

## 6.851 ADVANCED DATA STRUCTURES (SPRING'10)

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### Problem 6    Sample Solutions

#### Dynamizing static search structures.

(a) To perform a successor search we start from the root node, perform a search for a successor and follow the link to its left until we reach a leaf node.

The runtime recurrence is then:  $T(n) = S(\Theta(n^{1/c})) + T(\Theta(n^{1-1/c}))$

For fusion trees we have:

$$T(n) = O(\log_{\omega} n^{1/c}) + T(\Theta(n^{1-1/c}))$$

$$T(n) = O(c^{-1} \log_{\omega} n) + O(c^{-1} \log_{\omega} n^{1-1/c} + T(\Theta(n^{(1-1/c)^2})))$$

Hence

$$T(n) = O\left(\sum_{i=0}^{\infty} c^{-1} \log_{\omega} n^{(1-1/c)^i}\right) = O\left(\sum_{i=0}^{\infty} c^{-1} (1-1/c)^i \log_{\omega} n\right)$$

$$T(n) = O(c^{-1} c \log_{\omega} n) = O(\log_{\omega} n)$$

(b) The space recurrence is:  $C(n) = \Theta(n^{1/c})(C(n^{(1-1/c)}) + 1)$ . Since we have  $\Theta(n^{1/c})$  subtrees of size  $O(n^{(1-1/c)})$  plus  $\Theta(n^{1/c})$  for the space at the current level. We see that the recurrence solves to  $C(n) = O(n)$ .

(c) We will constrain the number of nodes in a subtree rooted at a node at depth  $d$  to be

$$k = \Theta(n^{(1-1/c)^d})$$

When inserting or deleting a node, we make sure that all the nodes on our path satisfy the given property. when merging or splitting a node with  $k$  children we have to reconstruct its parent. The node's parent will have  $\Theta(k^{c/(c-1)})$  descendants, and  $\Theta(k^{1/(c-1)})$  children. Thus, rebuilding the parent would take  $O(k^{b/c-1})$ .

At any given level, we have to rebuild the node only after  $\Theta(k)$  descendants have been inserted/removed. Hence the amortized cost is  $O(k^{\frac{b}{c-1}-1})$ . Choosing  $\frac{b}{c-1} - 1 \leq 0$ ,  $c \geq b + 1$  gives us  $O(1)$  amortized cost per level, and the total of  $O(\log \log n)$

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