

## 6.851 ADVANCED DATA STRUCTURES (SPRING'10)

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### Problem 8      *Due: Thursday, Apr. 8*

Be sure to read the instructions on the assignments section of the class web page.

**Cuckoo Hashing.** We pick two hash-functions  $f, g: [u] \rightarrow [m]$  uniformly at random. Let  $S$  be the set of keys we want to store by cuckoo hashing. We define the *cuckoo graph* as done in the lecture: its nodes are the cells of the table, and we have an edge  $(f(x), g(x))$ , for all  $x \in S$ . Assume further, that the size of the table is  $m = 6|S|$ .

Show that with probability at least  $1/2$  the cuckoo graph contains no cycle.

*Hint:* Use the analysis by counting (similar to the “2-cycle case” in the cuckoo hashing analysis).

**Conditional Expectations.** Let  $G$  be a simple graph with vertex set  $V$  and edge set  $E$ . A cut of a set of vertices  $V' \subseteq V$  is the number of edges that have one endpoint in  $V'$  and the other in  $V \setminus V'$ . The NP-complete **MaxCut** problem asks for the largest cut.

A simple randomized approximation problem works as follows: Throw for every vertex a coin. If we got “tails” we add it to  $V'$  otherwise not. In the end an edge is with probability  $1/2$  in the cut, so the expected value of the cut for  $V'$  is  $|E|/2$ . Since every cut is at most  $|E|$  we have a 2-approximation.

Use the concept of conditional expectations to de-randomize this algorithm.

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