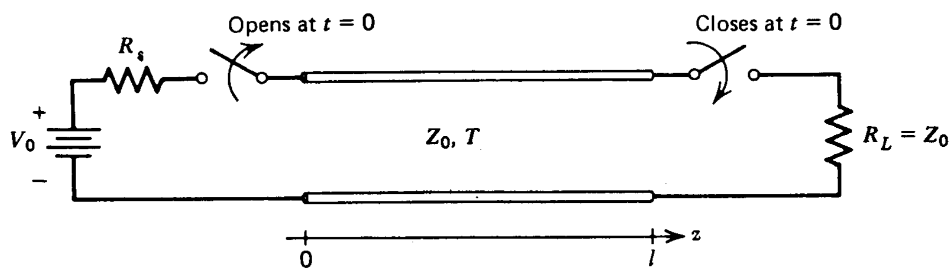
Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.013 Electromagnetics and Applications

Problem Set #8 Issued: 11/1/05
Fall Term 2005 Due: 11/9/05

Suggested Reading Assignment: 5.2.1, 5.2.2, 9.2
Quiz 2 will be on Thursday, November 17 at 10-11 a.m. It will cover material through P. S. #8, with a focus on sinusoidal steady state and transient waves on transmission lines; parallel plate, rectangular, and dielectric waveguides. **Quiz 2 Formula Sheets** (as attached to this problem set) will be provided.

Problem 8.1

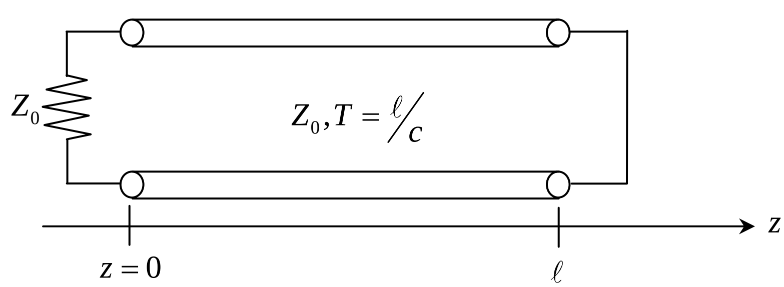
Switched transmission line systems with an initial dc voltage can be used to generate high voltage pulses of short time duration. The line shown is charged up to a dc voltage V_0 when at *t=0* the load switch is closed and the source switch is opened.



(a) What are the initial line voltage and current at t=0? What are V_+ and V_-?
(b) Sketch the time dependence of the load voltage.

Problem 8.10 in *Electromagnetic Field Theory: A Problem Solving Approach*, by Markus Zahn, 1987. Used with permission.

Problem 8.2



A transmission line of length ℓ and characteristic impedance Z_0 has a matched load at $z = 0$ and is short circuited at $z = \ell$. The speed of electromagnetic waves on the transmission line is c so that the one-way transit time is $T = \ell/c$. The transmission line is unexcited until at $t = 0$ it is hit by lightning so that the line voltage and current at $t = 0$ are

$$v(z, t = 0) = V_0 \qquad 0 < z < \ell$$
$$i(z, t = 0) = 0 \qquad 0 < z < \ell$$

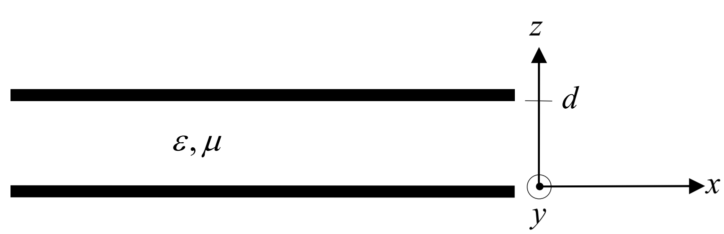
a) What are $V_+(t - z/c)$ and $V_-(t + z/c)$ at $t = 0$?
b) Plot the voltage and current at $z = 0$ as a function of time.
c) Plot the current at $z = \ell$ as a function of time.
d) Plot the voltage and current as a function of z for $0 < z < \ell$ at time $t = T/2$.

Problem 8.3
An air-filled metal rectangular waveguide has cross-sectional area dimensions a =2 cm and b = 1 cm.

a) What TE_{mn} and TM_{mn} mode have the lowest cut-off frequencies and what are these frequencies?

b) Over what frequency range will this waveguide operate at a single mode? What is the mode?

Problem 8.4



A parallel plate waveguide with spacing d, dielectric permittivity ε, and magnetic permeability μ supports TE_n and TM_n modes given by:

TE_n

$$\bar{E} = E_0 \sin k_z z \sin(\omega t - k_x x)\bar{i}_y$$

$$\bar{H} = \frac{E_0}{\eta k}\left[-k_z \cos k_z z \cos(\omega t - k_x x)\bar{i}_x + k_x \sin k_z z \sin(\omega t - k_x x)\bar{i}_z\right]$$

TM_n

$$\bar{E} = \frac{E_0}{k}\left[k_z \sin k_z z \sin(\omega t - k_x x)\bar{i}_x - k_x \cos k_z z \cos(\omega t - k_x x)\bar{i}_z\right]$$

$$\bar{H} = \frac{E_0}{\eta} \cos k_z z \cos(\omega t - k_x x)\bar{i}_y$$

$$k_z = \frac{n\pi}{d}, \quad k_x = \sqrt{\omega^2 \varepsilon\mu - k_z^2}$$

a) What are the surface charge densities on the *z=0* and *z=d* surfaces for TE_n and TM_n modes?

b) What are the surface current densities on the z=0 and z=d surfaces for TE_n and TM_n modes?

c) Find the equation for the magnetic field lines that go through coordinates (x_0, z_0) for the TE_n modes at *t=0* defined as:

$$\frac{dz}{dx} = \frac{H_z}{H_x}$$

Hint: $\int \cot u du = \ln(\sin u) + \text{Constant}$

d) Find the equation for the electric field lines that go through coordinates (x_0, z_0) for the TM_n modes at *t=0* defined as:

$$\frac{dz}{dx} = \frac{E_z}{E_x}$$

Hint: $\int \tan u du = -\ln(\cos u) + \text{Constant}$

e) **Extra Credit**: Using your favorite drawing program, draw some field lines of parts (c) and (d) that illustrate the fundamental shape of the TE_1 and TM_1 modes.