6.728 Applied Quantum and Statistical Physics:

Department of Electrical Engineering and Computer Science
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Quiz 1

Quiz Out: 10/16/06 Quiz Due: 10/18/06 at the beginning of class

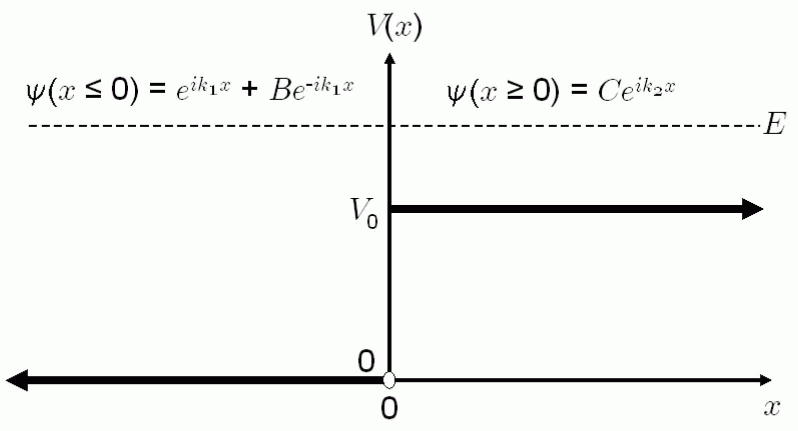
NAME:

Problem	Possible	Score
1	35	
2	35	
3	30	
Total	100	

Problem 1 (35 points)

Consider an electron that is incident from the left on the step barrier $V(x)$ where

$$V(x) = \begin{cases} 0 & \text{if } x < 0 \\ V_o & \text{if } x \geq 0 \end{cases}$$



The eigenfunction at a constant energy E can be written as

$$\psi(x) = \begin{cases} e^{ik_1x} + Be^{-ik_1x} & \text{if } x \leq 0 \\ Ce^{ik_2x} & \text{if } x \geq 0 \end{cases}$$

(a) Explain why the wavefunction $\psi(x)$ has the above form. What are the values of k_1 and k_2 in terms of E, V_o and fundamental constants?

(b) Find the transmitted probability current density, $J_{\text{trans}}(x)$ in terms of B, C, k_1, k_2 and fundamental constants.

(c) Find the values of B and C in terms of k_1 and k_2.

(d) Find the reflection R and transmission T coefficients. (Note you can check that you have the correct answer with the result on page 4 of the formula sheet.)

Problem 2 (35 points)

Consider the simple harmonic oscillator for a particle with mass m and oscillation frequency ω_o. The energy eigenstates are given by the set $\phi_n(x)$ which have eigenenergies $E_n = \hbar\omega_o(n + 1/2)$.

You are given a wavefunction whose initial state in time is

$$\Psi(x, t = 0) = c_o\phi_0 + c_3\phi_3$$

(a) What is the wavefunction $\Psi(x, t)$ for all time?

(b) What is the probability density of finding the particle at some position x_1 as a function of time?

(c) What is the expectation value of the Energy $\langle E \rangle$?

(d) Find $\langle x \rangle$, $\left\langle x^2 \right\rangle$, and $\left\langle x^3 \right\rangle$ in terms of c_0, c_3 and the constants of the system.

Problem 3 (30 points)

The Hamiltonian H is a function of momentum p and is given by

$$H = \frac{p^2}{2m} + \alpha\, p$$

where m is the mass of the particle and α is a constant with units of velocity.

(a) Use Ehrenfest's Theorem to find

$$\frac{d}{dt}\langle x \rangle \qquad \text{and} \qquad \frac{d}{dt}\langle p \rangle\,.$$

(b) Write down Schoedinger's Equation in x-space for this Hamiltonian.

(c) Find the eigenfunctions and eigenenergies.