Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 8 Solutions November 5, 2005

Problem 1: DC Generator

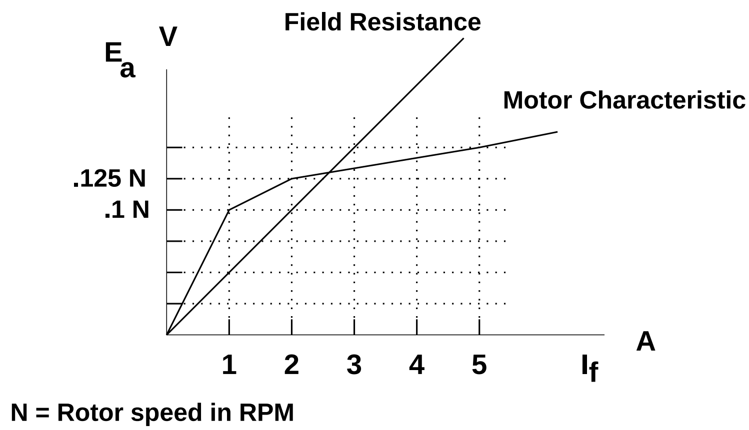

Figure 1: DC Generator Test Curve

Operating with no load, the situation is as shown in Figure 1. The equilibrium situation is that

$$R_f I_f = E_a - R_a I_f$$

The trick is to characterize E_a, which we may do using a piecewise-linear method:

$$\begin{array}{lll} \text{if} & 0 < I_f < I_1 & E_a = G\Omega I_f \\ \text{if} & I_1 < I_f < I_2 & E_a = E_1 + G_1\Omega\left(I_f - I_1\right) \\ \text{if} & I_2 < I_f & E_a = E_2 + G_2\Omega\left(I_f - I_2\right) \end{array}$$

Note that as Ω changes, so do the breakpoints E_1 and E_2 and the slopes $G\Omega$, $G_1\Omega$ and $G_2\Omega$.

In the first part of the problem we are simply interested in seeing if there *is* a solution: if there is any value for which the field current line crosses the voltage characteristic. This will be the case if:

$$G\Omega \geq R_f + R_a$$

Since

$$G = \frac{200}{\Omega_0}$$

We find the speed at which the machine will just self-excite as

$$\frac{\Omega_s}{\Omega_0} = \frac{R_f + R_a}{200}$$

This scaling works in RPM too, so the speed at which the thing will self-excite is:

$$N_s = 2000 \times \frac{75}{200} = 750\text{RPM}$$

In each of the (piecewise) linear ranges, we can characterize operation in the following way:

$$V = R_f I_f = E_n + R_n(I_f - I_n) - R_a\left(I_f + I_g\right)$$

where I_g is load current and $R_n = G_n\Omega$ may be used because we are running this at constant speed.

This becomes, for each region:

$$I_f = \frac{E_n - R_n I_n - R_a I_g}{R_a + R_f - R_n}$$

The no-load field current (and hence voltage) are found using this expression in the upper range with, obviously, zero load current. This evaluates to about 279 volts (which is consistent with the drawing in Figure 1.

To find the limits of the regions, this expression can be solved for the value of I_g which results in field current being the boundary current for that region, or:

$$I_{gn} = \frac{E_n - \left(R_a + R_f\right) I_n}{R_a}$$

And, once field current is found within each region the terminal voltage is simply:

$$V = E_n + R_n\left(I_f - I_n\right) - R_a I_g$$

The values of load current I_g that correspond to the two break points are 100 A (upper breakpoint) and 125 A (lower breakpoint). The full load voltage curve is shown in Figure 2.Note that the machine will not sustain load currents above 125 A: those will cause 'voltage collapse'.

To 'flat compound' the machine we must provide $\Omega G_s = 1\Omega$. At the operating point the *incremental* field characteristic is $\Omega G_f = \frac{50}{3}\Omega$, so the number of series field turns required is $N_s = 500 \times \frac{3}{50} = 30$.

Simulation is straightforward. Using I_f as the single state variable,

$$\frac{dI_f}{dt} = \frac{1}{L_f}\left(E_a - (R_a + R_f)I_f\right)$$

and E_a must satisfy the nonlinear relationship:

$$\begin{array}{lll} if & I_f < 1 & E_a = 200 I_f \\ if & 1 < I_f < 2 & E_a = 200 + 50(I_f - 1) \\ if & 2 < I_f & E_a = 250 + \dfrac{50}{3}(I_f - 2) \end{array}$$

The simulation is shown in Figure 3.

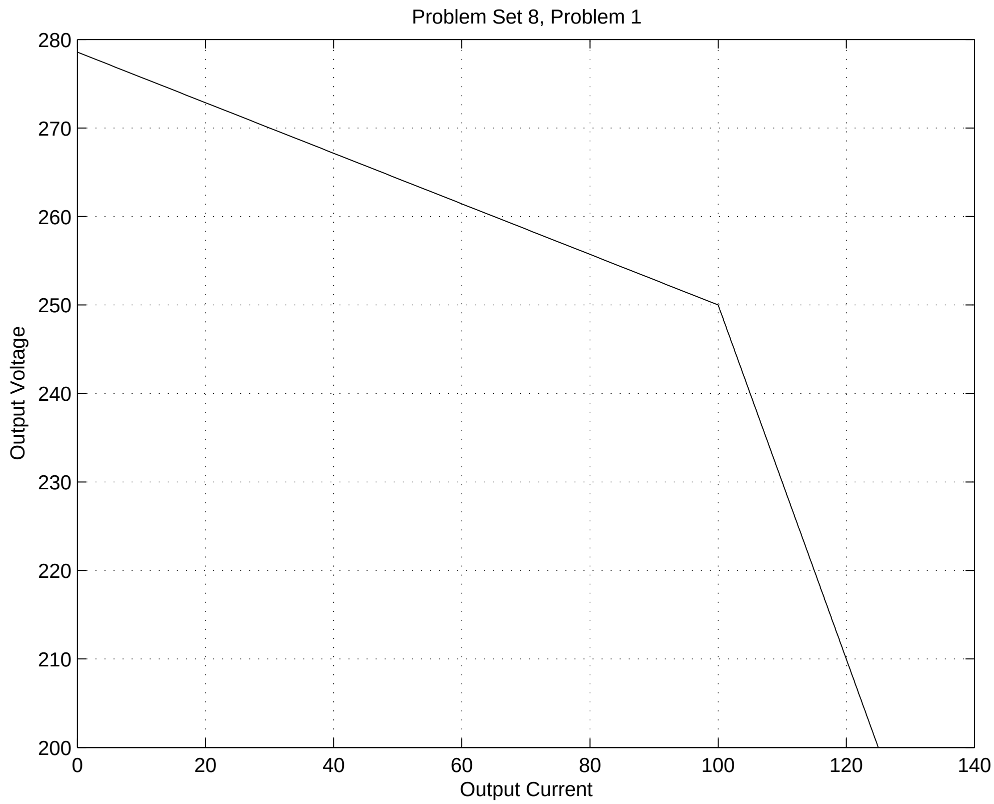

Figure 2: Voltage vs. load current

Problem 2: Compound Motor

With no series field winding, the machine is characterized by:

$$
\begin{aligned}
I_a &= \frac{V - G_f I_f \Omega}{R_a} \\
I_f &= \frac{V}{R_f} \\
T &= G_f I_f I_a
\end{aligned}
$$

With the machine in long shunt connection the equivalent expressions are:

$$
\begin{aligned}
I_a &= \frac{V - G_f I_f \Omega}{R_a + G_s \Omega} \\
I_f &= \frac{V}{R_f} \\
T &= \left(G_f I_f + G_s I_a \right) I_a
\end{aligned}
$$

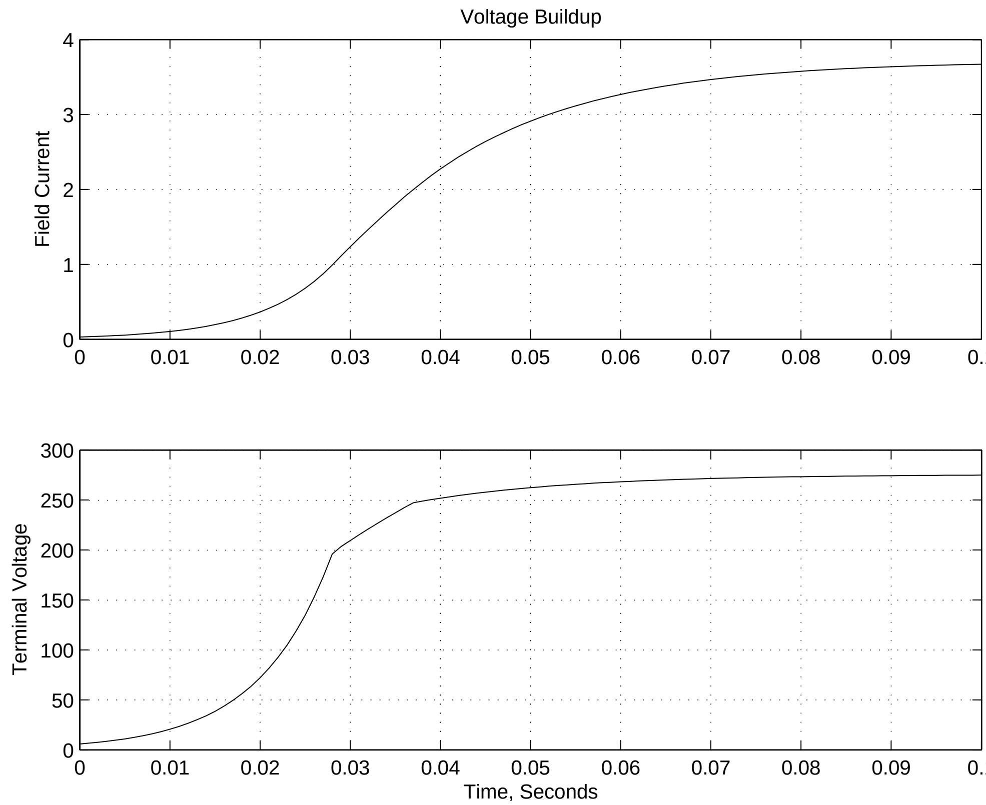

Figure 3: Voltage Buildup

Short shunt operation is one step more complicated (See Figure 4). We can write loop equations like this:

$$
\begin{aligned}
V &= (R_s + R_a + G_s\Omega)\, I_a + (R_s + G_f\Omega)\, I_f \\
0 &= (R_a + G_s\Omega)\, I_a + (G_f\Omega - R_f)\, I_f
\end{aligned}
$$

It is just as easy to let MATLAB solve this linear system. The results are shown in Figures 5, 6 and 7.

Problem 3: Losses The excitation can be split up into two traveling waves:

$$\cos kx \cos \omega t = \mathrm{Re}\left\{ \frac{1}{2} e^{j(\omega t - kx)} + \frac{1}{2} e^{j(\omega t + kx)} \right\}$$

We need to solve this problem for only one of these and then, if we are only interested in the average dissipation, multiply by two.

To start, note that below the sheet the complex amplitudes of the fields must be simply:

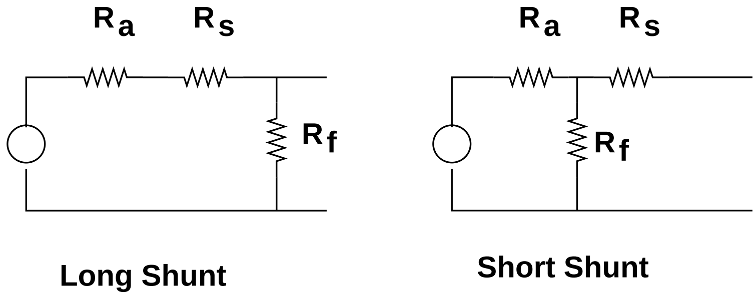

Figure 4: Compound Motor Hookups

$$\underline{S}_b = \frac{\underline{H}_y}{\underline{H}_x} = j$$

Next, see that any current in that sheet must be related to the flux density through the sheet:

$$\underline{K}_z = -\frac{\omega}{k}\mu_0\sigma_s\underline{H}_y = -j\frac{\omega}{k}\mu_0\sigma_s\underline{H}_{xb}$$

Then, since x- directed field above the sheet is equal to x- directed field below the sheet minus the z-directed surface current:

$$\underline{H}_{xa} = \underline{H}_{xb}\left(1 + j\frac{\omega}{k}\mu_0\sigma_s\right)$$

and then, since $H_y = jH_{xb}$, we have the ratio of fields at the top of the sheet:

$$\underline{S}_0 = \frac{\underline{H}_y}{\underline{H}_x} = \frac{j}{1 + j\frac{\omega}{k}\mu_0\sigma_s}$$

We can now transform this field ratio to the surface of the stator, using the form of expression shown in the problem statement. There is a bit of algebra to be done, but that same field ratio is found to be, iin a straightforward way:

$$\underline{S}_s = j\frac{\underline{S}_0 \cosh kg + j\sinh kg}{\underline{S}_0 \sinh kg + j\cosh kg}$$

Now we can turn this into a surface impedance by using:

$$\underline{Z}_s = \frac{\underline{E}_z}{-\underline{H}_x} = \frac{\omega}{k}\mu_0\underline{S}_s$$

Note that we are using a surface impedance which is the ratio of z- directed electric field to z- directed current in the region below the stator.

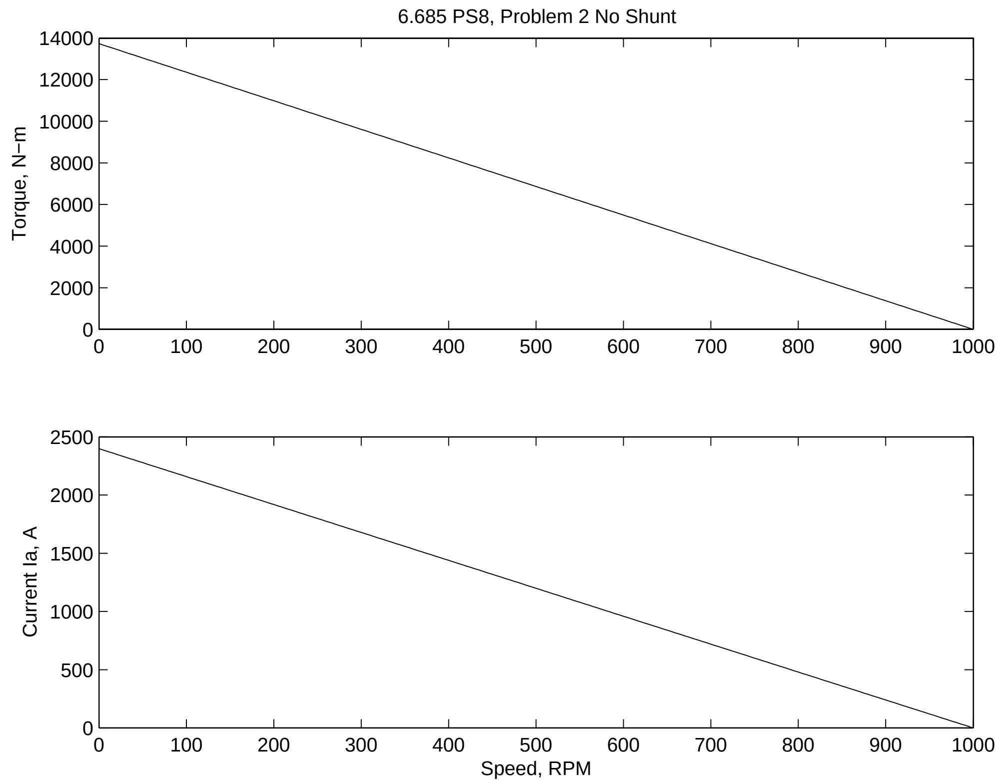

Figure 5: No Series Field

Loss density is found by using, for the current driven case:

$$P_c = \frac{1}{2}|K_z|^2 \mathrm{Re}\left\{\underline{Z}_s\right\}$$

In the case of a constrained flux, the stator provides the equivalent of a fixed electric field:

$$E_{zs} = -\frac{\omega}{k} B_y$$

And the loss expression becomes:

$$P_s = \frac{1}{2}\left(\frac{\omega}{k} B_y\right)^2 \mathrm{Re}\left\{\frac{1}{\underline{Z}_s}\right\}$$

The results are calculated in a script and are shown in Figure 8.

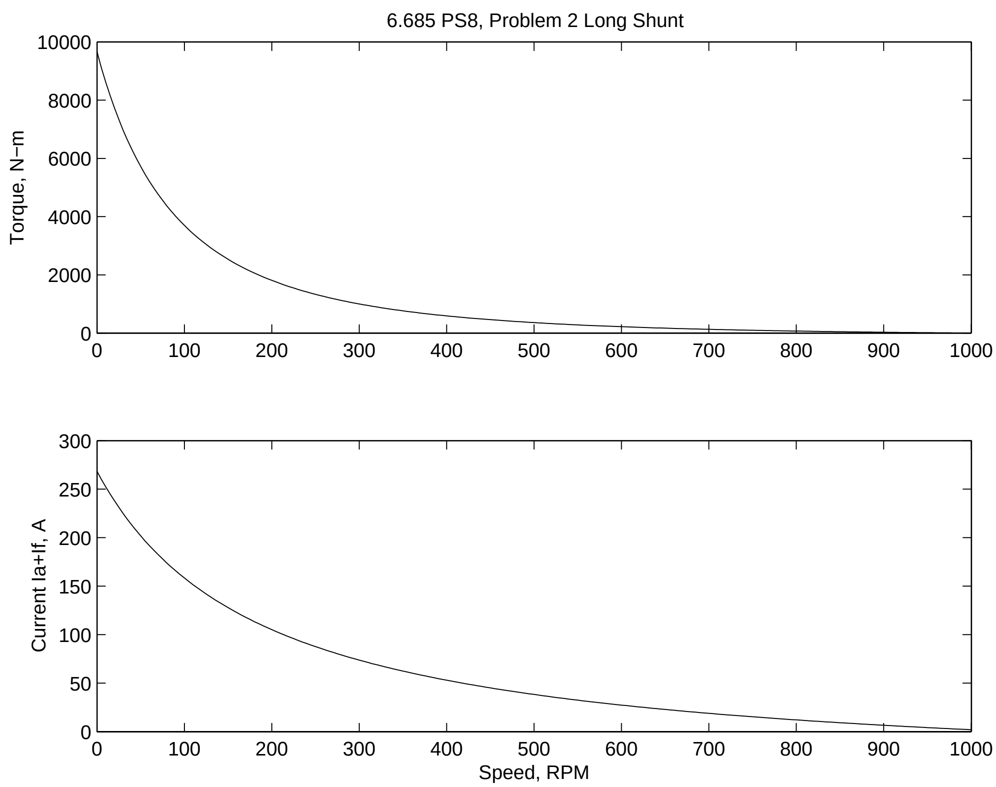

Figure 6: Series Field in Long Shunt Connection

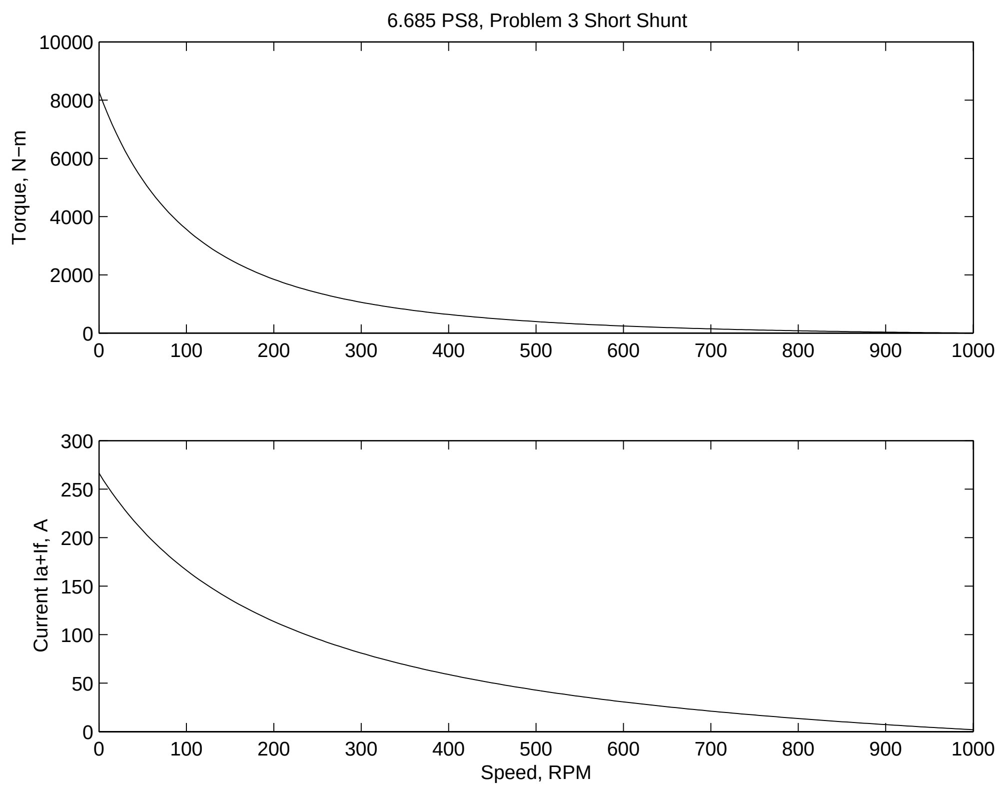

Figure 7: Series Field in Short Shunt Connection

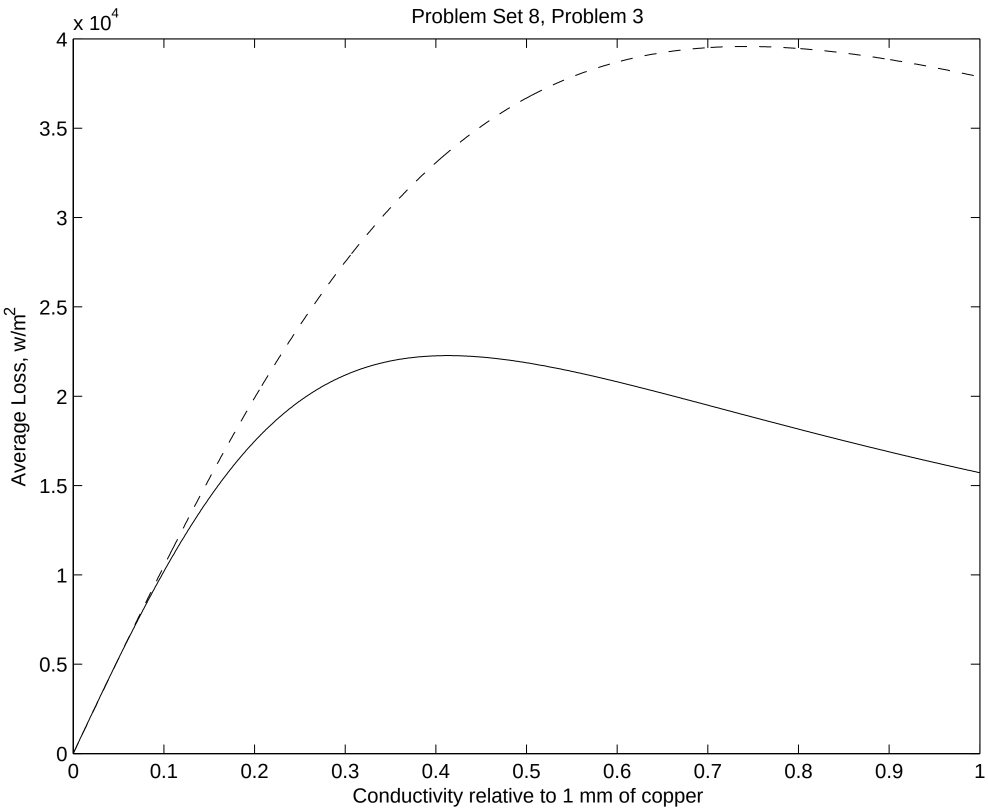

Figure 8: Loss vs. conductivity: Current excitation solid, Flux excitation dashed

```
% 6.685 Problem Set 8, Problem 1

% Getting DC Generator Output Voltage
% Parameters
Ra = 1;
Rf = 74;
E1 = 200;
E2 = 250;
I1 = 1;
I2 = 2;
R1 = 50;
R2 = 50/3;

If0 = (E2-R2*I2)/(Ra+Rf-R2)
V0 = E2 + R2 * (If0-I2)

Ig1 = (E1-(Ra+Rf)*I1)/Ra
Ig2 = (E2-(Ra+Rf)*I2)/Ra


I_g2 = 0:Ig2/100:Ig2;
I_f2 = (E2-R2*I2 - Ra .* I_g2) ./ (Rf+Ra-R2);
V_2 = E2 + R2 .* (I_f2 - I2);

Id = Ig1-Ig2;
dI = Id/200;
I_g1 = Ig2+dI:dI:Ig1;
I_f1 = (E1-R1*I1 - Ra .* I_g1) ./ (Rf+Ra-R1);
V_1 = E1 + R1 .* (I_f1 - I1);

I_g = [I_g2 I_g1];
V_g = [V_2 V_1];

figure(1)
plot(I_g, V_g)
title('Problem Set 8, Problem 1')
ylabel('Output Voltage')
xlabel('Output Current')
grid on
```

```
% Simulation for Problem 8.1
I_f0 = 6/200;
t0 = 0:.001:.1;
[t, I_f] = ode23('dcmsim', t0, I_f0);

Ea = zeros(length(t));
for i = 1:length(t)
 if I_f(i)<1,
    Ea(i) = 200 * I_f(i);
elseif I_f(i) < 2,
    Ea(i) = 200 + 50 *(I_f(i)-1);
else
    Ea(i) = 250 + (50/3) * (I_f(i)-2);
end
end
V = (100/101) .* Ea;

figure(1)
clf
subplot 211
plot(t, I_f)
title('Voltage Buildup')
ylabel('Field Current')
grid on
subplot 212
plot(t, V)
ylabel('Terminal Voltage')
xlabel('Time, Seconds')
grid on
-----------------
function dI_f = dcmsim(t, I_f)
% simulation script for Problem 8.1, voltage buildup
Ra = 1;
Rf = 74;
L = 1;
if I_f<1,
    Ea = 200 * I_f;
elseif I_f < 2,
    Ea = 200 + 50 *(I_f-1);
else
    Ea = 250 + (50/3) * (I_f-2);
end
dI_f = (1/L)*(Ea-(Ra+Rf)*I_f);
```

```
% Compound Motor

Rf = 300;
Rs = 2;
Ra = .25;
Nf = 500;
Ns = 20;
Nz = 1000;
omt = 2*pi*Nz/60;
Vt = 600;
If0 = Vt/Rf;

Gf = Vt/(omt*If0);
Gs = Gf*Ns/Nf;
N = 0:5:Nz;
om = (pi/30) .* N;

% Part 1: No Shunt
If = Vt/Rf;
Ia = (Vt - Gf*If .* om) ./ Ra;

T = Gf * If .* Ia;
figure(2)
subplot(211)
plot(N, T);
title('6.685 PS8, Problem 2 No Shunt')
ylabel('Torque, N-m');
subplot(212)
plot(N, Ia)
ylabel('Current Ia, A')
xlabel('Speed, RPM')

%Part 2: Long Shunt
If = Vt/Rf;
Ia = (Vt - Gf*If .* om) ./ (Ra + Rs + Gs .* om);

T = (Gf .* If + Gs .* Ia) .* Ia;
figure(3)
subplot(211)
plot(N, T);
title('6.685 PS8, Problem 2 Long Shunt')
ylabel('Torque, N-m');
subplot(212)
plot(N, Ia+If)
ylabel('Current Ia+If, A')
```

```
xlabel('Speed, RPM')

%Part 3: Short Shunt

for i = 1:length(N);
    M = [Rs+Ra+Gs*om(i) Rs+Gf*om(i);Ra+Gs*om(i) Gf*om(i)-Rf];
    I = M\[Vt;0];
Ia(i) = I(1);
If(i) = I(2);
end

T = (Gf .* If + Gs .* Ia) .* Ia;
figure(4)
subplot(211)
plot(N, T);
title('6.685 PS8, Problem 3 Short Shunt')
ylabel('Torque, N-m');
subplot(212)
plot(N, Ia+If)
ylabel('Current Ia+If, A')
xlabel('Speed, RPM')
```

```
% PS8, Problem 3

Ks = 80000;                     % amplitude of current source drive
Bs = .1;                        % and of alternate flux source

lambda = .05;                   % wavelength
g = .005;                       % gap
f=1000;                         % frequency
rsig = (0:.001:1);              % range of relative conductivities
sigs = .001*6e7 .* rsig;        % surface conductivities
muzero = pi*4e-7;

k = 2*pi/lambda;                % here is wavenumber
om = 2*pi .* f;                 % and frequency in radians/second

% first get surface coefficient at top of sheet
S0 = j ./ (1 + j*(muzero .* sigs ./ k) .* om);
% then the same at the stator surface
S = j .* (S0 .* cosh(k*g) + j*sinh(k*g)) ./ (S0 .* sinh(k*g) + j*cosh(k*g));
% now the surface impedance
Zs = (muzero .* om ./ k) .* S;
% Here is loss when driven by a current source
% Note this loss is half the peak value because drive is a cosine in space
P_c = .25*Ks^2 .* real(Zs);
% and here is loss when driven by a 'flux source'
P_v = .25 .* ((om ./ k) .* Bs) .^2 .* real(1 ./ Zs);

figure(6)
plot(rsig, P_c, rsig, P_v, '--')
title('Problem Set 8, Problem 3')
ylabel('Average Loss, w/m^2')
xlabel('Conductivity relative to 1 mm of copper')
```