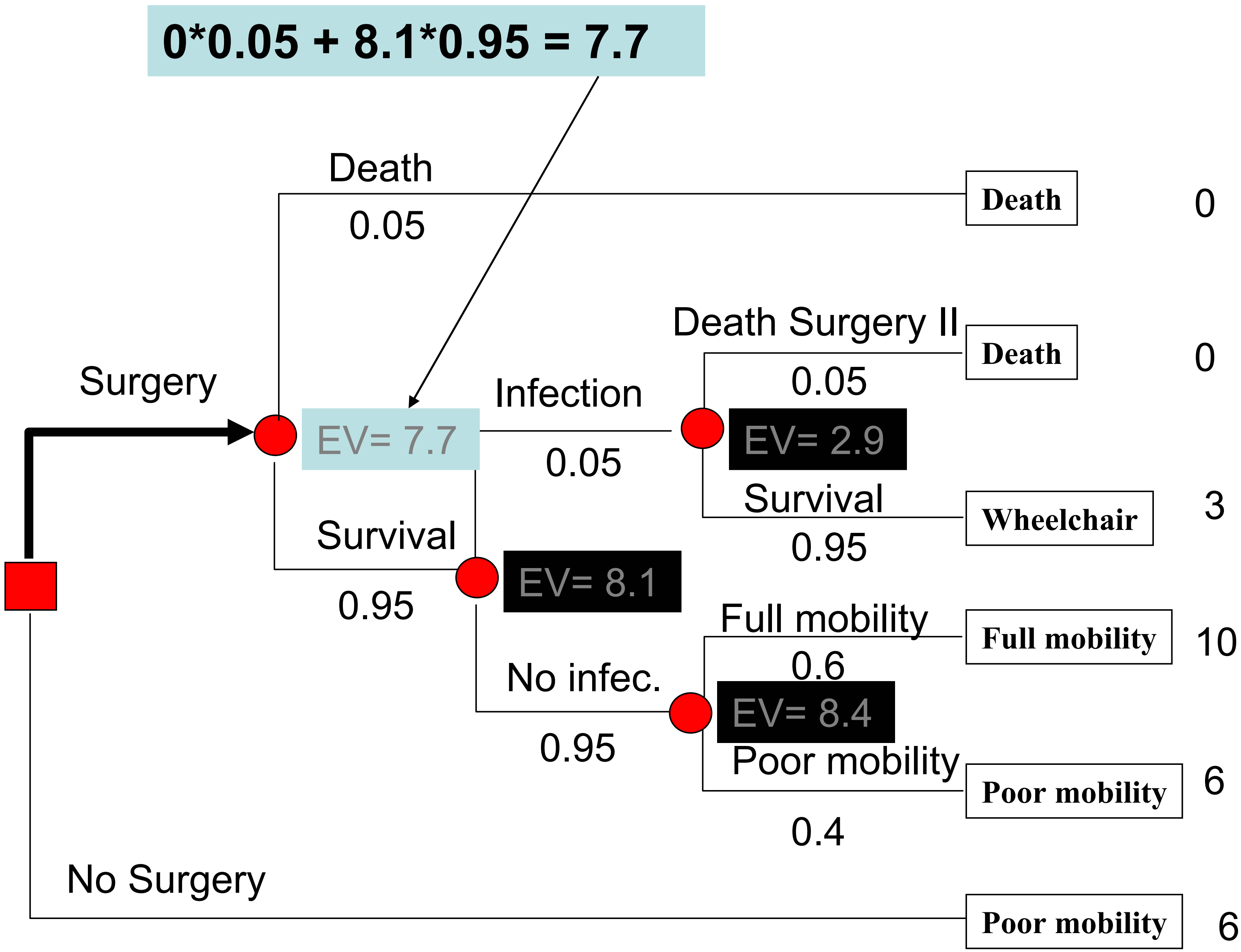
0.6*10 + 0.4*6 = 8.4



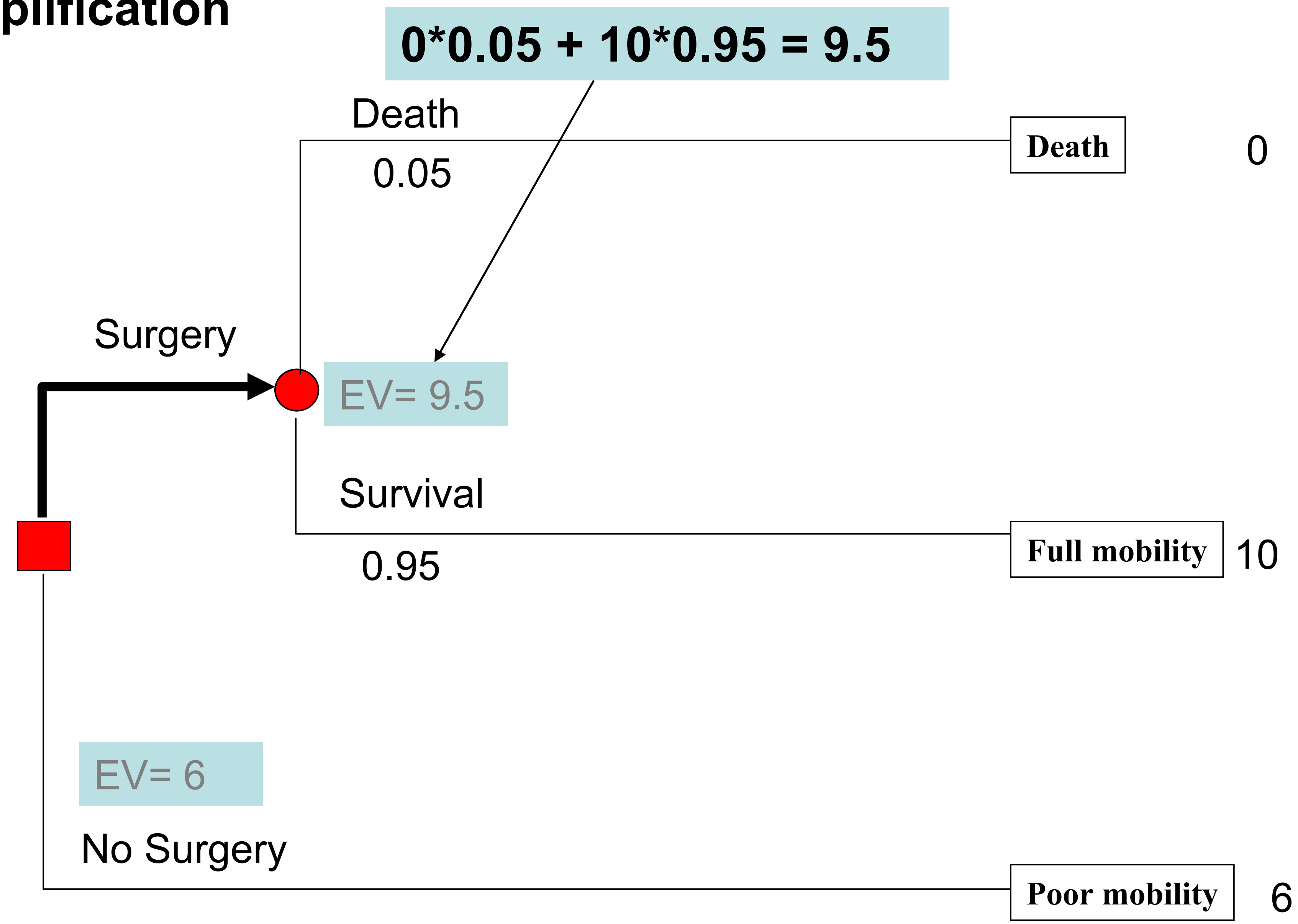

Death
0.05
Death
0
Surgery
Death 2
0.05
Death
0
Infection
0.05
EV= 2.9
Survival
0.95
Wheelchair
3
Survival
0.95
Full mobility
0.6
Full mobility
10
No infec.
0.95
EV= 8.4
Poor mobility
0.4
Poor mobility
6
No Surgery
Poor mobility
6



2.8*0.05 + 8.4*0.95 = 8.1
Death
0.05
Death
0
Death Surgery II
Death
0
Surgery
Infection
0.05
EV= 2.9
Survival
Wheelchair
3
Survival
0.95
EV= 8.1
Full mobility
Full mobility
10
No infec.
0.95
EV= 8.4
Poor mobility
Poor mobility
6
No Surgery
Poor mobility
6

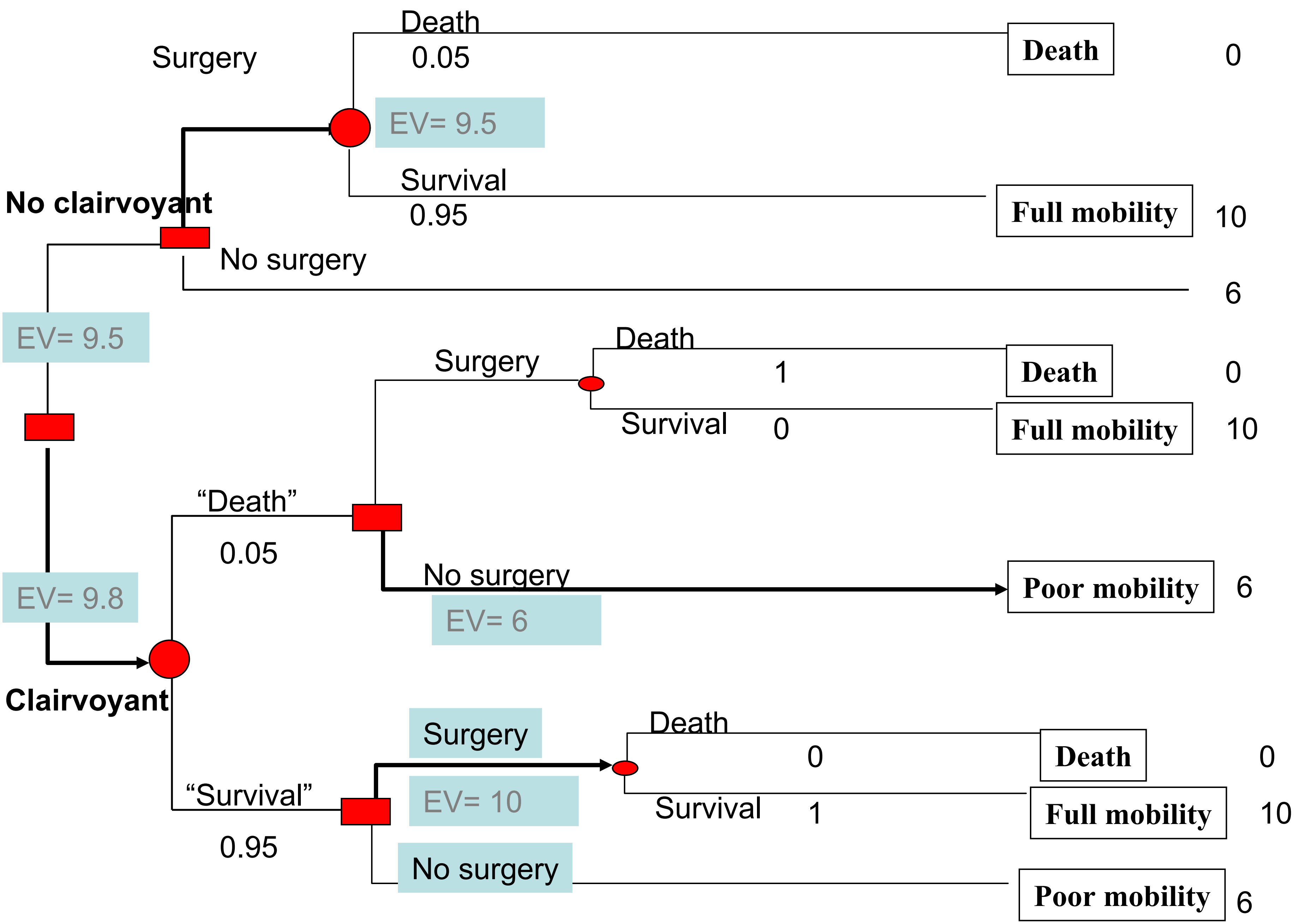

0*0.05 + 8.1*0.95 = 7.7
Death
0.05
Death
0
Death Surgery II
Death
0
Surgery
Infection
0.05
EV= 7.7
0.05
EV= 2.9
Survival
Wheelchair
3
Survival
0.95
0.95
EV= 8.1
0.95
Full mobility
Full mobility
10
0.6
No infec.
EV= 8.4
0.95
Poor mobility
Poor mobility
6
0.4
No Surgery
Poor mobility
6

Simplification

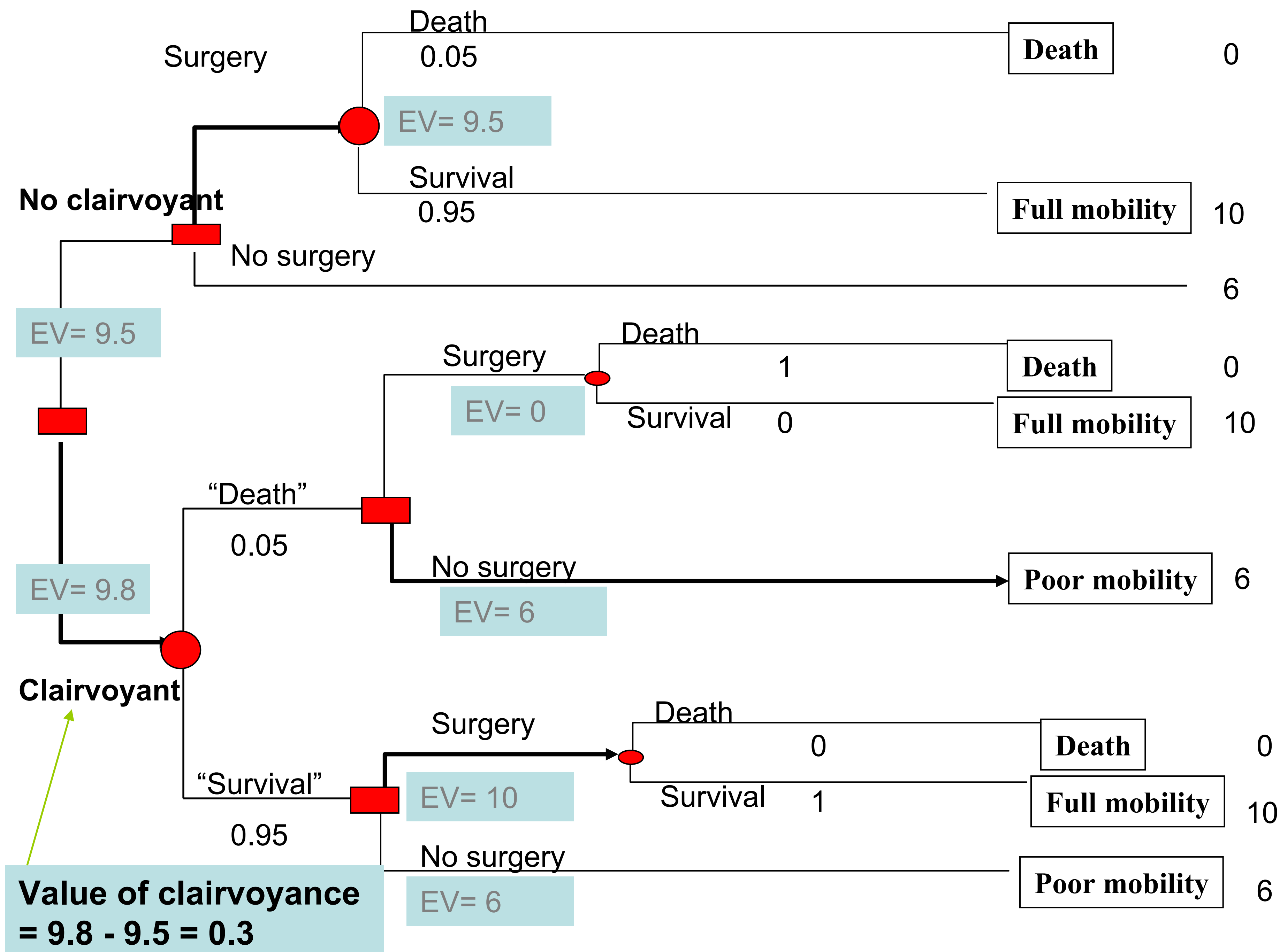


Value of Information

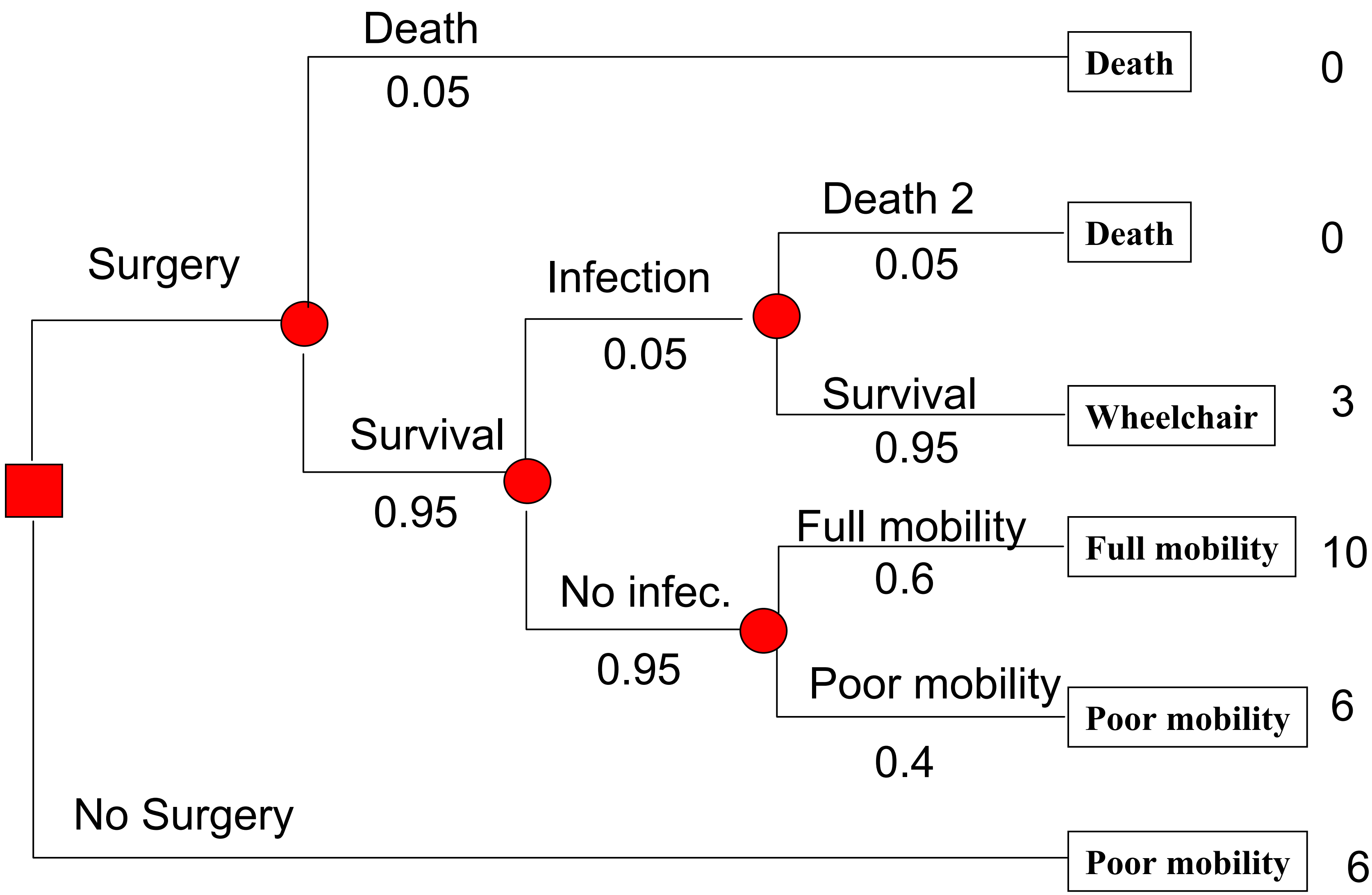
- Value of “Clairvoyance” (e.g. perfect prognostic system)
- If someone knows exactly what will happen if you make a certain decision, how much is that worth?
- E.g., if someone knows for sure whether the patient will die or survive following surgery, how much is that worth?
- It is usually calculated as the difference between the expected value with clairvoyance and without clairvoyance



Death
0.05
Surgery
EV= 9.5
Death
0
Survival
0.95
Full mobility
10
No clairvoyant
No surgery
6
EV= 9.5
Death
Surgery
1
Death
0
Survival
0
Full mobility
10
"Death"
0.05
No surgery
Poor mobility
6
EV= 6
EV= 9.8
Clairvoyant
Surgery
Death
0
Death
0
"Survival"
EV= 10
Survival
1
Full mobility
10
0.95
No surgery
Poor mobility
6

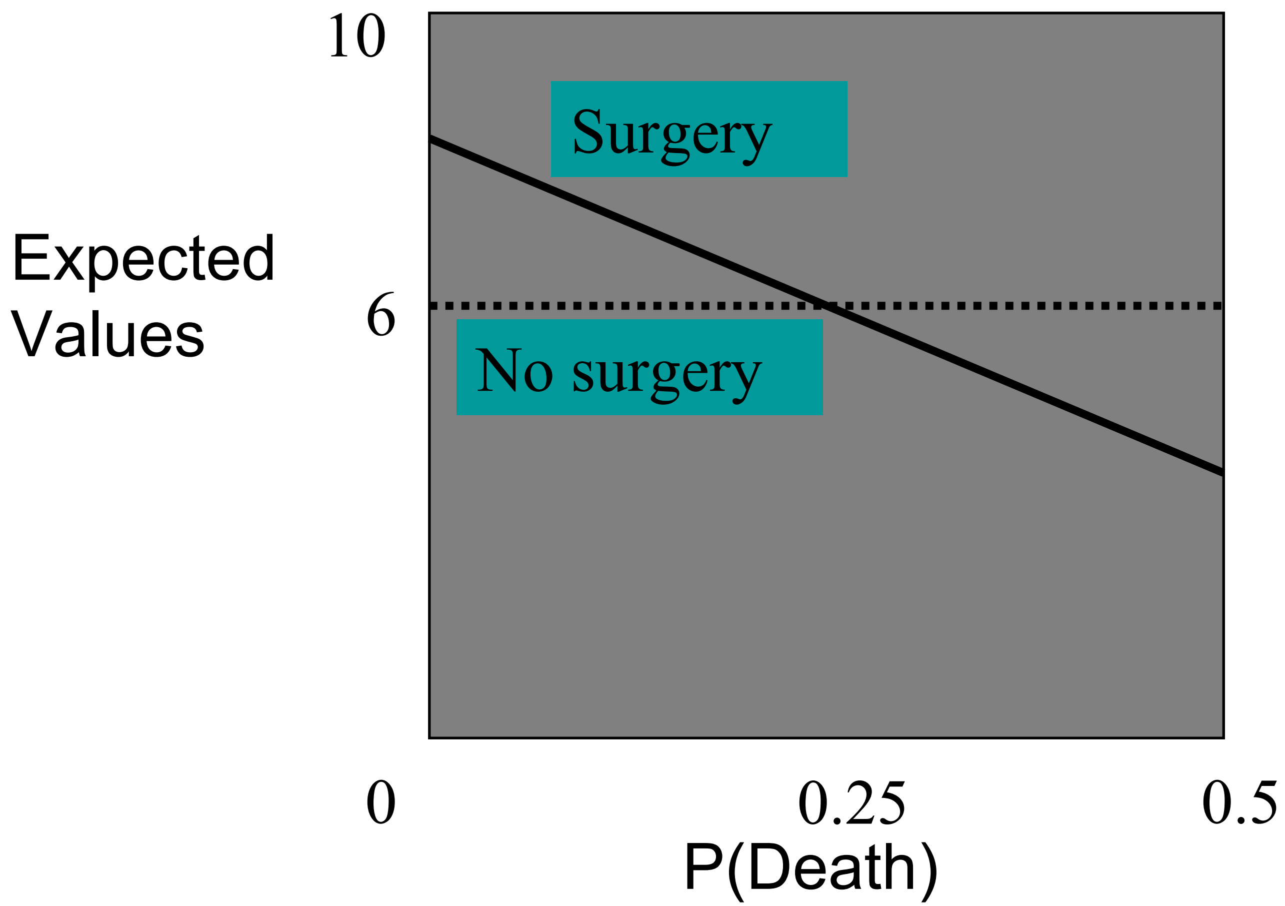


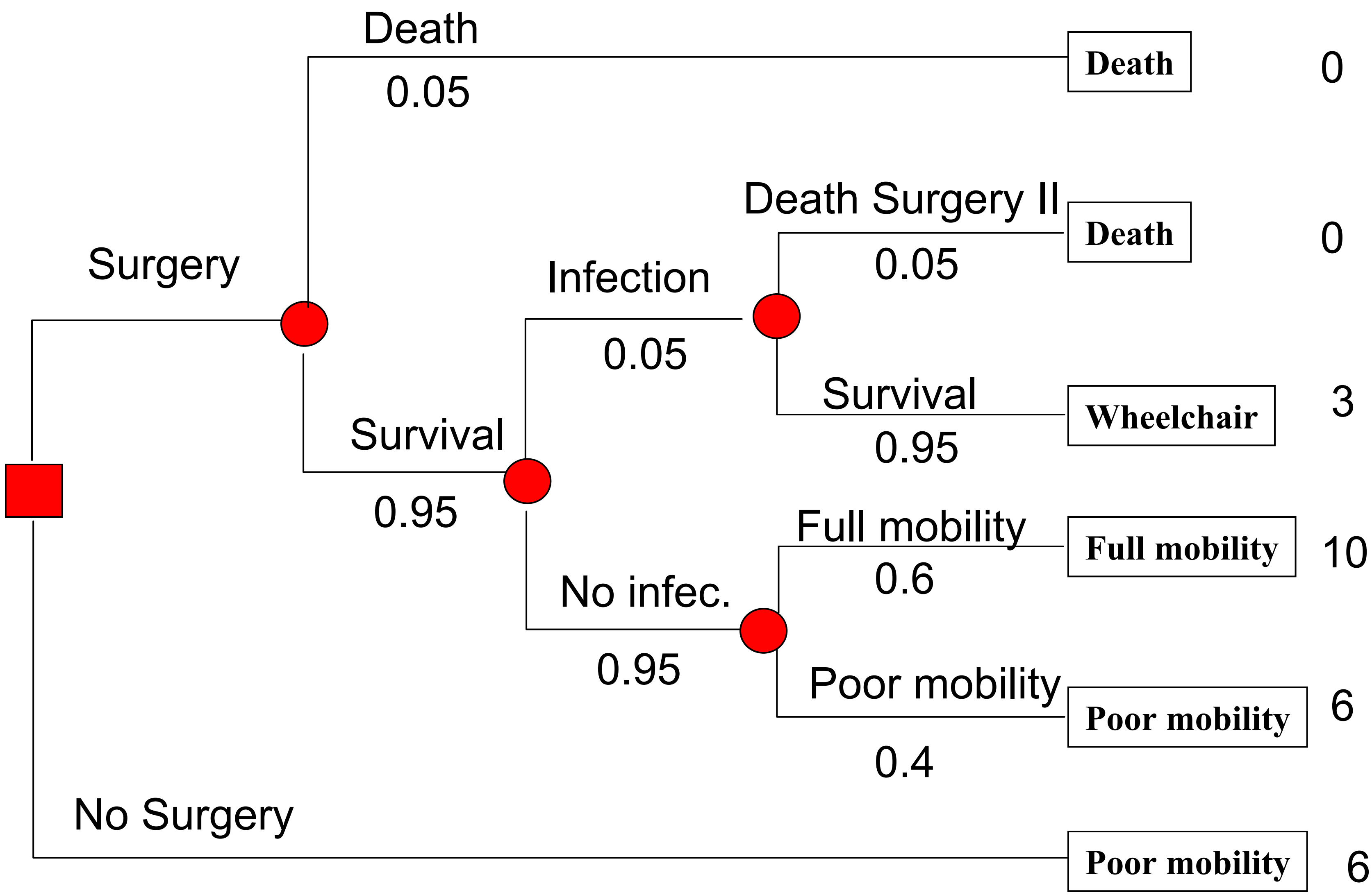
Death
0.05
Surgery
Death
0
EV= 9.5
Survival
0.95
Full mobility
10
No clairvoyant
No surgery
6
EV= 9.5
Death
Surgery
1
Death
0
EV= 0
Survival
0
Full mobility
10
"Death"
0.05
No surgery
Poor mobility
6
EV= 9.8
EV= 6
Clairvoyant
Death
Surgery
0
Death
0
"Survival"
EV= 10
Survival
1
Full mobility
10
0.95
No surgery
Poor mobility
6
EV= 6
Value of clairvoyance
= 9.8 - 9.5 = 0.3


Death
0.05
Death
0
Death 2
0.05
Death
0
Surgery
Infection
0.05
Survival
0.95
Wheelchair
3
Survival
0.95
Full mobility
0.6
Full mobility
10
No infec.
0.95
Poor mobility
0.4
Poor mobility
6
No Surgery
Poor mobility
6

Sensitivity Analysis

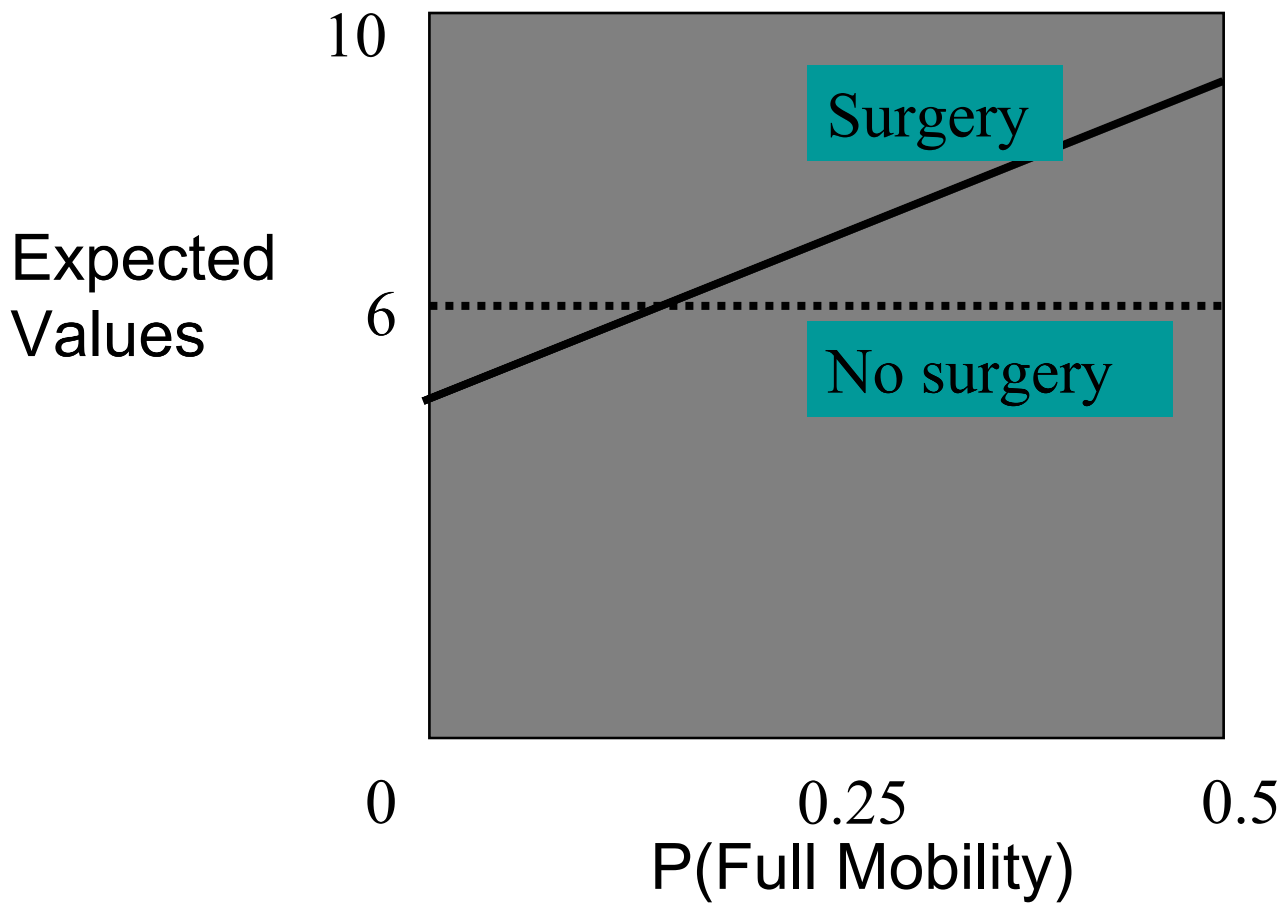
- Effect of probabilities in the decision




Death
0.05
Death
0
Death Surgery II
Death
0
0.05
Surgery
Infection
0.05
Survival
Wheelchair
3
0.95
Survival
0.95
Full mobility
Full mobility
10
0.6
No infec.
0.95
Poor mobility
Poor mobility
6
0.4
No Surgery
Poor mobility
6

Sensitivity Analysis

- Effect of probabilities in the decision

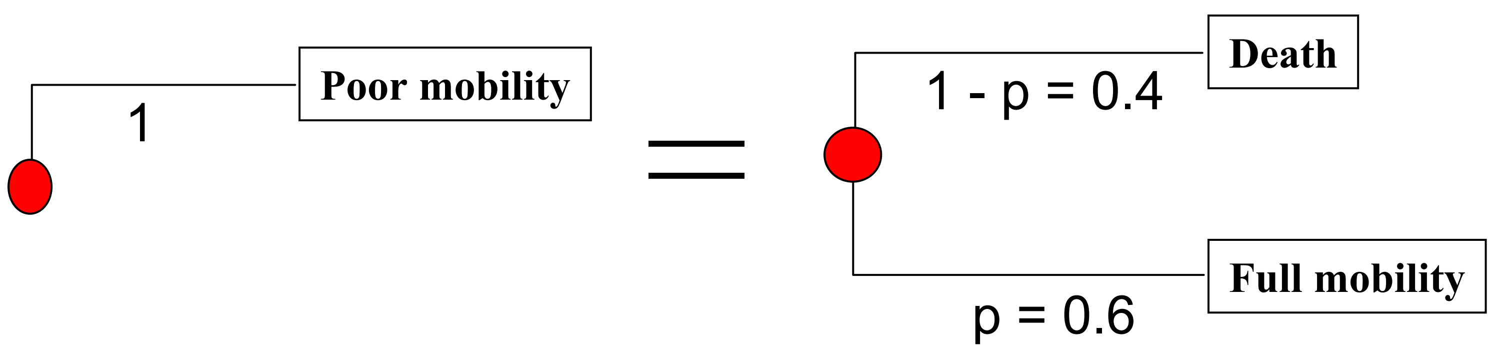


Utilities

- Quantitative measure of desirability of a health state, from patient’s perspective
- Methods
  - standard gamble
  - time-tradeoff
  - visual-analog scale
  - others

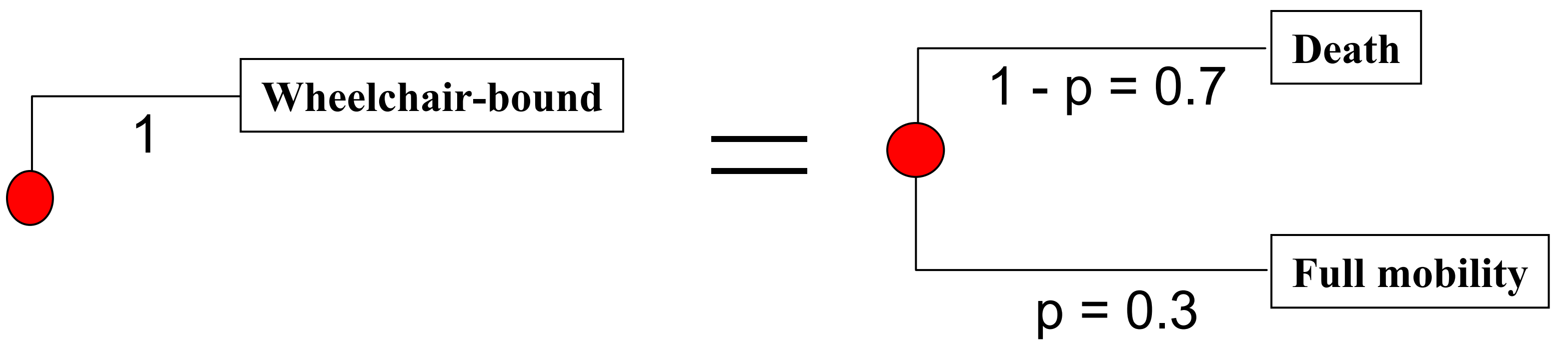
Standard Gamble

- What chances (p) are you willing to take (between best and worst case scenarios) so that you would not be living with **poor mobility**?



Standard Gamble

- What chances (p) are you willing to take (between best and worst case scenarios) so that would not be living **wheelchair-bound**?



Time Trade-Off
Visual Analog Scale

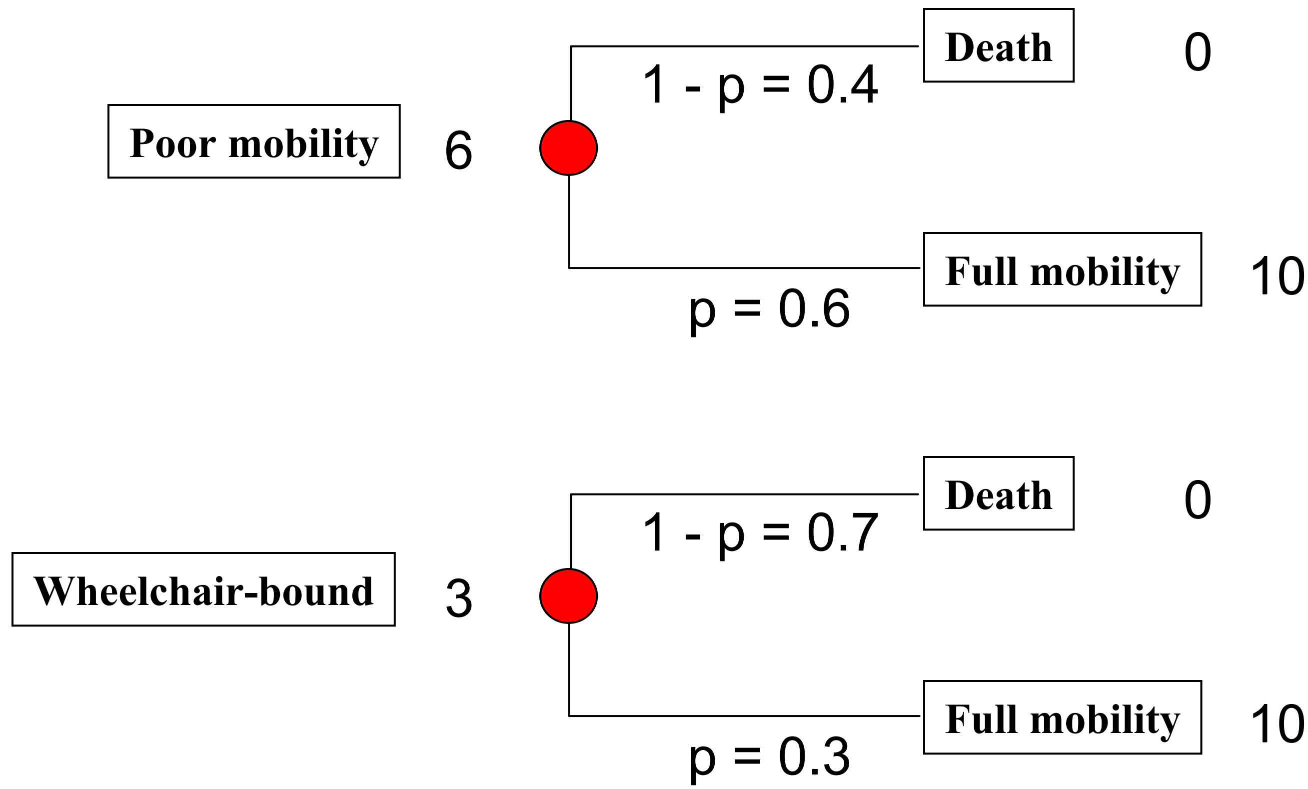
Time Trade-Off

- 10 years wheelchair-bound = 3 years full mobility
- does not involve gambles, so does not assess risk attitude

Visual Analog Scale

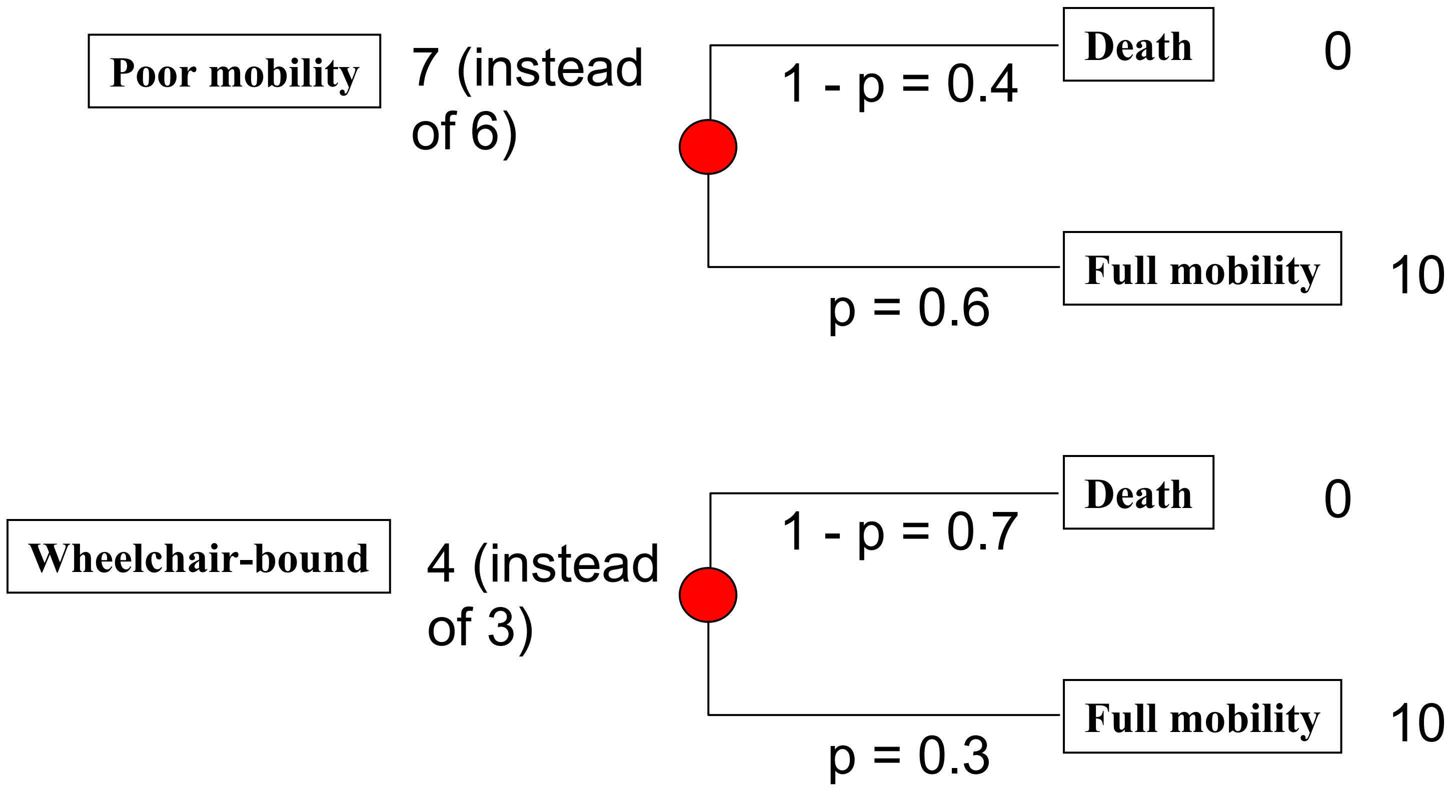
Risk Neutral Individual

(Utility = Expected Value)



Risk Averse Individual

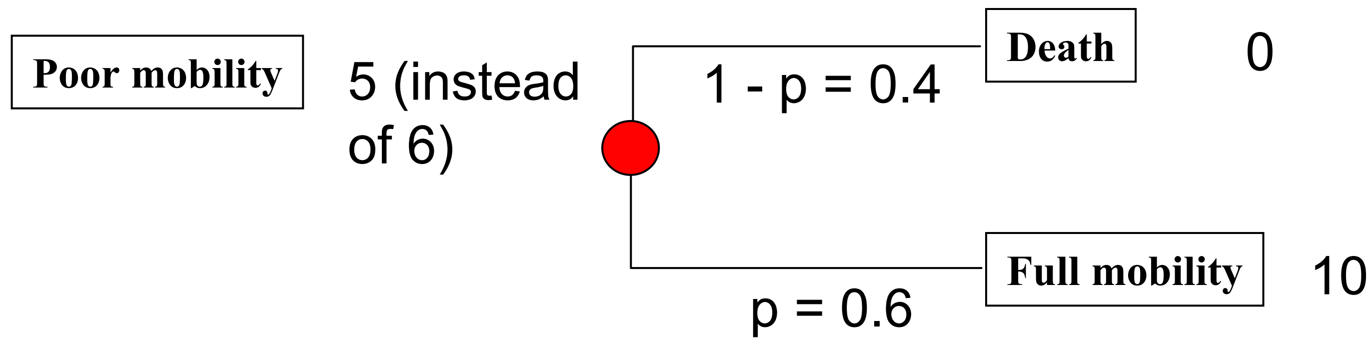
(Utility > Expected Value)

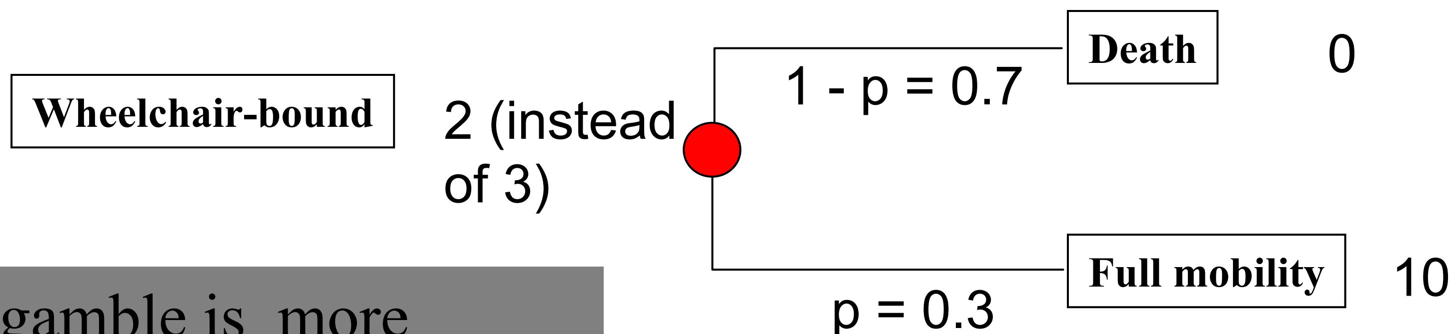


"A sure outcome is better than the gamble"

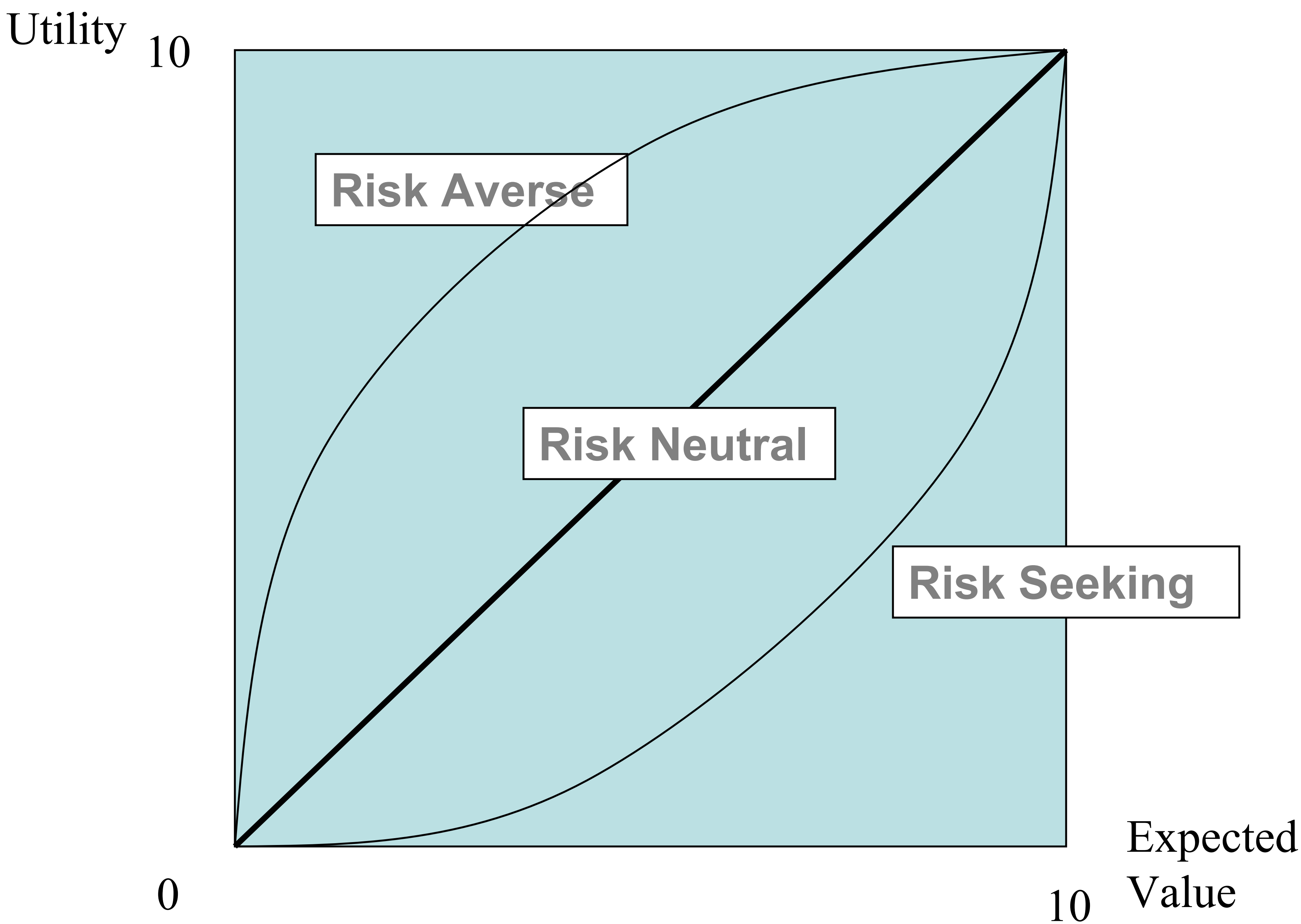
Risk Seeking Individual

(Utility < Expected Value)





"The gamble is more valuable than a sure (somewhat bad) thing"


Utility
10
Risk Averse
Risk Neutral
Risk Seeking
Expected Value
0
10

Summary

- Use conditional probabilities to assign probabilities to branches
- Use some utility scale that is consistent
- Calculate expected values
- Choose the max expected value
- Find out value of information
- Perform sensitivity analysis