MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 5 6.632 Electromagnetic Wave Theory
Spring Term 2003

Reading assignment: Section 3.3, 3.4 J. A. Kong, "*Electromagnetic Wave Theory*"

Problem P5.1

Let a plane wave be incident on a plane boundary from the inside of a negative uniaxial crystal (see textbook, page 147). Consider the special case in which the optic axis is perpendicular to the plane of incidence. Find the range of θ such that there is only total internal reflection for the ordinary wave and the transmitted waves are extraordinary waves.

Problem P5.2

For a highly conducting earth, assume $\sigma/\omega\epsilon \gg 1$, where σ is the conductivity of the earth. Consider a TM wave incident upon the boundary of the earth surface.

(a) Write the magnetic and electric field components for $R^{\mathrm{TM}} \neq 0$ and then set $R^{\mathrm{TM}} = 0$. Show that

$$k_z = k_{zR} + ik_{zI} = \frac{\omega\epsilon}{\sigma}\sqrt{\frac{\omega\mu\sigma}{2}}(1-i)$$

which propagates towards the surface and attenuates away from the surface. This is known as the Zenneck wave. Is the Zenneck wave a fast wave? A fast wave has $\mathrm{Re}\,(k_x) < k$.

(b) Define a surface impedance

$$R_s - iX_s = k_z/\omega\epsilon$$

What is R_s and X_s?

(c) For an inductive surface characterized by $R_s = 0$, write down $\overline{H}$ with $R^{\mathrm{TM}} = 0$ and discuss the behavior of the resulting wave.

Problem P5.3

Consider a solid-state Fabry-Perot etalon filter made of an eight-layer stratified medium. Regions 1, 3, 5, and 7 are made of magnesium fluoride (refractive index $n = 1.35$) and are a quarter-wavelength thick. Regions 2, 4, and 6 are made of zinc sulfide (refractive index $n = 2.3$). Regions 2 and 6 are a quarter-wavelength thick, but region 4 is a half-wavelength thick. What are the reflectivity and transmissivity for a plane wave normally incident upon this stratified medium? Explain why the structure can be used for filtering purposes.

Problem P5.4

(a) A plane wave in free space is incident at an angle θ on a half-space. For large $\epsilon \gg \epsilon_o$ show that the transmitted wave is almost perpendicular to the boundary by finding θ_t.

(b) A plane wave in free space is incident at an angle θ on a conducting half-space. For large $\sigma/\omega\epsilon_o$ show that the transmitted wave is almost perpendicular to the boundary by finding θ_t.