EQS Tutorial

10/18

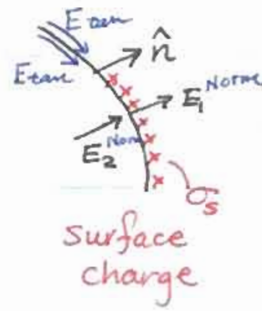
Objectives: Ⓘ Boundary Conditions for EQS
Ⓘ Electrical Properties of Materials
Ⓘ Debye Length
Ⓘ Dipole source @ center of sphere
Ⓥ Charge Relaxation

I. Boundary Conditions for EQS: ($\frac{\partial}{\partial t}$ for magnetic related terms $\to 0$).

ⓐ Gauss's Law: $\nabla \cdot \varepsilon E = \rho_e \Rightarrow \hat{n} \cdot (\varepsilon_1 \vec{E}_1 - \varepsilon_2 \vec{E}_2) = \sigma_s$.

$$\varepsilon_1 E_1^{Norm} - \varepsilon_2 E_2^{norm} = \sigma_s$$

$$D_1^{Norm} - D_2^{Norm} = \sigma_s$$



ⓑ Faraday's Law: $\nabla \times E \simeq 0 \Rightarrow \hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$

$$E_1^{tan} = E_2^{tan}$$

ⓒ Conservation of Charge: $\nabla \cdot J = -\frac{\partial \rho_e}{\partial t} \Rightarrow \hat{n} \cdot (\sigma_1 \vec{E}_1 - \sigma_2 \vec{E}_2) = -\frac{\partial}{\partial t} \sigma_s$

Note: Assuming that $J = \sigma E$ only right now.

II. Electrical Properties of Materials:

Conductor: $\left.\begin{array}{l} \sigma \to \infty \\ \vec{E}_{net} = 0 \end{array}\right\} J = \sigma E$ (J is finite)

Insulator: $\left.\begin{array}{l} \sigma = 0 \\ E \text{ is finite} \end{array}\right\} J = 0$

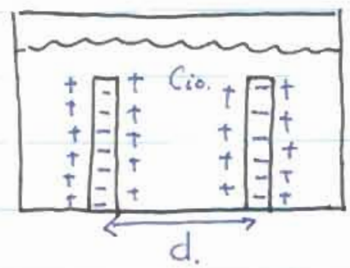
Dielectric: $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_0$

Dielectric is a poor conductor, but an efficient storage substance for electromagnetic fields.

III. Debye Length: Why is it inversely proportional to concentration in bath?

"Charge Shielding"

$$\frac{1}{K} = \sqrt{\frac{\varepsilon R T}{2 z_+^2 F^2 C_{io}}}$$ for case where $z_+ = -z_-$



Case I: $C_{io} = 0.01\,M$

Surrounding '+' charge: +3

Rod: −5 Coulomb.

$\boxed{\text{Net charge: } -2 \text{ Coulomb}}$

Case II: $C_{io} = 0.001\,M$

Surrounding '+' charge: +0.3

Rod: −5 Coulomb

$\boxed{\text{Net charge: } -4.7 \text{ Coulomb.}}$

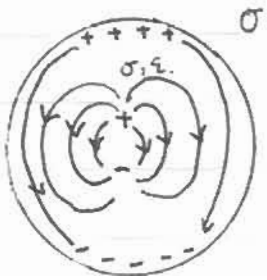
$$F = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2}$$

$$E \propto \frac{1}{r^2}$$

$$V \propto \frac{1}{r}$$

- To feel same amount of electric force, when q_1 & q_2 ↑, r ↑, ∴ debye length would increase
- concentration ion ↑, net charge ↓ if fixed charge is constant, debye length ↓

IV. Dipole Source @ center



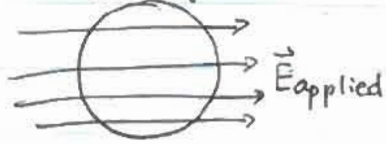
σ = 0

- Induced surface charge due to charge relaxation.
- E & Φ due to independent contribution of dipole source & uniform field (Superposition ☺)
- @ R → ~~∞~~ far field, field appears uniform.

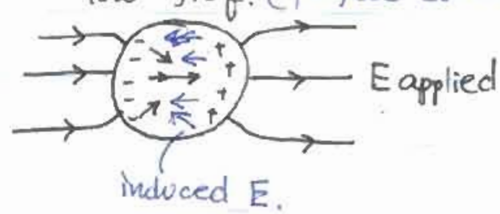
V. Charge Relaxation:

- Relating to frequency
  - when there's an applied E field @ low frequency, meaning $\frac{1}{f} = T \gg \frac{\varepsilon}{\sigma}$, a good conductor appears as a perfect conductor.
  - Otherwise, the applied E field passes through the good conductor.

high freq.



low freq. (perfect conductor).



Inside perfect conduct, $\vec{E}$ applied & Induced $\vec{E}$ cancel each other.

- Relating to Poisson & Laplace Equation

  If $T \gg \frac{\varepsilon}{\sigma}$ such as heart beat where T = 1s ≫ 1ns

  ↑ charge relaxation time.

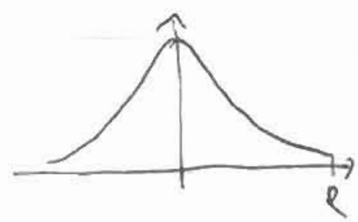
  $\nabla \cdot J = -\frac{\partial \rho}{\partial t} \simeq 0$

  $\nabla \cdot \sigma E = 0$, $E = -\nabla \Phi$

  $\sigma \nabla^2 \Phi = 0$ use Laplace

- Relating to Diffusion

  Diffusion $\tau_{diff} \propto \frac{L^2}{D}$



Charge Relaxation

$\tau = \frac{\varepsilon}{\sigma}$