6.851 Advanced Data Structures (Spring'10)

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Problem 2 *Due: Thursday, Feb. 18*

Be sure to read the instructions on the assignments section of the class web page.

Query time kd-trees. We consider a kd-tree that stores n points in the plane. Every node v of the kd-tree represents a region $\texttt{region(v)}$ in the induced subdivision of the plane.

1. Show that the boundary of an (axis-parallel) query rectangle can intersect at most $O(\sqrt{n})$ such regions.
2. Show that $\Omega(\sqrt{n})$ is a lower bound for the maximal number of regions that intersect the boundary of an axis-parallel query rectangle by defining a set of n points and a query rectangle appropriately.

Segment stabbing. Let S be a set of disjoint line segments in the plane.

1. Develop a data structure that can report all $s \in S$ that are hit by a vertical ray emanating from (x, y) towards ∞, that is

$$\texttt{Above(x, y)} := \{\texttt{s} \in \texttt{S} \mid \texttt{s} \cap \{(\texttt{x}, \texttt{y}') \mid \texttt{y} \leq \texttt{y}'\} \neq \emptyset\}.$$

   Query times should be $O(\log n + k)$.

2. Develop a data structure that can report all $s \in S$ that are hit by a line segment with endpoints (x, y_1) and (x, y_2), that is

$$\texttt{Between(x, y}_1\texttt{, y}_2\texttt{)} := \{\texttt{s} \in \texttt{S} \mid \texttt{s} \cap \{(\texttt{x}, \texttt{y}') \mid \texttt{y}_1 \leq \texttt{y}' \leq \texttt{y}_2\} \neq \emptyset\}$$

   Query times should be $O(\log^2 n + k)$.

Hint: Modify a segment tree.

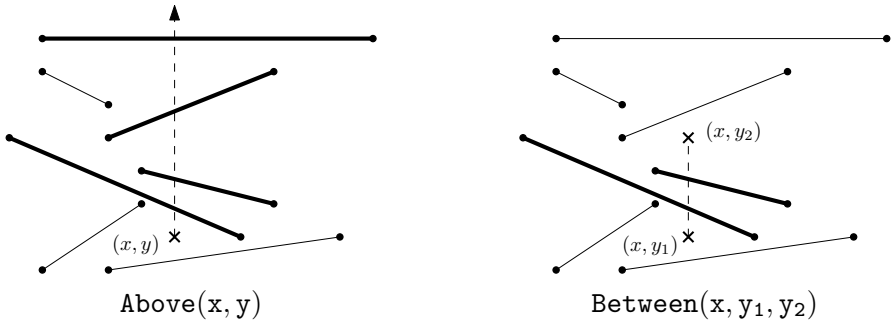

Figure 1: Illustration what the queries should report (thick lines).

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