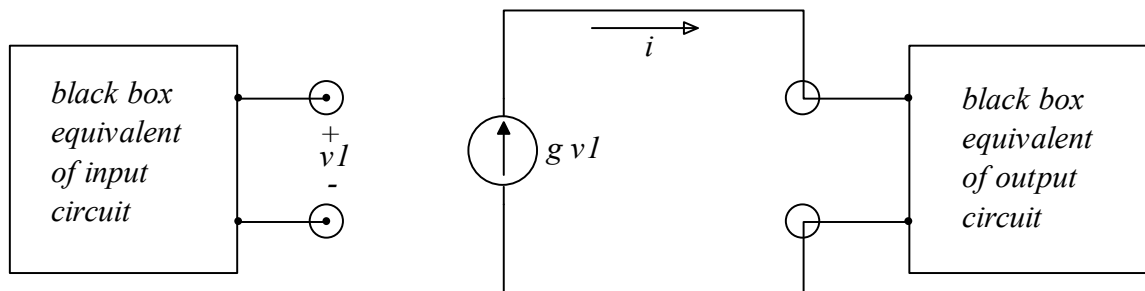**Dependent Sources:**
Introduction and analysis of circuits containing dependent sources.

So far we have explored time-independent (resistive) elements that are also linear. We have seen that two terminal (one port) circuits can be modeled by simple circuits (Thevenin or Norton equivalent circuits) and that they have a straight line *i-v* characteristic.
Here we introduce the idea of a dependent source. We will see that the use of dependent sources permits the use of feedback. Feedback can be used to control amplifiers and to build interesting transducers.

Dependent Sources
A dependent source is one whose value depends on some other variable in the circuit. An illustrative example of a dependent source is,

Here we see that there is an "input" circuit that develops a voltage, *v1*.

In a separate part of the circuit there is a linear, voltage-dependent, current source that delivers a current given by

$$i = g\,v1 \tag{1.1}$$

Where *g* is a constant with the units of A/V.
So the current that flows into the output circuit depends on the measurement of a voltage on the input circuit.

Now clearly we could mimic a dependent source by looking at a meter and changing a potentiometer (for example) in relation to the reading. Here we will introduce circuits that carry out this function without any intervention.
Notice that the above circuit is still linear since the output current depends linearly on the measured voltage. For now we will concern ourselves with only linear dependent sources. Later, we will see examples of non-linear dependent sources where the analysis will be somewhat more complex.

There are four general classes of linear dependent sources. Their names, acronyms and associated symbols are:
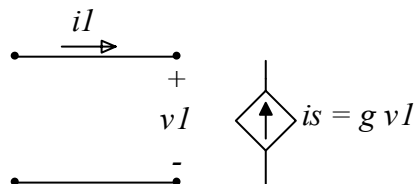
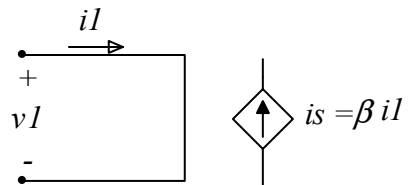Voltage Controlled Voltage Source: VCVS

$$i1$$

$$+$$
$$v1 \qquad vs = A\,v1$$
$$-$$
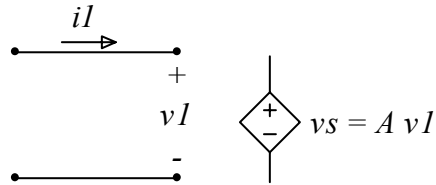
Current Controlled Voltage Source: CCVS

$$i1$$

$$+$$
$$v1 \qquad vs = r\,i1$$
$$-$$
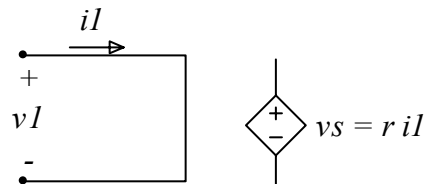
Voltage Controlled Current Source: VCCS.

$$i1$$

$$+$$
$$v1 \qquad is = g\,v1$$
$$-$$

Current Controlled Current Source: CCCS

$$i1$$

$$+$$
$$v1 \qquad is = \beta\,i1$$
$$-$$

The parameters A, r, g, β, are real numbers, and $v1$, $i1$ are voltages/currents in some circuit.

Circuit Analysis with Linear Dependent Sources.

Linear dependent sources provide no new complications to circuit analysis. Kirchhoff's laws still apply, and formal circuit analysis goes ahead just as before. The dependent source only introduces a constraint on the solution.
The simplest example is where the measurement and dependent source are in two isolated circuits.

Let's consider the current amplifier circuit shown on Figure 1 . The circuit has one independent current source and one dependent current source. The dependent current source is a CCCS. We would like to determine the voltage vc as indicated.
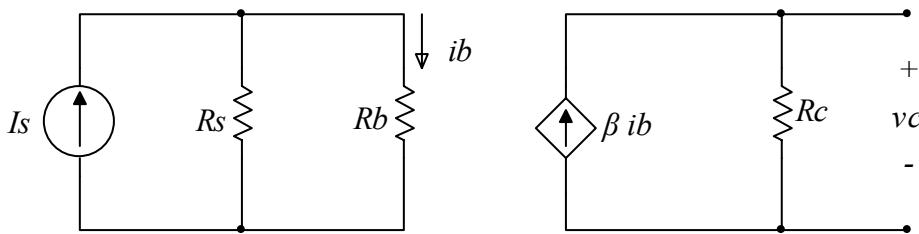

Figure 1. Current Amplifier Circuit

The left hand circuit is a current divider, and

$$ib = \frac{Rs}{Rs + Rb} Is \qquad (1.2)$$

The right hand circuit is a current source. The output voltage vc is given by

$$vc = \beta \; ib \; Rc \qquad (1.3)$$

So now we see that the output voltage vc depends on the measured current ib of the input circuit. Combining Equations (1.2) and (1.3) we obtain

$$vc = \beta \underbrace{\frac{RsRc}{Rb + Rc}}_{gain} Is \qquad (1.4)$$

So, the overall circuit behaves as an amplifier with the gain dependent on the resistors and the proportionality constant β.

Let's now consider the slightly more interesting circuit shown on Figure 2.
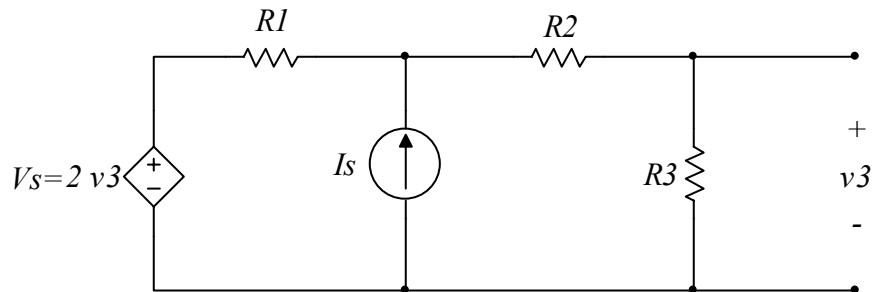

Figure 2. Circuit with dependent voltage source

Let's use nodal analysis to solve for the currents and voltages in this circuit.

Figure 3 shows the nodes of interest, the variables and the polarities.
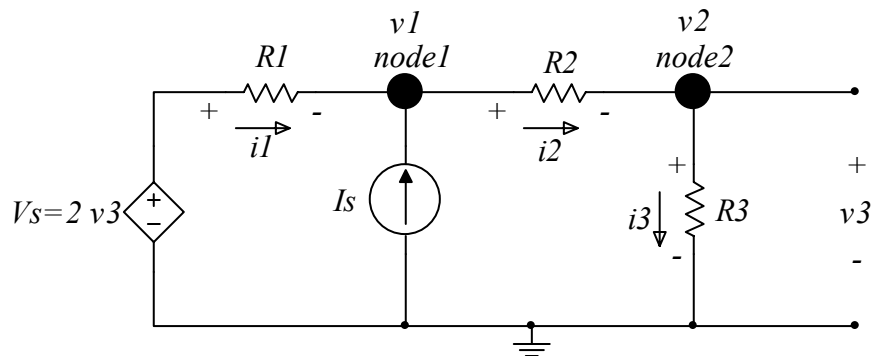

Figure 3. Nodal analysis of circuit with dependent sources

KCL at node1 gives

$$i1 + Is - i2 = 0$$

$$\frac{Vs - v1}{R1} + Is - \frac{v1 - v2}{R2} = 0 \qquad (1.5)$$

KCL at node2 gives

$$i2 - i3 = 0$$

$$\frac{v1 - v2}{R2} - \frac{v2}{R3} = 0 \tag{1.6}$$

In matrix form, Equations (1.5) and (1.6) become

$$\begin{pmatrix} \dfrac{1}{R2} + \dfrac{1}{R3} & -\dfrac{1}{R2} \\[2ex] -\dfrac{1}{R2} & \dfrac{1}{R1} + \dfrac{1}{R2} \end{pmatrix} \begin{pmatrix} v1 \\ v2 \end{pmatrix} = \begin{pmatrix} Is + \dfrac{Vs}{R1} \\[2ex] 0 \end{pmatrix} \tag{1.7}$$

and the solution is given by

$$v1 = \frac{(R2 + R3)(IsR1 + Vs)}{R1 + R2 + R3} \tag{1.8}$$

$$v2 = \frac{R3(IsR1 + Vs)}{R1 + R2 + R3} \tag{1.9}$$

Now need to include the constraints of the dependent sources. These constraints are

$$v2 = v3 \tag{1.10}$$

And

$$Vs = 2v3 \tag{1.11}$$

Substituting Equations (1.10) and (1.11) into Equations (1.8) and (1.9) we obtain

$$v1 = \frac{IsR1(R2 + R3)}{R1 + R2 + R3} \tag{1.12}$$

$$v2 = \frac{IsR1R3}{R1 + R2 + R3} \tag{1.13}$$

Analysis of Circuits with Dependent Sources Using Superposition

When employing the principle of superposition to a circuit that has dependent and independent sources we proceed as follows:

- Leave dependent sources intact.
- Consider one independent source at the time with all other independent sources set to zero.

Let's explore this with the following example:
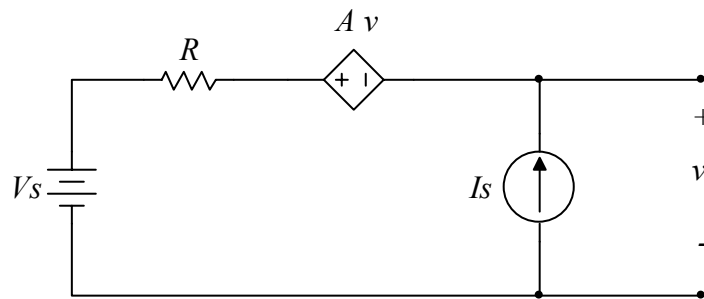
For the circuit on Figure 4 calculate the voltage v.

Figure 4. Circuit with dependent source. Analysis using superposition

We proceed by first considering the effect of the current source acting alone. The circuit of Figure 5 shows the corresponding circuit for which the independent voltage source Vs has been suppressed.
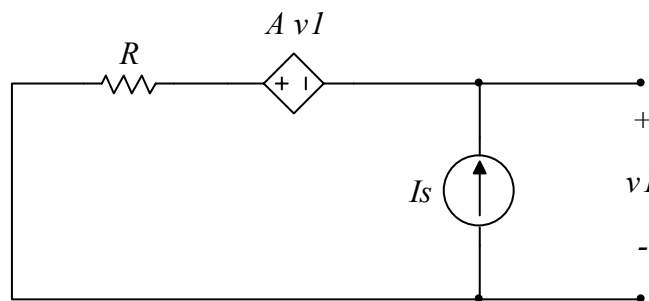
Figure 5. Circuit with the voltage source suppressed

By applying KVL we obtain:

$$v1 - IsR + Av1 = 0 \qquad (1.14)$$

And v1 becomes

$$v1 = \frac{IsR}{1+A} \qquad (1.15)$$

Next we evaluate the contribution to the output with the independent voltage source acting alone. The corresponding circuit is shown on Figure 6.
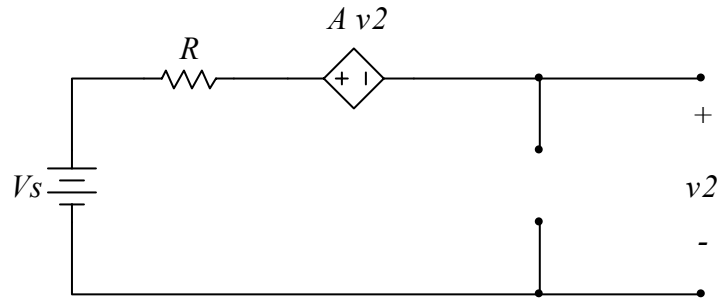

Figure 6. Circuit with the current source suppressed.

Again applying KVL we have

$$Av2 + v2 - Vs = 0 \tag{1.16}$$

(Note that the voltage drop across R is zero since there is no current flowing in the circuit.)
And v2 becomes

$$v2 = \frac{Vs}{1+A} \tag{1.17}$$

And so the total voltage is written as the superposition of *v1* and *v2*.

$$\begin{aligned} v &= v1 + v2 \\ &= \frac{1}{1+A}\left(Vs + IsR\right) \end{aligned} \tag{1.18}$$

Let's now look at the slightly more complicated circuit shown on Figure 7 with multiple dependent and independent sources. We will determine the voltage *vo* by using superposition.
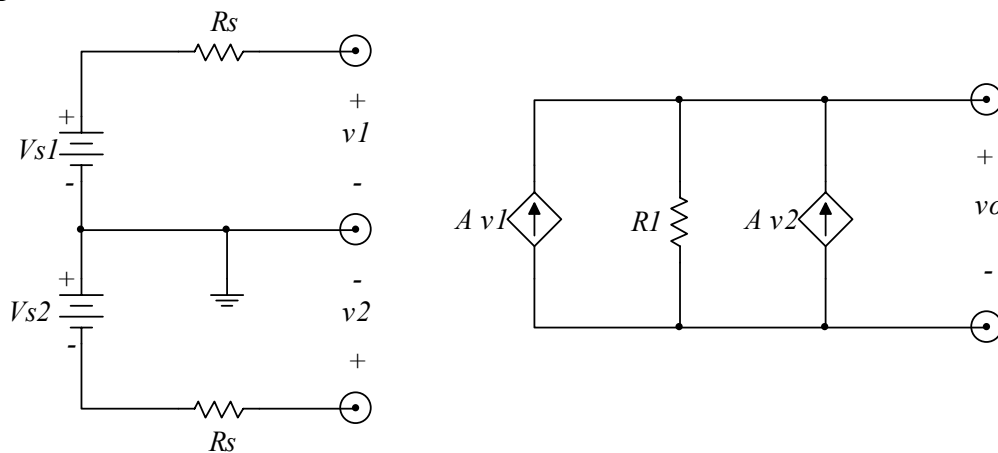
Figure 7. Circuit with dependent and independent sources

The procedure is the same as before: leave dependent sources intact, calculate the contribution of each independent source acting alone.

Figure 8 shows the circuit with *Vs2* suppressed. The indicated output *vo1* is the contribution of voltage source *Vs1*.
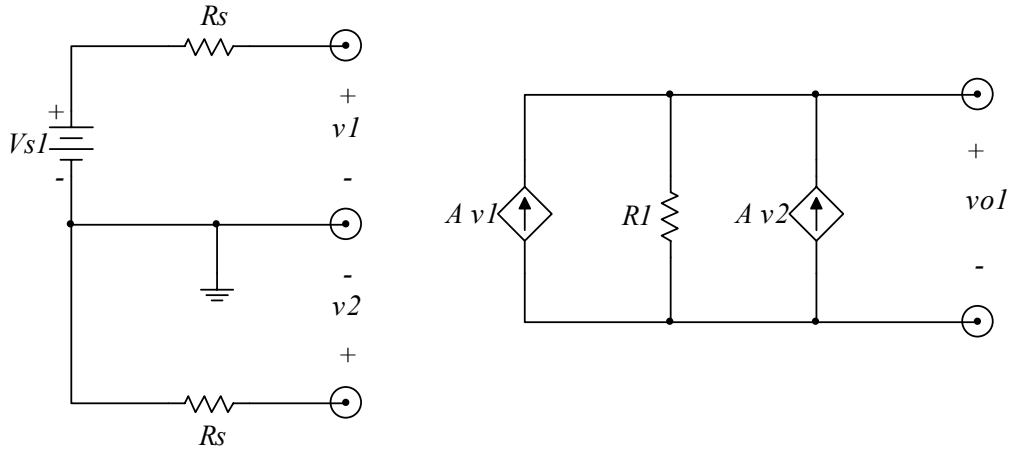

Figure 8.

Since the voltage source *Vs2* has been suppressed, the voltage *v2* is zero. Therefore the current provided by the voltage controlled current source *Av2* is zero. This is shown schematically on the circuit of Figure 9.
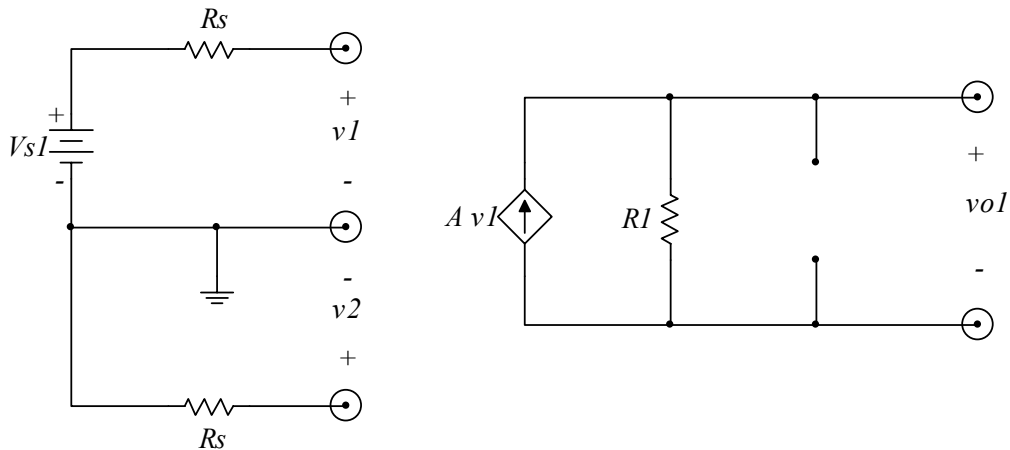

Figure 9

Therefore the voltage *vo1* is

$$vo1 = A\,v1\,R1 = A\,Vs1\,R1 \qquad (1.19)$$

Next we will suppress the voltage source *Vs1* as shown on the circuit of Figure 10.

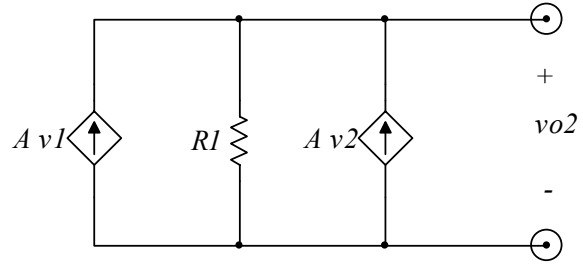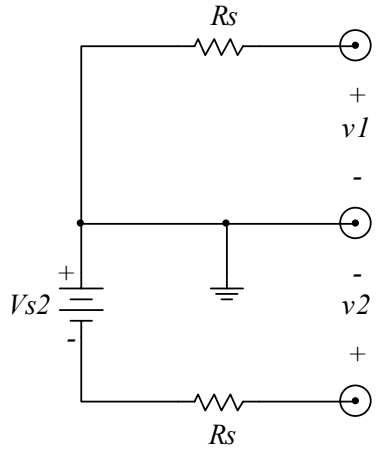**Figure 10**

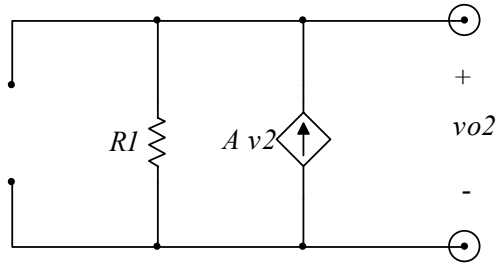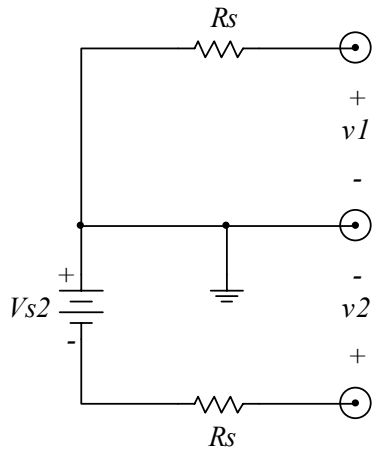Now the voltage $v1 = 0$ and $v2 = -Vs2$, and the circuit reduces to the one shown on Figure 11

Figure 11

The voltage *vo2* is now

$$vo2 = -A \ Vs2 \ R1 \tag{1.20}$$

And by combining Equations (1.19) and (1.20) the voltage *vo* is

$$vo = vo1 + vo2 = A(Vs1 - Vs2)R1 \tag{1.21}$$

Determining the Thevenin/Norton equivalent circuit of circuits containing dependent sources.

For a given two-terminal port the equivalent circuit is defined exactly as in the case of independent sources. Measure or calculate the open-circuit voltage and the short circuit resistance. These two points define the *i-v* characteristics of the port. The characteristic resistance is the ratio of the open circuit voltage to the short circuit current.

Given a schematic, the characteristic resistance can also be found by suppressing all *independent* sources and calculating the effective resistance between the terminals. NOTE, do **not** suppress the *dependent* sources.

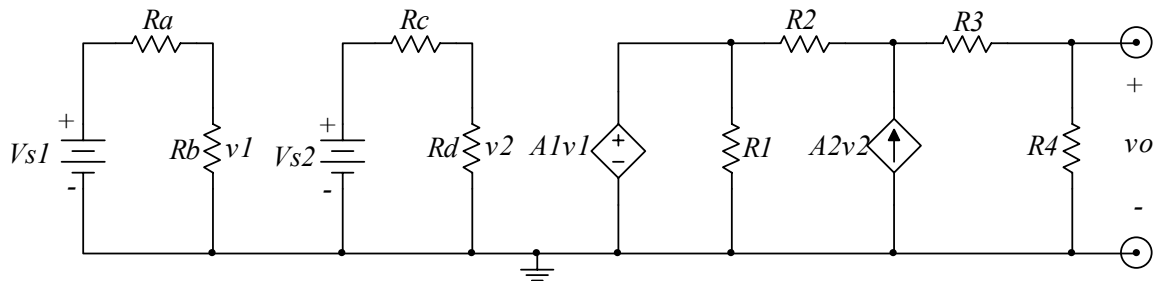Let's consider the circuit shown on Figure 12. We would like to calculate the voltage *vo*.

Figure 12

We will do this by first finding the Thevenin equivalent circuit seen by resistor *R4*. We will calculate the equivalent circuit across the terminals *X-Y* as shown on the circuit of Figure 13.
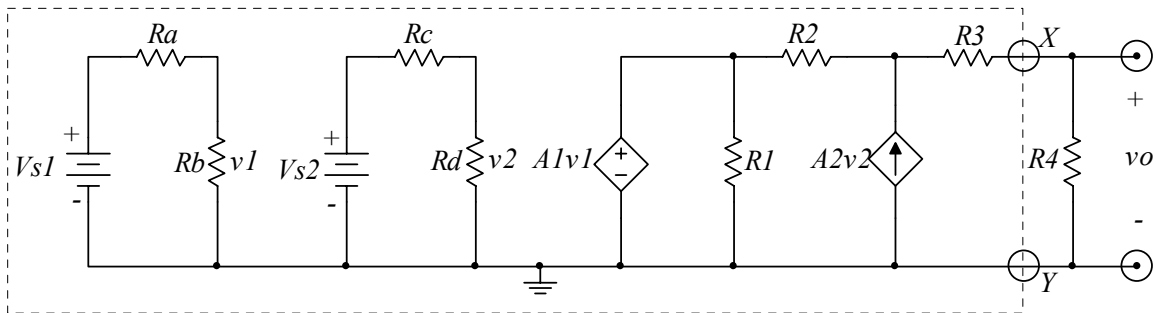

Figure 13

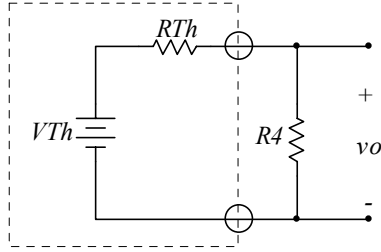In turn we will find *vo* by considering the simple voltage divider circuit shown on Figure 14.

Figure 14

First let's calculate the Thevenin equivalent resistance *RTh* seen at port *X-Y* (Figure 15).
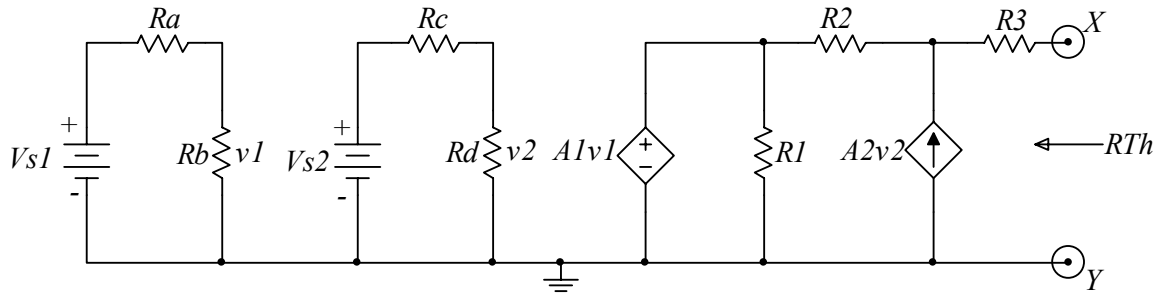

Figure 15

In order to calculate *RTh* we will suppress all independent sources in our circuit. (Note: leave dependent sources intact). Figure 16 shows the circuit with the independent sources suppressed.
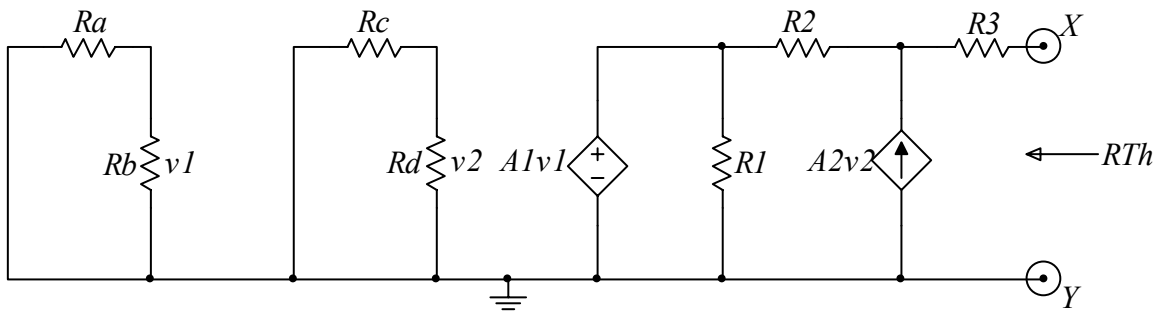

Figure 16

Note that sending both *Vs1* and *Vs2* to zero causes *v1* and *v2* to also go to zero. Therefore, the dependent sources also go to zero and the corresponding circuit is now shown on Figure 17 where the dependent sources are now zero.
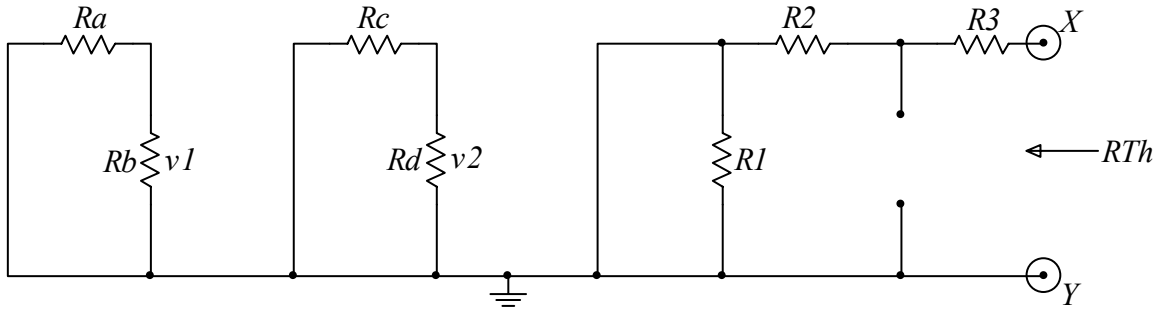
Figure 17

So we now see that the Thevenin equivalent resistance across terminals *X-Y* is

$$RTh = R2 + R3$$ (1.22)

Next we will calculate the Thevenin voltage *VTh*, or the open circuit voltage, across terminals *X-Y*.

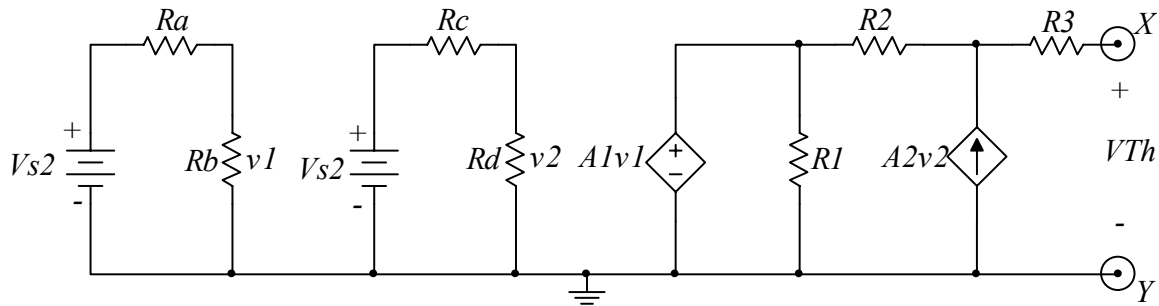**Figure 18**

We can calculate *VTh* by employing any of the circuit analysis methods. We will use superposition for this case. *VTh=VTh1+VTh2* as shown on corresponding circuits of Figure 19.

Figure 19
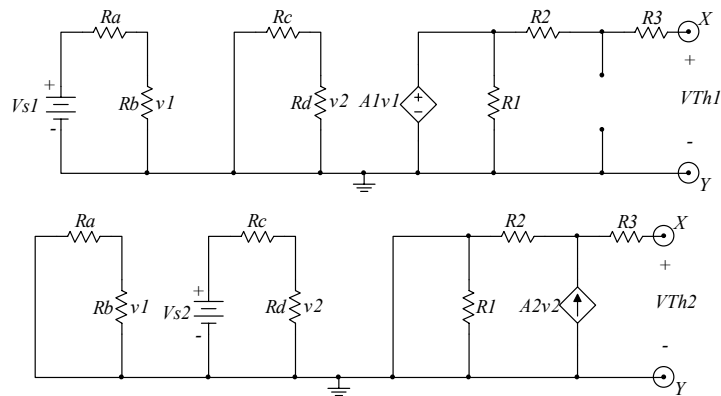
From Figure 19 we see that,

$$VTh1 = A1v1$$
$$= A1Vs1\frac{Rb}{Ra+Rb}$$

(1.23)

and

$$VTh2 = A2v2\,R2$$
$$= A2Vs2\frac{Rd}{Rc+Rd}R2$$

(1.24)

And superposition gives

$$\boxed{\begin{aligned}VTh &= VTh1+VTh2 \\ &= A1Vs1\frac{Rb}{Ra+Rb}+A2Vs2\frac{Rd}{Rc+Rd}R2\end{aligned}}$$

(1.25)

From the equivalent circuit shown on Figure 14 the desired voltage is

$$vo = \left(A1Vs1\frac{Rb}{Ra+Rb}+A2Vs2\frac{Rd}{Rc+Rd}R2\right)\frac{R4}{R4+R2+R3}$$

(1.26)

Next, let's analyze a circuit that contains only dependent sources. For the circuit shown on Figure 20 we will determine the Thevenin and Norton equivalent circuits across terminals *a-b*.
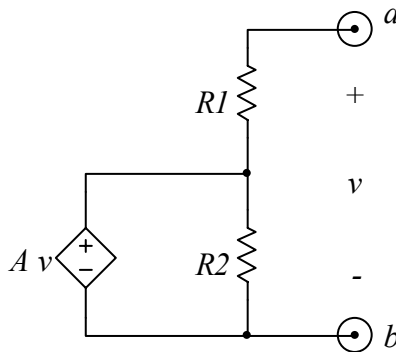

Figure 20

The equivalent circuit for this has to be a resistance since with no input there can not be an output.

Normally with a two terminal device we would measure the open-circuit voltage and the short circuit current and from these two measurements determine the *i-v* characteristic.

For this circuit, the short-circuit current is zero since shorting the terminals forces *v* to zero.

The open circuit voltage is not so obvious, one might think that the output voltage is undefined in the open circuit arrangement and thus any voltage would be possible. But let's look more carefully. In the open circuit configuration the current through *R1* is zero, so there is no voltage drop across *R1*. By KVL this requires that *v* be equal to the voltage drop across *R2*. However, KVL requires that the voltage drop across *R2* equal *Av*, or that $v = Av$. This can be true in only two cases:

(1) in the case that A=1, or
(2) in the general case that $v = 0$.

Let's consider the more general case.

The circuit is still a linear, resistive circuit, and so we still need only 2 points to define the *i-v* characteristic. Since both the open circuit and short circuit measurements probe the same point (*i=0* and *v=0*) we need to define another point. In order to find the second point we may fix either the current or the voltage at the output and determine the other. If we set the voltage across the terminals *a-b* to *Vt* (it does not matter what value we use), then we have the circuit shown on Figure 21. The current flow indicated by the arrow results from the application of the test voltage *Vt*.
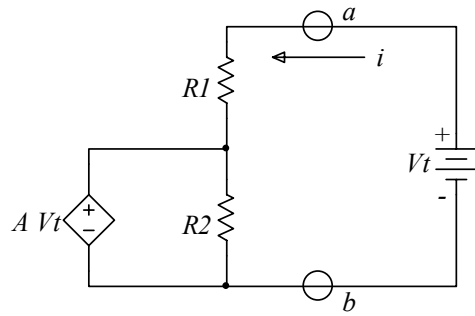
Figure 21

By applying KVL around the loop formed by *Vt*, *R1* and *R2* we have

$$iR1 + AVt - Vt = 0 \qquad (1.27)$$

$$i = Vt\frac{1-A}{R1} \qquad (1.28)$$

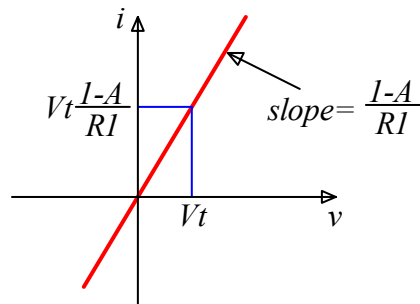The *i-v* characteristic curve is now shown on Figure 22

Figure 22

And the Thevenin equivalent resistance is

$$RTh = \frac{R1}{1-A} \qquad (1.29)$$

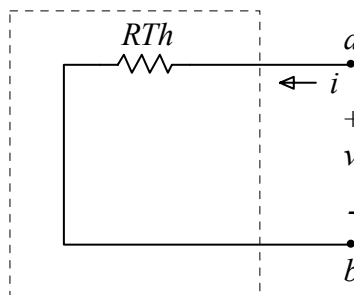And so the Thevenin equivalent circuit is as shown on Figure 23

Figure 23

Similarly we could apply a test current source *It* and determine the resulting voltage *v*. This is shown on Figure 24.
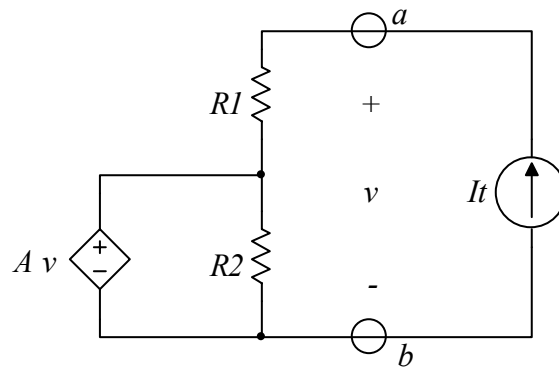

Figure 24

Again applying KVL around the loop we have

$$ItR1 + Av - v = 0 \tag{1.30}$$

And

$$v = It \frac{R1}{1 - A} \tag{1.31}$$

Again indicating that the equivalent resistance (Thevenin/Norton) is

$$RTh = RN = \frac{R1}{1 - A} \tag{1.32}$$

Applications of dependent sources

Dependent sources provide a convenient means of:

1. converting between voltage and current

2. changing resistance

Since dependent sources often appear in the part of the circuit that we are using to make a measurement, they also enable the input and output characteristics of a device to be optimized separately.
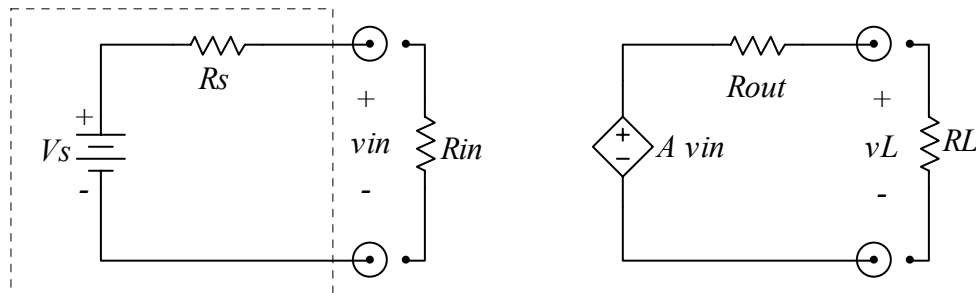
Look at the amplifier circuit shown on Figure 25.

Figure 25. Amplifier circuit

The circuit at the far left is a Thevenin equivalent of a voltage source. So this can stand in place of any voltage source regardless of the actual complexity of the physical source. The resistor *Rin* is used to measure the voltage *vin* that is provided by this source. Since *Rin* is the basis of a voltage measurement we desire that *Rin>>Rs*. This is a general design criteria that we have seen before.

The circuit at the right is a Thevenin equivalent voltage source driving a load. Here the good design characteristics require that *Rout<<RL*.

By breaking the circuit into four components as shown on Figure 26 we will be able to investigate the details of each part for a deeper understanding.
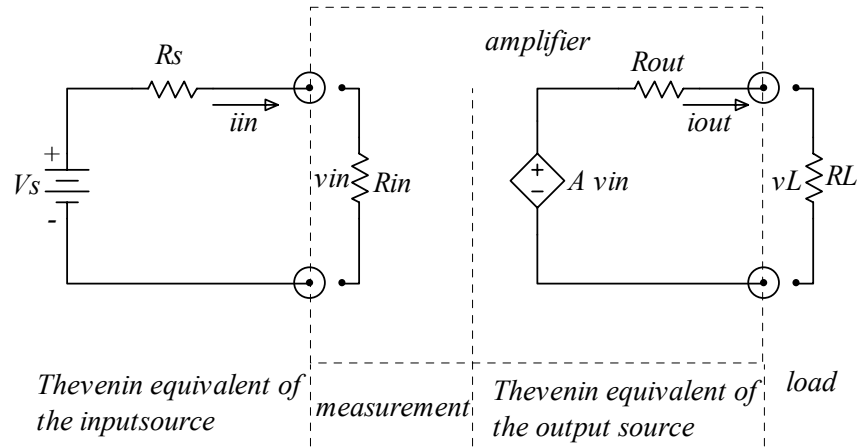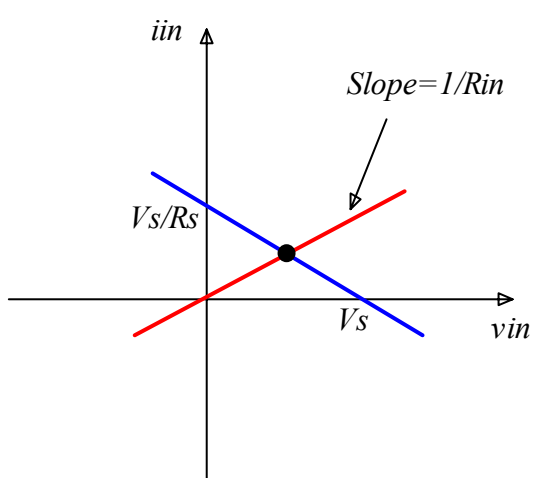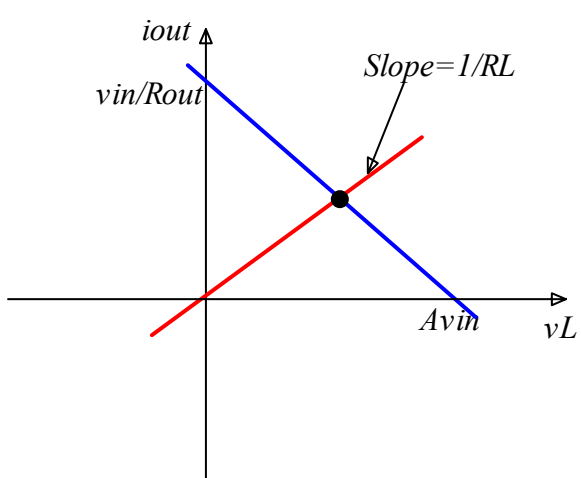
Figure 26. Detailed breakdown of the amplifier circuit

By describing the circuit in terms of the Thevenin equivalent circuits we have provided the most compact form possible for this application. Since it is a linear circuit, the *i-v* characteristic will provide additional insight.


i-v characteristic of input

i-v characteristic of output

Here then we see that depending on the choice of the resistors and the parameter *A* we can build an amplifier that detects a voltage and delivers power. Notice that *Vs* would often be a time varying voltage (a signal) and so the operating points would slide back and forth, but the slopes would not change.
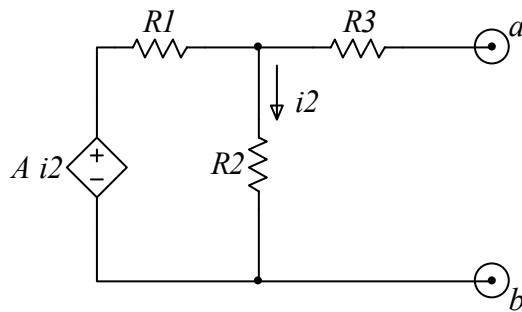
(We will continue with more details on dependent sources including the concept of feedback next class)
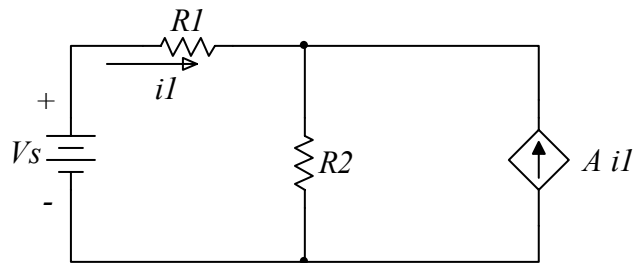
Problems:

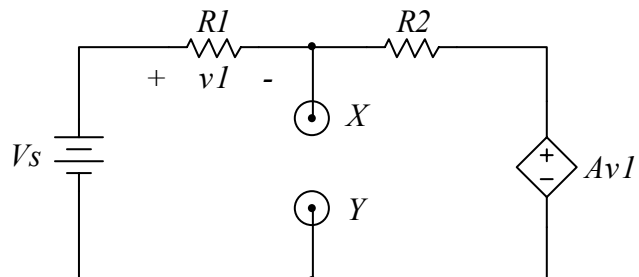1. Determine the Thevenin equivalent circuit across terminals *X-Y*.
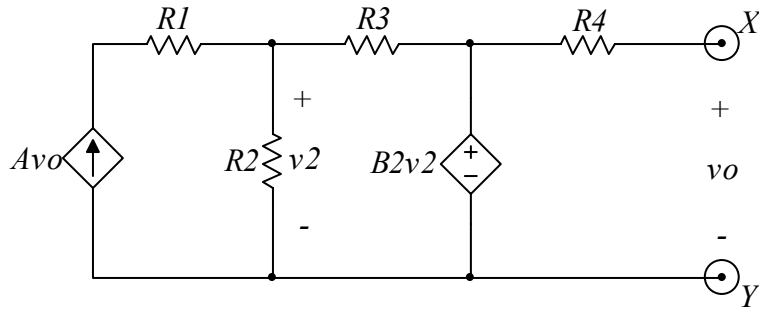
3. Obtain the Thevenin equivalent circuit seen at port a-b

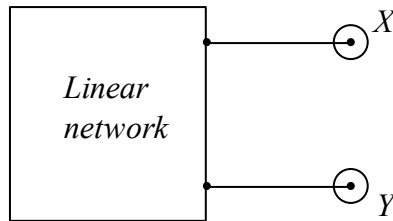4. For the circuit below determine the current *i1*.

5. Obtain the Thevenin equivalent circuit seen at port a-b

6. A circuit is represented by the network as shown on the following figure. Our task is to determine the Thevenin equivalent circuit by performing a set of measurements on it.
First we connect a 1kΩ resistor across the terminals and measure a voltage of 2Volts across *X-Y*.
Next we connect a 20kΩ resistor and measure a voltage of 10Volts across *X-Y*. From these data determine the Thevenin equivalent circuit.

7. Determine the Thevenin equivalent circuit seen by resistor *RL*.