Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.685 Electric Machinery

Quiz 2 Solutions **December 14, 2005**

Problem 1: From the Homework:

1. Magnetic field is in the direction perpendicular to the paper and is:

$$H_z = \frac{I_0}{D}$$

so that the pertinent value of the Maxwell Stress Tensor is:

$$T_{xx} = -\frac{\mu_0}{2}\left(\frac{I_0}{D}\right)^2$$

Since $T_{xy} = T_{xz} = 0$ on all interesting surfaces and $T_{xx} = 0$ in the space to the right of the block, total force on the block is

$$f_x = -wDT_{xx} = \frac{\mu_0}{2} I_0^2 \frac{w}{D}$$

2. voltage is

$$V = \frac{d\lambda}{dt}$$

and

$$\lambda = wx\mu_0 \frac{I_0}{D}$$

, so that

$$V = \mu_0 \frac{w}{D} I_0 u$$

3. Mechanical power delivered is

$$P_m = f_x u = \frac{\mu_0}{2} I_0^2 \frac{w}{d} u$$

Electrical power is

$$P_e = VI = \mu_0 I_0^2 \frac{w}{D} u$$

4. Electrical efficiency is

$$\eta = \frac{P_m}{P_e} = \frac{1}{2}$$

for all speeds, so it must be /1/2 overall.

5. Velocity and position are, starting at zero:

$$\begin{aligned} u &= \frac{f_x}{M} t = \frac{\mu_0 I_0^2}{2M} \frac{w}{D} t \\ x &= \frac{1}{2} \frac{f_x}{M} t^2 = \frac{\mu_0 I_0^2}{4M} \frac{w}{D} t^2 \end{aligned}$$

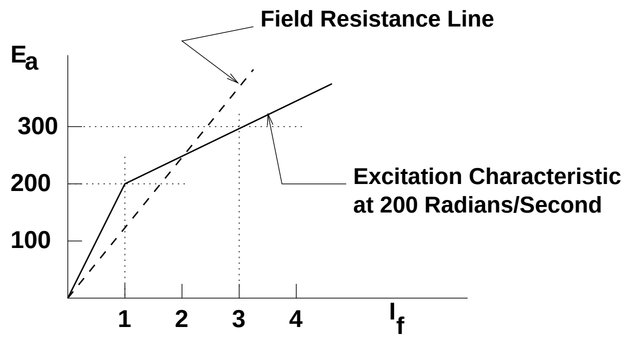

Figure 1: Problem 3: Induction Motor Equivalent Circuit

Problem 2: DC Generator The solution to this problem is described as the intersection of a line that describes field current in terms of internal voltage (the dotted line in Figure 1 and the curve that describes internal voltage in terms of field current (the solid lines in the same figure). Where they intersect is a solution, and if the slope of the resitance curve is higher than that of generation the intersection is stable. The field resistance characteristic has a constant slope of 125Ω, while the generation characteristic is proportional to machine speed.

1. The minimum speed interaction is described when the first segment of the generation curve matches the resistance curve, and excitation (and internal voltage) will grow until field current is 1 A. At that point:

$$E_a = 126V$$

Noting that $E_a = \Omega I_f$ and the equilibrium point is at $I_f = 1$, $\Omega = 126$ Radians/second.

2. At the steady speed of 200 Radians/Second, the equilibrium will be between the two characteristics:

$$\begin{aligned} E_a &= 200 + 50\,(I_f - 1) \\ E_a &= 126 I_f \end{aligned}$$

The solution is what is drawn in Figure 1, and is at $E_a = 248.7$ and $I_f = 1.97$.

3. At that condition, *power* into the machine is just what is required to overcome dissipation:

$$P = I^2 R = 1.97^2 \times 126 = 489\text{watts}$$

Torque is:

$$T_m = \frac{P}{\Omega} = \frac{500}{200} = 2.44\text{Nm}$$

Problem 3: Induction Machines Note that we can turn the source and two inductances on the left hand side of the induction machine equivalent circuit into the equivalent shown in Figure 2, we have for the locked rotor case:

$$\underline{I} = \underline{I}_2 = \frac{110}{j2 + 2}$$

Air-gap power is:

$$P_{ag} = 3\frac{110^2 2}{2^2 + 2^2} = \frac{6 \times 110^2}{8}$$

Torque is:

$$T = \frac{p}{\omega_e} P_{ag} = \frac{2}{110}\frac{6 \times 110^2}{8} = \frac{12}{8} \times 110 = 165\text{Nm}$$

Running light, the right-hand branch disappears and

$$I = \frac{121}{11 + 1.1} = 10A$$

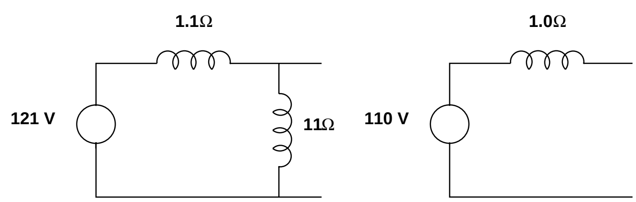

Figure 2: Problem 3: Simplification to Induction Motor Equivalent Circuit

Problem 4: Permanent Magnets The field from the permanent magnets is, over the magnets themselves,

$$B_0 = \pm B_r \frac{h_m}{h_m + g} = \pm\frac{1}{2}T$$

Axial field induced is

$$E_z = B_r \Omega R = \pm 12.5\text{V/m}$$

As there are two meters of affected wire, voltage induced is ± 25 volts. The last 'trick' here is to determine the *sign* of voltage induced. To do this consider Figure 3. Write Faraday's Law around a loop consisting of the terminals and the loop of wire, starting from the positive terminal:

$$\oint \vec{E} \cdot d\ell = -\frac{d}{dt} \int \int \vec{B} \cdot dA$$

Since the initial rate of change of flux is negative the voltage must be positive.

The result is shown in Figure 4.

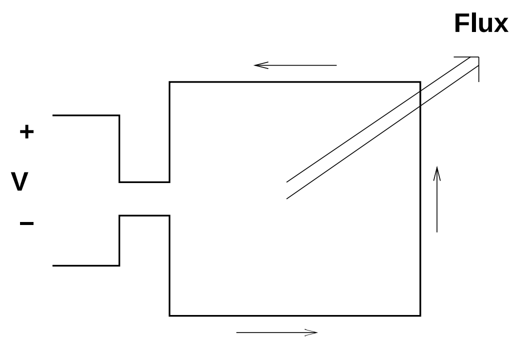

Figure 3: Problem 2: Faraday's Law Problem

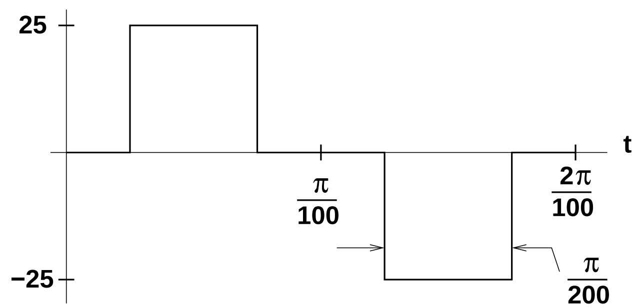

Figure 4: Problem 2: Search Coil Voltage