BE.430/2.795//6.561/10.539/HST.544

Homework Set 4

Handed out: Friday Oct. 8th

Due: Friday Oct. 15 by 5pm

: (a) Chapter 1, Section 1.0 to1.3 in the Grodzinsky text (but you can skip material on Magnetoquasistatics); Read and work examples 1.3.1 to review solutions of Laplace's Equation (for the electrical potential Φ) in spherical coordinates, which you'll need for problem 2, and 1.3.2 for additional insights in cylindrical coordinates.

1. Problem 1.3.3 (Grodzinsky text):

This problem examines the charge, potential, and electric field distribution at the interface between charged surfaces (e.g., cells, proteins, electrodes, channels in bioMEMs devices, etc.) and electrolyte solutions (e.g., physiological buffers). **Do parts (a)-(d):** Note – analytical solutions can be found for all answers here. This is one of the most important examples involving the electrical double layer at bio-surfaces.

2. Problem 1.3.4 (Grodzinsky text):

For this problem, first consider the case of zero frequency (dc). That is, do the whole problem (parts a,b,c) for the case in which all point and dipole current sources are <u>steady</u>.

<u>New part (d):</u> At the end, argue that your solutions to parts (a,b,c) are still correct for the case in which a heart beats at ~1 Hz. You will need to estimate charge relaxation time constants and compare to characteristic times of interest.

<u>New part (e):</u> Describe how you would solve for the electric field and potential outside the spherical person of part (c); i.e., the field generated in all space by the beating heart within the person. To do this, write the boundary conditions that you would use at r = R and r = ∞, and state the form of the potential that you would try in the region r > R. (DO NOT SOLVE).

<u>Notes/Hints for this Problem</u>: (a) You'll only need to consider the three most common Laplacian potentials in spherical coordinates, which have the form

$$\Phi \sim \textbf{(A/r)} \text{ [point charge]} + \textbf{(B r cos}\theta\textbf{)} \text{ [uniform field]} + \textbf{C (cos}\theta\textbf{/r}^2\textbf{)} \text{ [dipole]}.$$

These solutions can be derived using separation of variables. In Prob. 1.3.4, you'll need to solve for the constants A,B,C by satisfying the boundary conditions on either Φ or its normal derivative (normal component of the E field). While separation of variables is not guaranteed to

give you all the solutions to Laplace, if you do find one (or a superposition of several solutions) that satisfies the BCs, then this solution is guaranteed to be Unique (see Uniqueness Problem 1.3.5...which you are not asked to solve). (b) Read through and work Example 1.1.1 (pg. xxviii) on the point charge dipole. Parts (a) and (b) of this problem asks you to derive the form of the potential for a point current source and a point current-dipole source, by analogy to a point charge source and a point charge-dipole source, respectively, in an infinite medium. Part (c) then brings in the boundary conditions invoked for the spherical person.

3. **Problem 1.6.1** (Grodzinsky text):

Reading assignment to prepare for this problem: (a) Read sections 1.4 through 1.6 of the text. (b) Read and work through example 1.3.2 (p. 40) on charge relaxation. While Prob. 1.6.1 concerns a moving fluid, the velocity profile is given, so you are asked to find solutions in the frame of the moving fluid. The math should be straight forward...no tricks here! It's all concepts. (Remember the class discussion!)