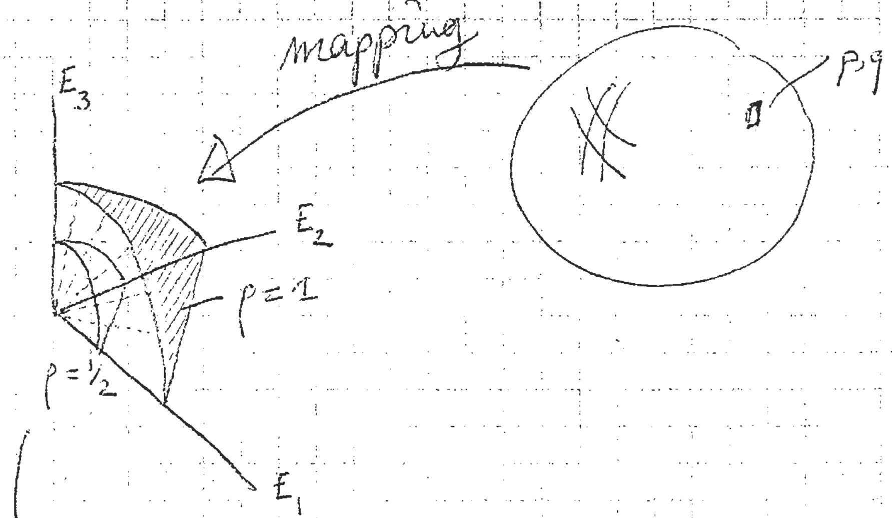
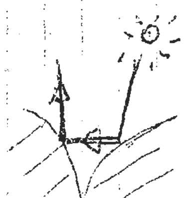


we can fill in the 3D LUT!

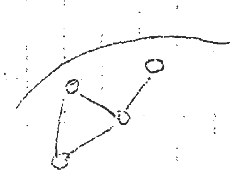
$\boxed{6.866 \quad 09/30}$

Now, if the surface has a constant albedo, 2 meas are enough but 3 meas gives error info. (onion slice in E^3 space) thus we can sort out outliers (shade, bogues, etc.) as well as mutual illumination.

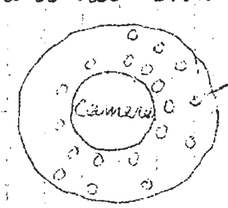


2 images, what about more?

ex @ CMU: half a sphere with light sources



another example: a circle of LEDs:



and you get a better fit.

How does it work? with a Lamb. surface, we want to minimize

$$\min_{\underline{n}} \sum_{i=1}^{N} (\underline{n}.\underline{s}_i - E_i)^2$$

The calculus is straight forward : $\frac{d}{d\underline{n}}(\ ) = 0$. i.e $\begin{pmatrix} \frac{d}{dn_x} \\ \frac{d}{dn_y} \\ \frac{d}{dn_z} \end{pmatrix} = 0$.

Just to experience the notation:

$$\frac{d}{d\underline{a}}(\underline{a}\cdot\underline{b}) = \frac{d}{d\underline{a}}(a_x b_x + a_y b_y + a_z b_z) = \begin{vmatrix} b_x \\ b_y \\ b_z \end{vmatrix} = \underline{b}$$

$$\frac{d}{d\underline{a}}(a^T M b) = \frac{d}{d\underline{a}}(\underline{a}\cdot(Mb)) = Mb.$$

$$\frac{d}{d\underline{a}}(a^T M a) =$$

Now: $\frac{d}{d\underline{n}}\left(\sum(\underline{n}.\underline{s}_i - E_i)^2\right) = 0$

means: $\sum(\underline{n}.\underline{s}_i - E_i)\frac{d}{d\underline{n}}(\underline{n}.\underline{s}_i - E_i) = 0$

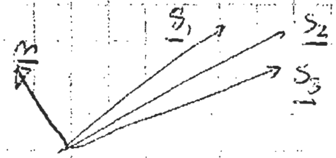
or: $\sum(\underline{n}.\underline{s}_i - E_i)\ \underline{s}_i = 0$ thus $\sum(\underline{n}.\underline{s}_i)\vec{s_i} = E_i . \underline{s}_i$

thus: $\sum(\underline{s}_i s_i^T).\underline{n} = \sum E_i \underline{s}_i$

$$\left(\sum(\underline{s}_i \underline{s}_i^T)\right)\underline{n} = \sum E_i \underline{s}_i \quad \text{or} \quad \boxed{\mathcal{S}\underline{n} = \mathcal{E}} \qquad n = \mathcal{S}^{-1}\mathcal{E}$$

$$\det(\mathcal{S}) \neq 0$$

⊗ The surface normale is a sum of light directions - INTERESTING



When is $\det(S) \neq 0$? when we have 3 independent light sources.

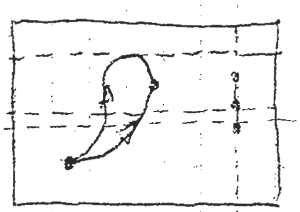
$$([\underline{s}_1 \; \underline{s}_2 \; \underline{s}_3] \neq 0)$$

Let's solve: $\min_{\underline{s}} \sum_{i=1}^{N} (\underline{n}_i \cdot \underline{s} - E_i)^2$ knowing $\underline{n}_i$, find $\underline{s}$

Same method: $\frac{d}{d\underline{s}}(\;) = 0. \implies N\underline{s} = \underline{E}'$

Shape from gradient

$$z(x,y) \leftarrow p(x,y)\, q(x,y) \qquad z_1 - z_0 = \int_{s_1}^{s_2} p\,dx + q\,dy$$



bad news: if we integrate step by step, the result depends on the path!

or we want $\oint (p\,dx + q\,dy) = 0$ (1)

for all paths

p. 472: Gauss's Integral Theorem

$$\iint_D \left(\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial y} \right) dx\,dy = \int_{\partial D} (Q\,dy - P\,dx) \qquad (2)$$

Thus by applying (2) onto (1):

$$\iint_D \left(\frac{\partial q}{\partial x} - \frac{\partial p}{\partial y}\right) dx\, dy = 0 \quad \text{for all } D$$

If we shrink D to a point: $\underline{\underline{\frac{\partial q}{\partial x} = \frac{\partial p}{\partial y}}}$ everywhere

integrability condition

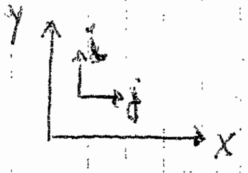
Thus: if $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ then $p_y = q_x$

There is too much (redundant) information $\Rightarrow$ least squares method

$$\min_{z_{ij}} \sum_i^n \sum_j^m \left((z_x)_{ij} - p_{ij}\right)^2 + \left((z_y)_{ij} - q_{ij}\right)^2$$

The coefficient matrix is huge but very sparse.

$z_x \approx \frac{1}{\varepsilon}(z_{i,j+1} - z_{ij})$ and $z_y \approx \frac{1}{\varepsilon}(z_{i+1,j} - z_{ij})$



$$\sum_{i,j}^{n,m} \left(\frac{z_{i,j+1} - z_{ij}}{\varepsilon} - p_{ij}\right)^2 + \left(\frac{z_{i+1,j} - z_{ij}}{\varepsilon} - q_{ij}\right)^2$$

$$\frac{d}{dz_{kl}}(\;\;) = 0$$

When $i=k$ and $j=l$: $2\left(\frac{z_{i,j+1} - z_{ij}}{\varepsilon} - p_{ij}\right)\left(-\frac{1}{\varepsilon}\right) + 2\left(\frac{z_{i+1,j} - z_{ij}}{\varepsilon} - q_{ij}\right)\left(-\frac{1}{\varepsilon}\right)$

when $i=k$ and $j+1=l$: $2\left(\frac{z_{kl}-z_{kl-1}}{\varepsilon}-p_{kl-1}\right)\left(\frac{1}{\varepsilon}\right)+2\left(\frac{z_{kl}-z_{k-1\,l}}{\varepsilon}-q_{k-1\,l}\right)\frac{1}{\varepsilon}$

and then we gather this up:

$$\underbrace{\frac{1}{\varepsilon^2}\left(4z_{kl}-z_{k,l+1}-z_{k,l-1}-z_{k-1,l}-z_{k+1,l}\right)}_{\frac{1}{\varepsilon^2}\begin{array}{|c|c|c|}\hline & -1 & \\ \hline -1 & 4 & -1 \\ \hline & -1 & \\ \hline\end{array}}+\underbrace{\frac{1}{\varepsilon}\left(p_{kl}-p_{kl-1}\right)}_{\approx p_x}+\underbrace{\frac{1}{\varepsilon}\left(q_{kl}-q_{k-1\,l}\right)}_{\approx q_y}=0$$

$\approx$ Laplacian $\underbrace{\begin{array}{|c|}\hline -1\\ \hline 2\\ \hline -1\\ \hline\end{array}}_{z_{xx}} + \underbrace{\begin{array}{|c|c|c|}\hline -1 & 2 & -1\\ \hline\end{array}}_{z_{yy}}$

$$\boxed{z_{xx}+z_{yy}=p_x+q_y}$$

answer experiment

Calculus: $\mathcal{O}(n^3)$...

Alternative: solve iteratively

$$z_{kl}=\underbrace{\frac{1}{4}\left(z_{kl+1}+z_{kl-1}+z_{k-1\,l}+z_{k+1\,l}\right)}_{\text{local average}}\underbrace{-\frac{1}{\varepsilon}\left(p_{kl}-p_{kl-1}\right)-\frac{1}{\varepsilon}\left(q_{kl}-q_{k-1\,l}\right)}_{\text{constant in the image}}$$

and it does converge!