Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.685 Electric Machines

Problem Set 6 Solutions — October 15, 2005

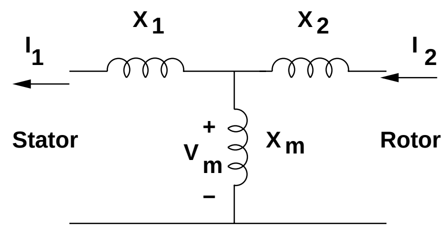

Figure 1: Doubly Fed Machine Single Phase Equivalent

Problem 1: 1. Assuming the stator is feeding a constant power:

With reference to Figure 1, which defines a few currents and voltages:

$$\begin{aligned} I_1 &= \frac{P}{\frac{3}{2}V_1} \\ V_m &= V_1 + jX_1 I_1 \\ I_2 &= I_1 + \frac{V_m}{jX_m} \\ \omega_r &= \omega - \omega_m \\ P_2 + jQ_2 &= \frac{\omega_r}{\omega}\frac{3}{2}V_2 I_2^* \end{aligned}$$

The result is plotted in Figure 2

2. If the output to the *system* is constant, we have

$$P_{\text{stator}} = P_{\text{system}} + P_{\text{rotor}}$$

And of course

$$P_{\text{rotor}} = \frac{\omega_r}{\omega} P_{\text{stator}}$$

The rest follows. Now we must calculate also stator power. The results are plotted in Figure 3

3. Kinetic energy stored at some speed is $E = \frac{1}{2}J\Omega^2$, so in spinning down from Ω_1 to Ω_2, the rotor delivers

$$E_1 - E_2 = \frac{J}{2}\left(\Omega_1^2 - \Omega_2^2\right)$$

Ten percent above and below synchronous speed are:

$$\begin{aligned} \Omega_1 &= 1.1 \times \frac{2\pi \times 60}{2} = 207.35\text{Rad/sec} \\ \Omega_1 &= 1.1 \times \frac{2\pi \times 60}{2} = 169.65\text{Rad/sec} \end{aligned}$$

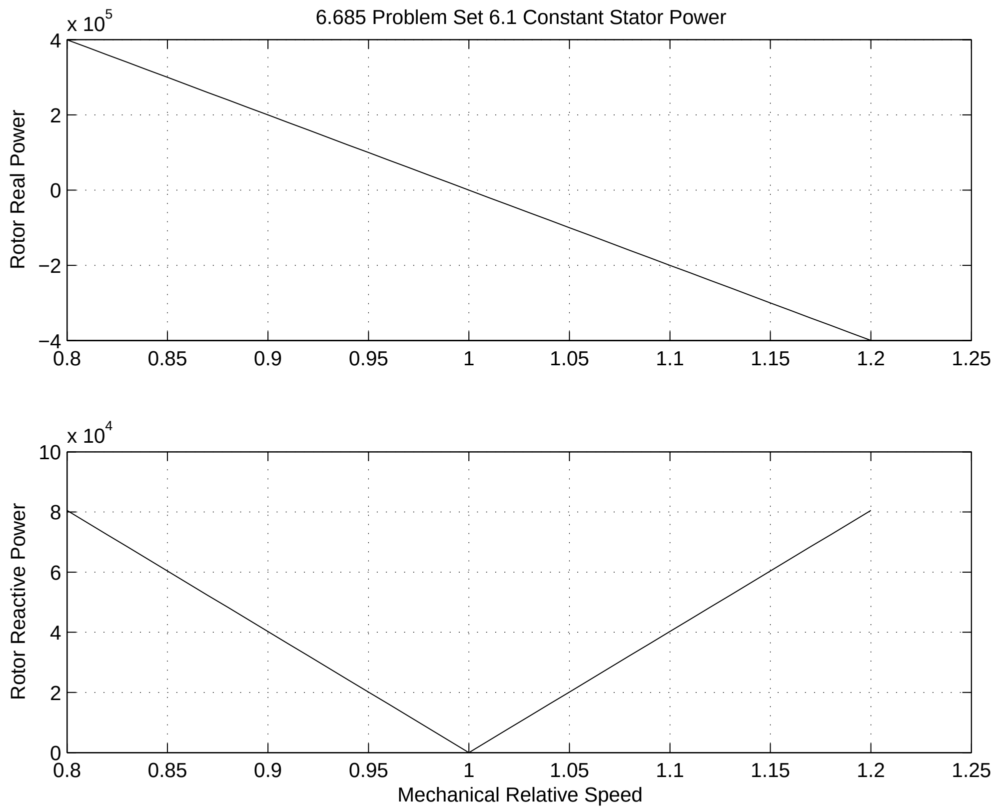

Figure 2: Rotor Input Real and Reactive: Constant Stator Output

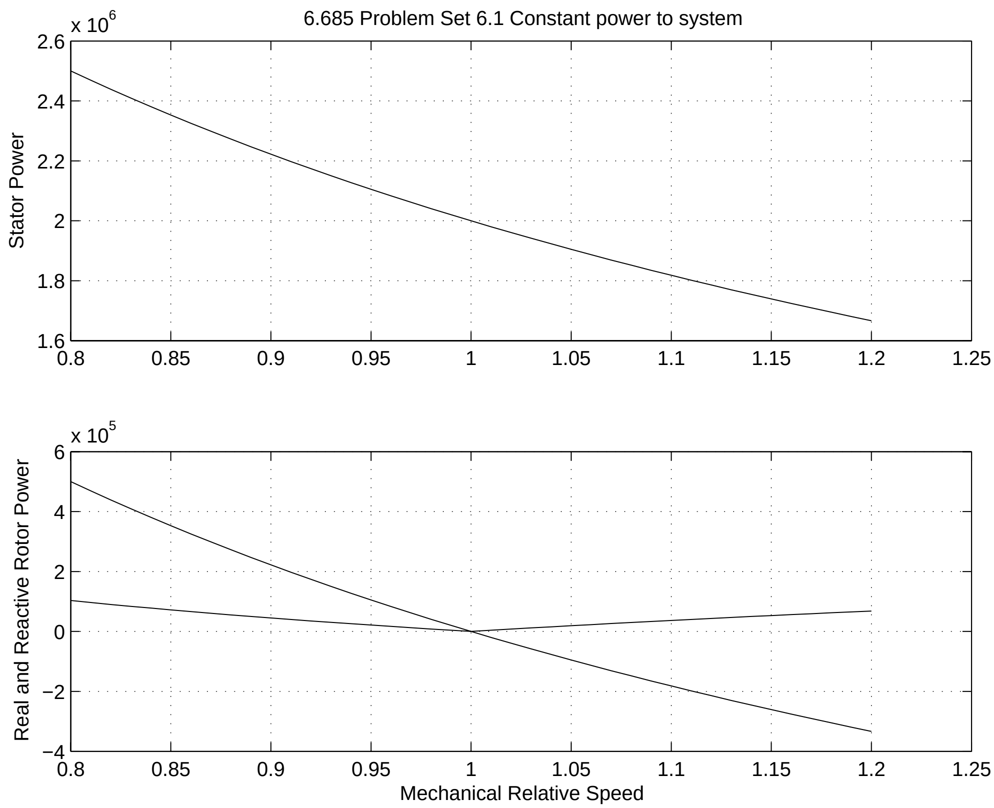

Figure 3: Rotor Input Real and Reactive: Constant Stator Output

Here, total energy delivered is 2 MW for 10 seconds or 20 MJ. Thus:

$$J = \frac{20 \times 10^6}{207.35^2 - 169.65^2} \approx 2814 kg - m^2$$

If we can assume variation of speed with time is about constant, our rotational speed is:

$$\omega_m = \omega_0 - 2\frac{\Delta}{T}t$$

where $\omega_0 = \omega + \Delta$ is the speed at the start of the transient and $\omega_0 - 2\Delta$ is the speed at time T.

Next, assuming we start out at a phase angle of the rotor of zero at the start of the transient, we have the absolute phase angle of the rotor as:

$$\phi = (\omega_0 - \omega)\, t + \frac{\Delta}{T}t^2$$

Voltage and current are calculated as above, but note that

$$\begin{aligned} V_r &= NV_2 \\ I_r &= \frac{1}{N}I_2 \end{aligned}$$

And then voltage and current at the rotor are:

$$\begin{aligned} v_r &= \Re V_r e^{j\phi} \\ i_r &= \Re I_r e^{j\phi} \end{aligned}$$

See Figure 4 for the output.

Problem 2 Induction Motor

This winding could be regarded in either of two ways. One is that it is a series connection of three windings with different pitch factors but all linking flux in the same phase. The winding factor is then the turns weighted average of the three pitch factors (since all windings have the same number of turns this is simply the average of pitch factors).

$$k_{pn} = \frac{1}{3}\left(\sin\left(n\frac{\alpha_1}{2}\right) + \sin\left(n\frac{\alpha_2}{2}\right) + \sin\left(n\frac{\alpha_3}{2}\right) + \right)$$

The second way of looking at this is to treat the winding as a full-pitched winding with three slots per pole per phase. Note that this is not at all what the winding is, but since it has the same active conductors as would a full pitch, m=3 winding, it must induce the same voltage and thus must have the same winding factor.

$$k_{bn} = \frac{\sin nm\frac{\gamma}{2}}{m \sin n\frac{\gamma}{2}}$$

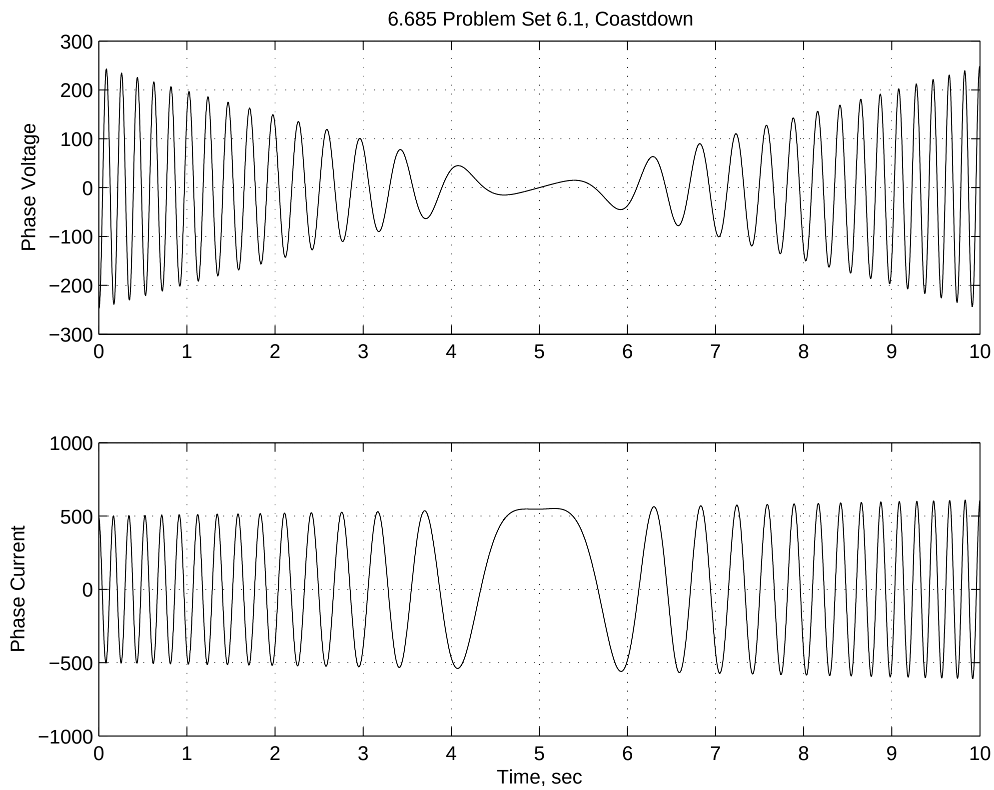

Figure 4: Rotor Current and Voltage Waveforms During Deceleration

The coil throw angles for the three coils are 7/9, 1 and 11/9 times π. The electrical slot angle is $2 * 360/36 = 20^\circ$. The winding factors for harmonics 1, 5 and 7 come out to be, by either method, .9598, .2176 and -.1774, respectively.

To find the inductance we note that this winding has six coils, each with 16 turns, so the total number of turns is $N_a = 96$. Inductance is, of course:

$$L_1 = \frac{3}{2}\frac{4}{\pi}\frac{\mu_0 R\ell N_a^2 k_w^2}{p^2 g} = \frac{6}{\pi} \times \frac{4\pi \times 10^{-7} \times .07239 \times .18098 \times .9598^2 \times 96^2}{4 \times .000254} \approx .2627\text{Hy}$$

Impedance is $X_1 = 377 \times L_1 \approx 99.05\Omega$

To get flux density, note that induced voltage is:

$$V = \frac{\omega}{p} 2R\ell N_a k_w B_r$$

Note that this is an expression for *peak* voltage in one phase, assuming that the value of B_1 is also peak. Peak phase voltage is:

$$V = \sqrt{\frac{2}{3}} V_{l-l}$$

if V_{l-l} is expressed as RMS, as we have done in the problem statement. Then we can invert all of this to get

$$B_1 = \frac{2}{377}\sqrt{\frac{2}{3}}\frac{480}{2 \times .0723 \times .18098 \times .9598 \times 96} \approx .8612\text{T}$$

```
% 6.685 Problem Set 6, Problem 1
% Parameters:
Xm=1.8;                  % magnetizing inductance
Xl=.009;                 % leakage inductance
Ns = 5;                  % turns ratio, primary to secondary

P = 2e6;                 % System output power
Vs = 600;                % line-line, RMS voltage
ome =2*pi*60;            % electrical frequency
omrr =.8:.01:1.2;        % mechanical frequency ratio
omr = 1 - omrr;          % rotor frequency ratio

V = sqrt(2/3)*Vs;        % line-neutral, peak

%Part 1: Stator provides constant power

I_1 = P/((3/2)*V);       % unity power factor: I_1 is real
Vm = V + j*Xl*I_1;       % voltage across magnetizing branch
I_2 = I_1 + Vm/(j*Xm);        % secondary current, referred to primary
V_2 = Vm + j*Xl*I_2;          % secondary voltage, referred to primary

Ir = I_2/Ns;             % current input to rotor
Vr = V_2*Ns .* omr;           % voltage input to the rotor

fprintf('Problem Set 6.1: Part 1: Constant Power from Stator\n')
fprintf('Primary Current = %g + j %g\n',real(I_1), imag(I_1));
fprintf('Primary Voltage = %g + j %g\n',real(V), imag(V));
fprintf('Magnetizing V   = %g + j %g\n',real(Vm), imag(Vm));
fprintf('Secondary I     = %g + j %g\n',real(I_2), imag(I_2));
fprintf('Secondary V     = %g + j %g\n',real(V_2), imag(V_2));

Pr = real((3/2)*conj(Ir) .* Vr);  % rotor real power input
Qr = imag((3/2)*conj(Ir) .* Vr).* sign(omr);  % rotor imaginary power input

figure(1)
subplot 211
plot(omrr, Pr)
title('6.685 Problem Set 6.1 Constant Stator Power')
ylabel('Rotor Real Power')
grid on
subplot 212
plot(omrr, Qr)
ylabel('Rotor Reactive Power')
xlabel('Mechanical Relative Speed')
grid on
```

```
% Part 2: Constant power output

P_s = P ./ omrr;
I_1 = P_s ./ ((3/2)*V);     % unity power factor: I_1 is real
Vm = V + j*Xl .* I_1;   % voltage across magnetizing branch
I_2 = I_1 + Vm ./(j*Xm); % secondary current, referred to primary
V_2 = Vm + j*Xl .*I_2;  % secondary voltage, referred to primary

Ir = I_2 ./Ns;            % current input to rotor
Vr = V_2 .* Ns .* omr;        % voltage input to the rotor

Pr = real((3/2) .*conj(Ir) .* Vr);  % rotor real power input
Qr = imag((3/2) .*conj(Ir) .* Vr) .* sign(omr);  % rotor imaginary power input

figure(2)
subplot 211
plot(omrr, P_s)
title('6.685 Problem Set 6.1 Constant power to system')
ylabel('Stator Power')
grid on
subplot 212
plot(omrr, Pr, omrr, Qr)
grid on
ylabel('Real and Reactive Rotor Power')
xlabel('Mechanical Relative Speed')

% Part 3: Coastdown
T = 10;                          % duration of transient
t = 0:.001:T;                     % over this time
dom = .1;                        % relative speed deviation
a = 2*dom*ome/T;                 % acceleration rate
omm = ome*(1+dom) - a .* t;      % mechanical speed
ommr = omm ./ ome;               % relative speed
omr = 1 - ommr;                  % rotor relative frequency
th = -ome*dom .* t + .5*a .* t .^2; % rotor angle

P_s = P ./ ommr;                 % stator power for this transient
I_1 = P_s ./ ((3/2)*V);          % unity power factor: I_1 is real
Vm = V + j*Xl .* I_1;            % voltage across magnetizing branch
I_2 = I_1 + Vm ./(j*Xm);         % secondary current, referred to primary
V_2 = Vm + j*Xl .*I_2;           % secondary voltage, referred to primary

Ir = I_2 ./Ns;                   % current input to rotor
Vr = V_2 .* Ns .* omr;           % voltage input to the rotor
```

```
va = real(Vr .* exp(j .* th));
ia = real(Ir .* exp(j .* th));

figure(3)
subplot 211
plot(t, va)
title('6.685 Problem Set 6.1, Coastdown')
ylabel('Phase Voltage')
grid on
subplot 212
plot(t, ia)
ylabel('Phase Current')
xlabel('Time, sec')
grid on
```

```
% 6.685 Problem Set 6, Problem 2

% dimensions
R = .0254*5.7/2;            % rotor radius
L = .0254*7.125;                % rotor length
g = .0254*.010;             % air-gap
p=2;                        % number of pole pairs
Nc = 16;                    % turns/coil
Ng = 6;                     % number of coil groups
muzero= pi*4e-7;
n = [1 5 7];                % consider these harmonics

alf1 = (7/9)*pi;            % coil throw angles for 3 coils
alf2 = pi;
alf3 = (11/9)*pi;
gamma = pi/9;               % slot angle

kp = (1/3) .* sin (n .* pi/2) .* (sin(n .* alf1/2) + sin(n .* alf2/2) + sin(n

kb = sin(n .* 3*gamma/2) ./ (3 .* sin(n .* gamma/2))

La = (3/2)*(4/pi)*(muzero*R*L/(p^2 *g)) * (Nc*Ng)^2 * kp(1) ^2

Xa = 377*La

B1 = (p/377)*sqrt(2/3)*480/(2*R*L*Ng*Nc*kp(1))
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