Toward More Sustainable Infrastructure

Part II

Comparing Economic and Financial Impacts Over the Life of Proposed Projects

Engineering Economics and Project Evaluation

Structure of Part II

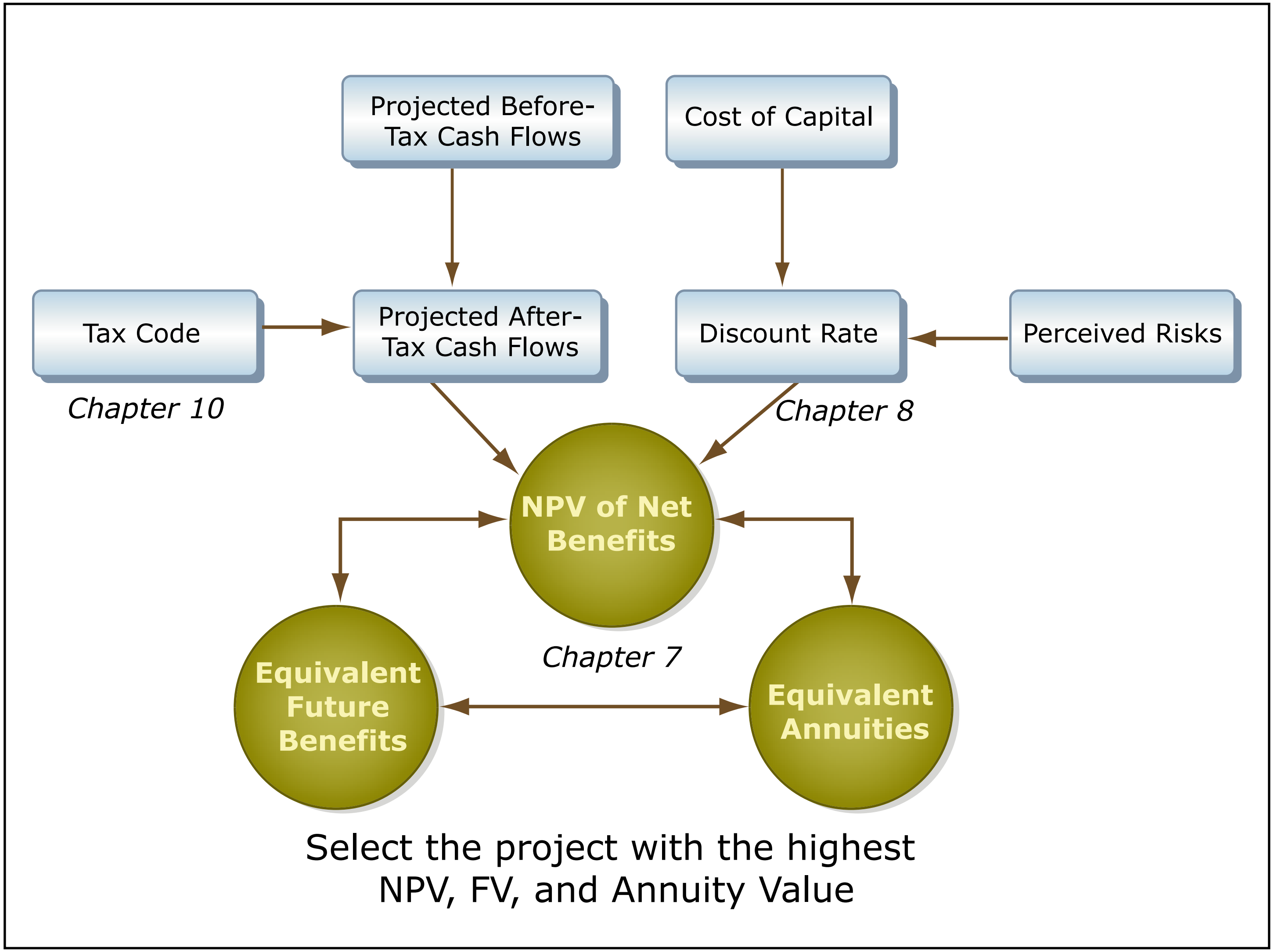


Image by MIT OpenCourseWare.

Toward More Sustainable Infrastructure: Chapter 7

Equivalence of Cash Flows

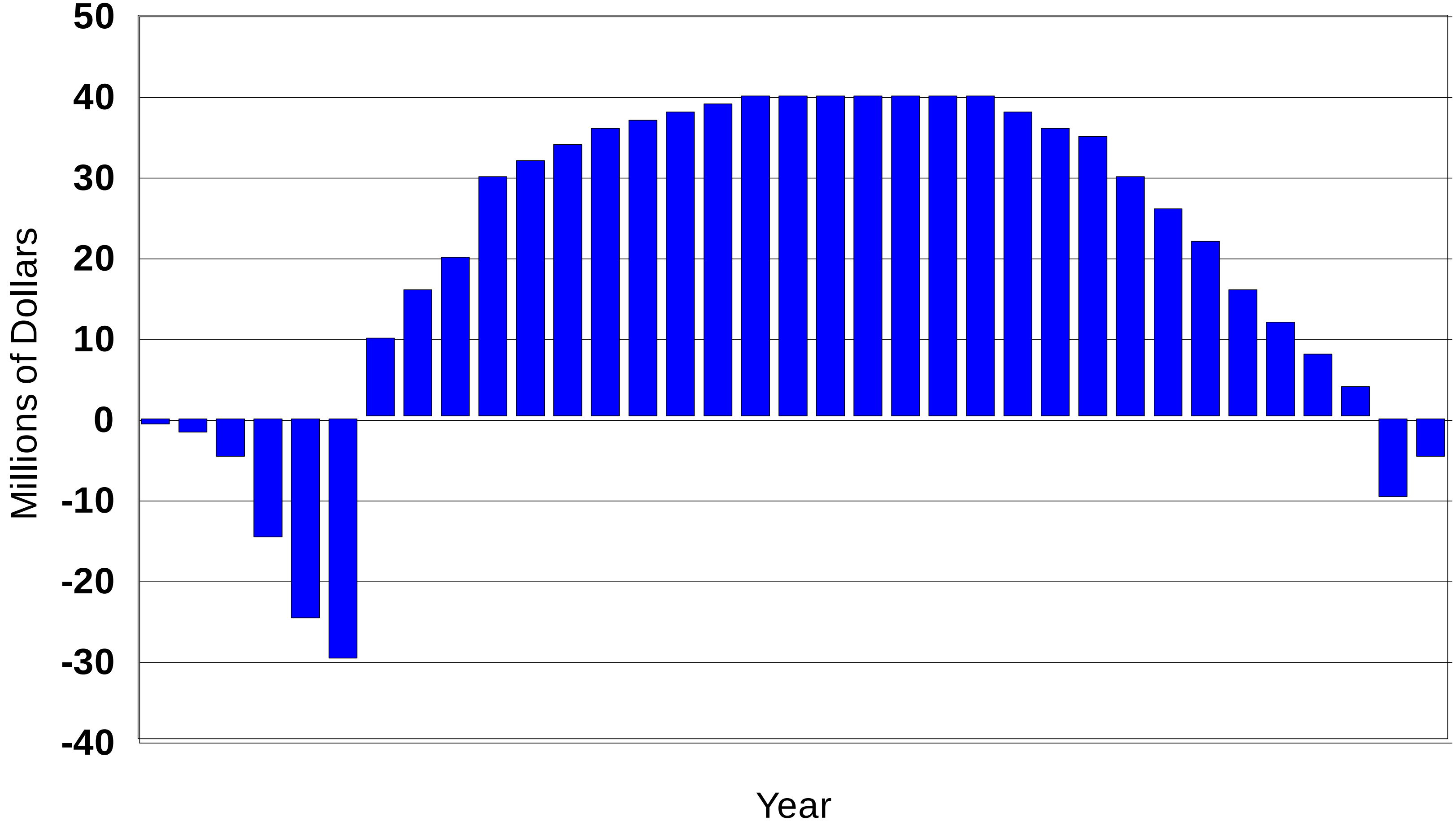
- Time Value of Money
- Equivalence of Cash Flows
- Equivalence Relationships
- Equivalence & Financing
- Equivalence & Costing

The Concept of Equivalence

"Economic equivalence is established, in general, when we are indifferent between a future payment, or a series of future payments, and a present sum of money".

W.G. Sullivan, E.M. Wicks, and J.T. Luxhoj, **Engineering Economy**, 12th ed., Prentice Hall, 2003 p. 76

Cash Flow of a Typical CEE Project



Equivalence is a Critical, Complex, and Interesting Concept

- Why is equivalence critical?

    It is much more convenient to compare present values than the complex cash flows associated with competing projects.

- Why are equivalence relationships complex – and interesting?

    What is equivalent for you might not be equivalent for me! This is often the basis for planning, financing, and negotiation.

Using Equivalence

- If we have an appropriate discount rate, we can convert any arbitrary stream of cash flows to various equivalent (but more easily understood) cash flows:
  - P = present value
  - F = future value at time t
  - A = annuity of A per period for N periods
- To make these conversions, we first need to understand the "time value of money"

Time Value of Money

- \$1 today is worth more than \$1 next year. How much more depends upon opportunities that are available (and how much we want to “discount” future costs and benefits)

- If we invest in a government bond paying i% per year interest, then the money will grow to \$1+i in one year and \$1 * $(1+i)^t$ after t years

- Likewise, \$1 at the end of t years is equivalent to having \$1/$(1+i)^t$ today and investing the money in bonds paying i% interest.

Net Present Value (NPV)

The NPV (or "present worth") is an estimate of the present value of future costs and benefits:

Given:

$C(t)$ = Costs during period t

$B(t)$ = Benefits during period t

Net benefits during period t = $B(t) – C(t)$

Discount Rate = i

Then

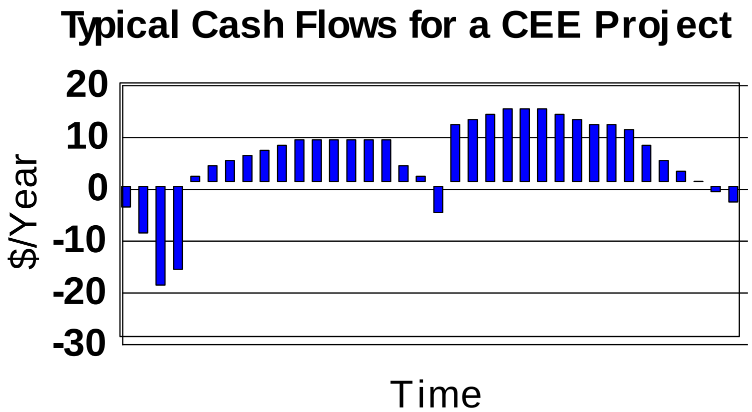
$NPV(t) = (B(t) – C(t))/(1 + i)^t$ after t years

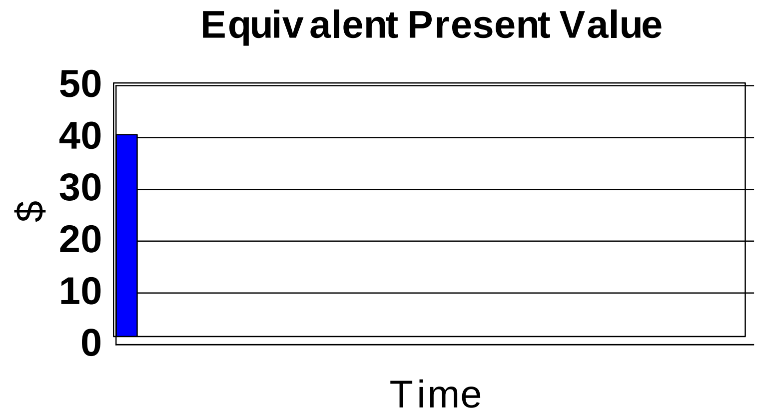
$NPV(project) = \Sigma((B(t) – C(t))/(1 + i)^t)$

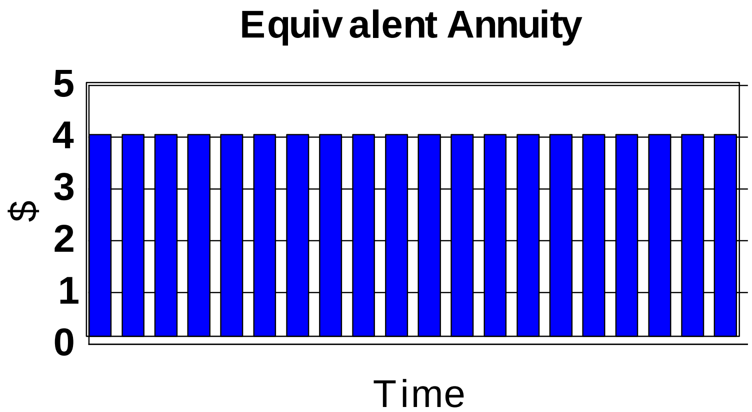
PV of $1,000 Received at Time t

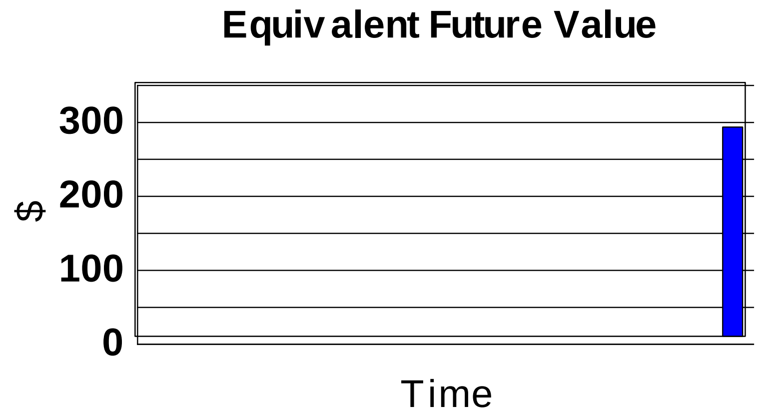
	5 Yrs	10 Yrs	20 Yrs	50 Yrs
1%	$950	$910	$820	$610
5%	$780	$610	$380	$88
10%	$620	$380	$150	$9
20%	$400	$160	$26	$0.11

Equivalence of Cash Flows





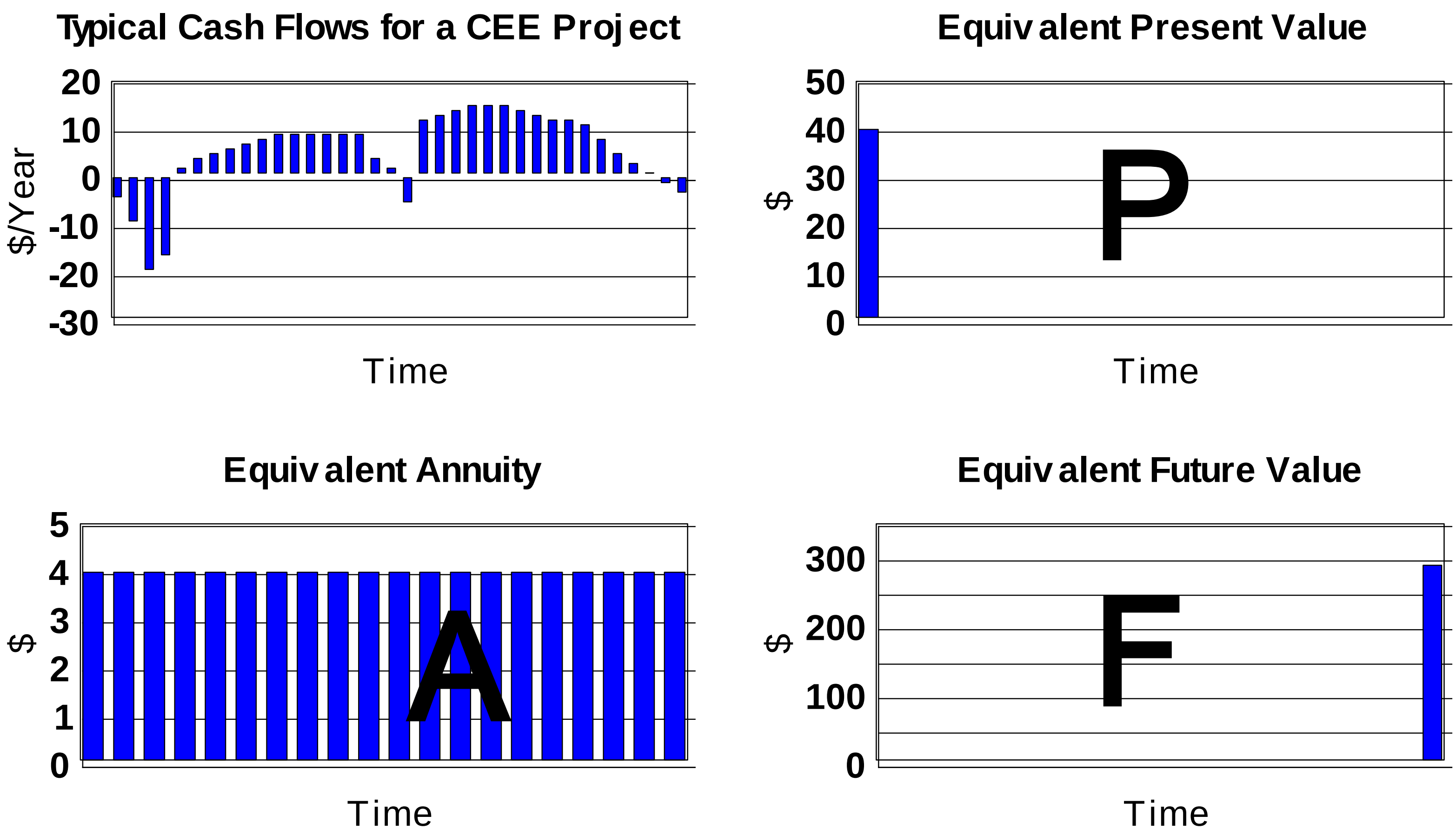




Meaning of PV of a Time Stream of Cash Flows

- PV > 0
  - This project is better than making an investment at i% per year for the life of the project
  - This project is worth further consideration
- PV < 0
  - This project does not provide enough financial benefits to justify investment, since alternative investments are available that will earn i% (that is the meaning of "opportunity cost")
  - The project will need additional, possibly non-cash benefits to be justified

Equivalence of Cash Flows



P

$$P*(1+i)^N = P*(F/P,i,N)$$

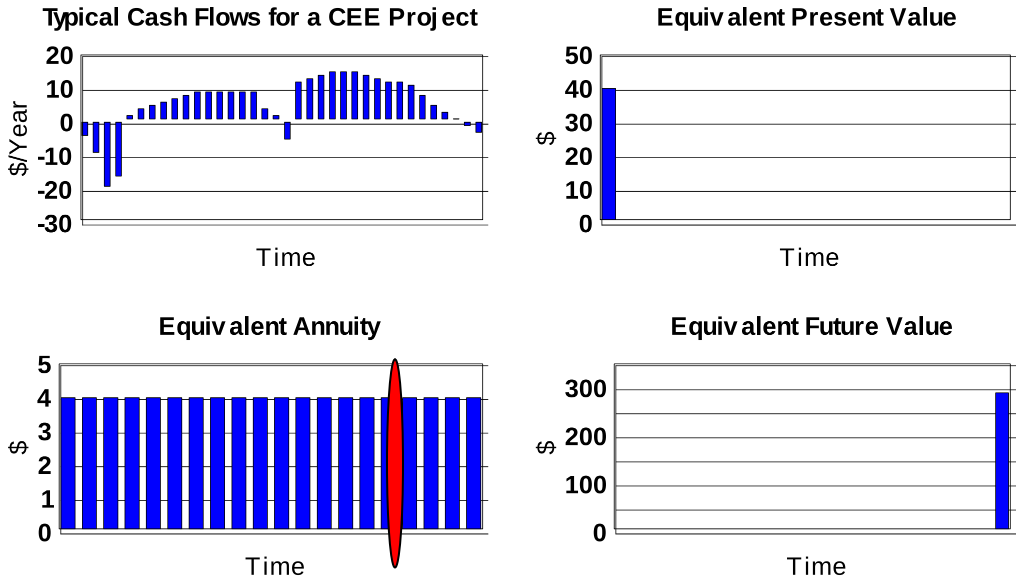
F

P

$$F/(1+i)^N = F*(P/F,i,N)$$

F

Equivalence of Cash Flows



(F/A,i,N)

The future value of an annuity will be the sum of the future value of the individual payments

$$F(A(t)) = A(t) * (1+i)^{N-t}$$

$$F(N) = A[(1+i)^{N-t} + \ldots + (1+i)^{N-t} + \ldots + (1+i)^{0}]$$

……

$$(F/A,i,N) = [(1+i)^{N-1})]/i$$

This results in a geometric sequence with a simple sum.

Other Factors

- $(A/F,i,N) = 1/(F/A,i,N) = i \,/\, [(1+i)^N - 1]$
- $(P/A,i,N) = (F/A,i,N)/(1+i)^N = [(1+i)^N - 1] \,/\, [i * (1+i)^N]$
- $(A/P,i,N) = 1/\,(P/A,i,N)$

For Large N – Some Very Useful Approximations

(the “Capital Worth” method)

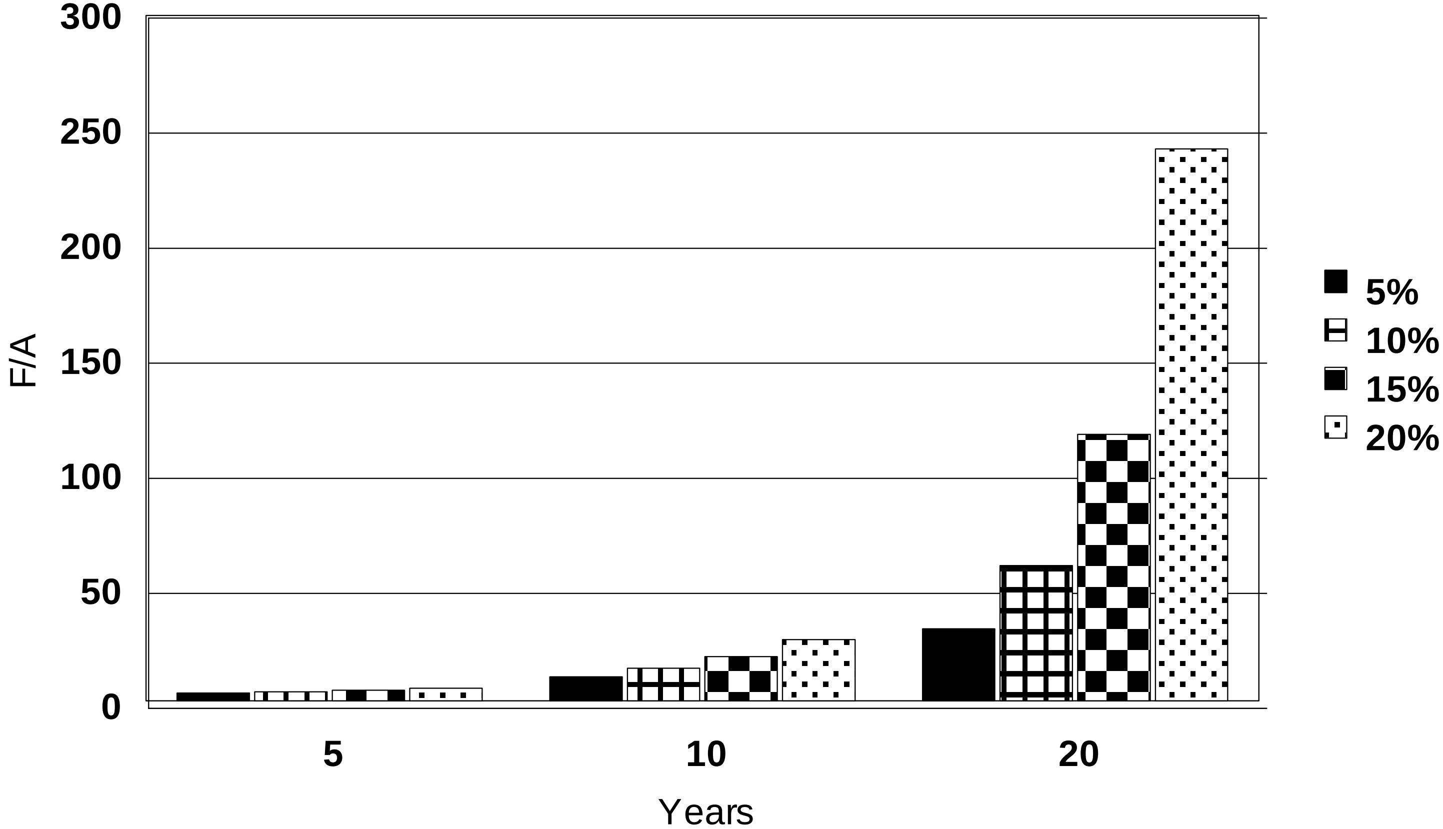
- (P/A,i,N) ~ 1/i

- (A/P,i,N) ~ i

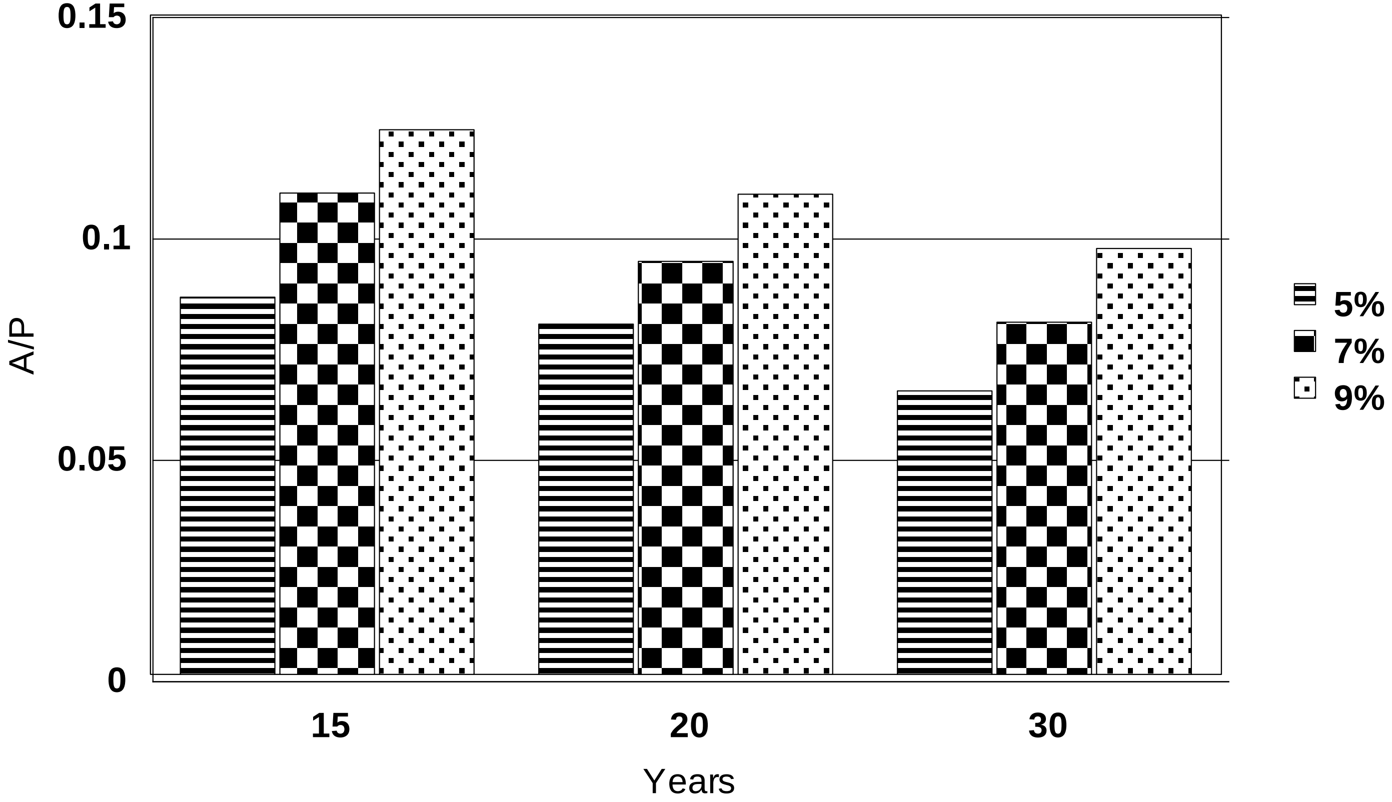
Equivalence Factors

- [F/P,i,N] = future value F after N periods given present value P and discount rate i
- [P/F,i,N] = present value given future value F, i, & N
- [F/A,i,N] = "uniform series compound amount factor"
  - How large will my IRA be after contributing $A at i% for N years?
- [A/F,i,N] = "sinking fund payment"
  - Annual savings to have a downpayment of a house in N years
- [A/P,i,N] = "capital recovery factor"
  - What will the mortgage payments be?
- [P/A,i,N] = "uniform series present worth factor"
  - My business makes $A/year - should I sell for $X?

Uniform Series, Compound Amount Factor [F/A,i,N]



Uniform Series, Capital Recovery Factor [A/P,i,N]



Capital Recovery Factor [A/P,i%,N] for selected interest rates i% and years N

Years	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
5	0.2184	0.2246	0.2310	0.2374	0.2439	0.2505	0.2571	0.2638
10	0.1172	0.1233	0.1295	0.1359	0.1424	0.1490	0.1558	0.1627
15	0.0838	0.0899	0.0963	0.1030	0.1098	0.1168	0.1241	0.1315
20	0.0672	0.0736	0.0802	0.0872	0.0944	0.1019	0.1095	0.1175
25	0.0574	0.0640	0.0710	0.0782	0.0858	0.0937	0.1018	0.1102
30	0.0510	0.0578	0.0651	0.0726	0.0806	**0.0888**	0.0973	0.1061

TMSI, Table 7.6

What Will My Mortgage Payment Be?

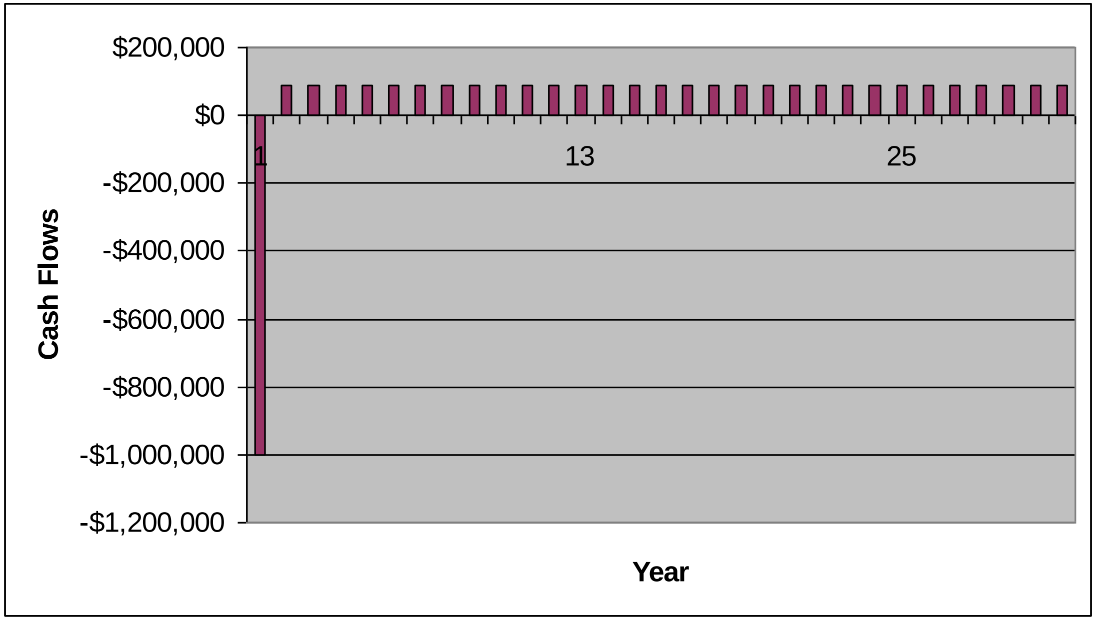
Mortgage Amount: $1,000,000 at 8% for 30 years

Payment is an annuity A that is worth $1,000,000 to the bank

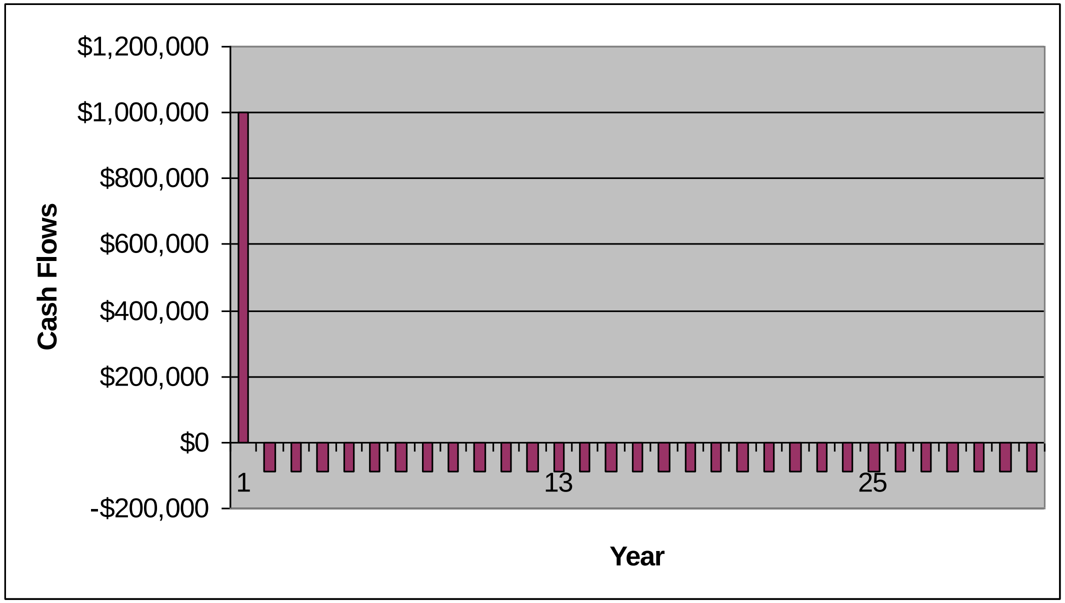
Payment = $1,000,000 * [A/P,i%,N]

= $1,000,000 *(0.088827) = $88,827 per year

Bank's Perspective:



Borrower's Perspective:



Year	Mortgage balance	Payment	Interest	Principal
0	$1,000,000	$88,827	$0	$0
1	$1,000,000	$88,827	$80,000	$8,827
2	$991,173	$88,827	$79,294	$9,534
3	$981,639	$88,827	$78,531	$10,296
4	$971,343	$88,827	$77,707	$11,120
5	$960,223	$88,827	$76,818	$12,010
…				
16	$760,317	$88,827	$60,825	$28,002
…				
29	$158,403	$88,827	$12,672	$76,155
30	$82,248	$88,827	$6,580	$82,248
31	$0			

Refinancing a Mortgage

- What will the payment be if the mortgage is refinanced after 15 years at 6% for 20 years?
- Step 1: what is the remaining balance?
- Step 2: what would the new mortgage payment be on that balance?

Refinancing a Mortgage

Step 1: what is the remaining balance?

- Use the table: $760 thousand
- Calculate the value of the remaining payments:

Balance = $88.8 thousand * [P/A,8%,15]

= $88.8 thousand * 8.5595

= $760 thousand

Step 2: what would the new mortgage payment be on that balance?

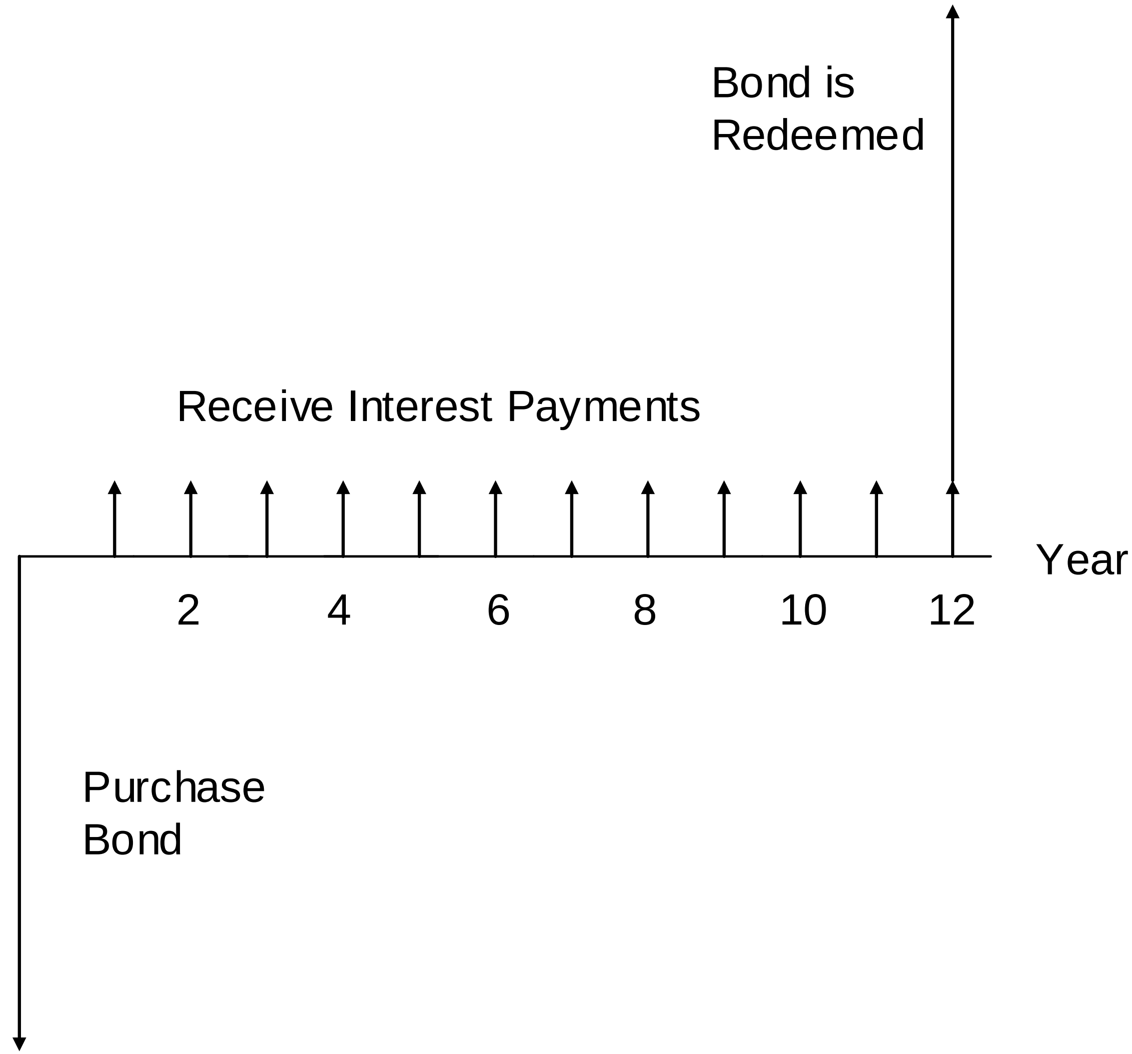
New Payment = $760 thousand * [A/P,6%,20]

= $760 thousand * .0872 = $66.3 thousand

Equivalence Factors –
How Do I Get Them?

- Use the tables at the back of the book
- Use a financial calculator at a bank or investment company's website
- Use the financial functions on a spreadsheet
- Create your own spreadsheet
- Just remember the basics: $P = F/(1+i)^n$

Cash Flow Diagram: 12-year bond purchased at the beginning of year one



Value of 30-year Bond Paying 6% for an Investor With a Discount Rate of 6%

Value of interest payments:

Present value = $1000(6%)[P/A,6%,30]

= $60 (13.7648)

= $826

Value of bond redemption:

Present value = $1000 [P/F,6%,$1000]

= $1000 (0.1741)

= $174

Total Value = $826 + $174 = $1000

Value of 30-year Bond Paying 6% for an Investor With a Discount Rate of 5%

Value of interest payments:

Present value = $1000(6%)[P/A,5%,30]

= $60 (15.3725)

= $922

Value of bond redemption:

Present value = $1000 [P/F,5%,30]

= $1000 (0.2314)

= $231

Total Value = $922 + $231 = $1153

Value of 30-year Bond Paying 6% if interest rates fall to 5% after 20 years (For an Investor With a Discount Rate of 5%)

Value of interest payments:

Present value = $1000(6%)[P/A,5%,10]

= $60 (7.7217)

= $463

Value of bond redemption:

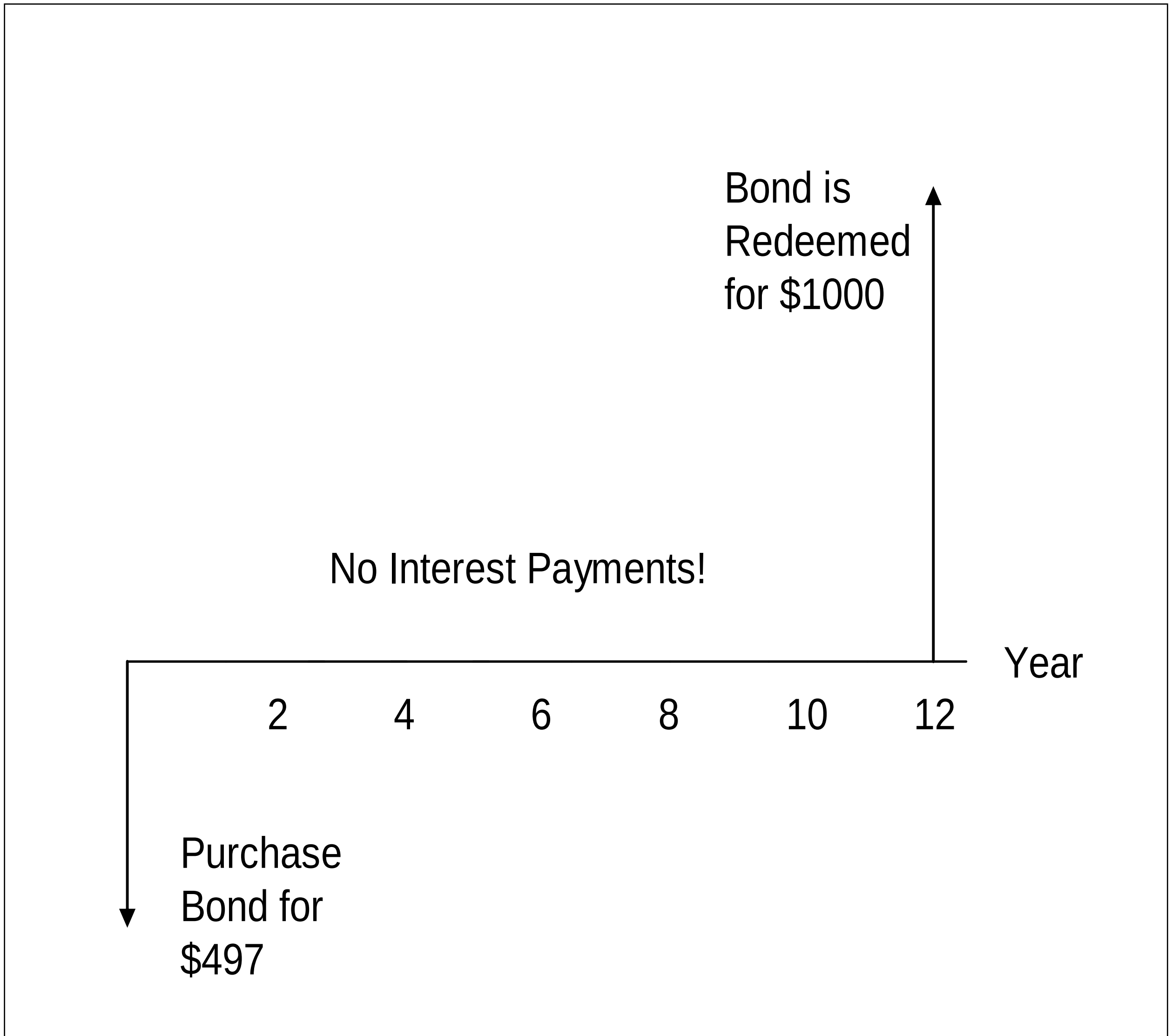
Present value = $1000 [P/F,5%,10]

= $1000 (0.6139)

= $614

Total Value = $463 + $614 = $1077

Cash Flow Diagram: 12-year zero coupon bond paying 6% interest



Value of 12-year Zero Coupon Bond for an Investor With a Discount Rate of 5% or 6%

Value of interest payments:

Present value = 0 (i.e. no interest paid)

Value of bond discounted at 6%:

Present value = $1000 [P/F,6%,12]

$= \$1000/(1.06)^{12}$

~ $500

Value of bond discounted at 5%:

$= \$1000/(1.05)^{12}$

~ $550

Saving for Retirement

- How much will be needed to live on?
  - $80,000 per year after age 60
- What investment return do they hope for?
  - 8%
- How much will they need to retire?
  - $1 million
- How much do they need to save each year if their investments grow at 8% for 30 years?
  - Annual savings = $1,000,000 [A/P,8%,30]
    ~ $8,800/year
- Suppose they work for 40 more years?
  - Annual savings = $1,000,000 [A/P,9%,40] ~ $3,900/year

Toll-based Financing

- Cost of bridge: $50 million
- Annual operating & maintenance: $3 million
- Financed by selling 4% bonds
- Expected traffic volume: 5-10 million cars/yr.

- Question: will a toll of $2 cover the costs of construction, operations, and maintenance?

Toll-based Financing

- Annual revenue: $10 - $20 million
- Net revenue after operating and maintenance costs: $7 to $17 million
- Annual carrying cost on bonds: 4% ($50 million) = $2 million
- Conclusion: a $2 toll will be sufficient
- A sinking fund could be established to pay off the bonds in 30 years:

  $50 million/[F/A,4%,30] = $50 million/56.08

  < $1 million extra per year

Discrete vs. Continuous Compounding

- Nominal Rate of Interest: interest rate if compounded annually

- Effective Rate of Interest: interest rate received over the course of a year if compounded more frequently

Discrete vs. Continuous Compounding

- $1000 at nominal rate of 12%, compounded annually:
  - $120 interest at end of one year

- $1000 at 12%, compounded semiannually:
  - $60 interest on the original $1000 at end of 6 months, which is reinvested
  - $60 interest on the original $1000 at end of 12 months
  - 12%($60) = $3.60 interest on the interest at the end of 12 months
  - Total interest = $123.60
  - Effective rate of interest = 12.36%

Discrete vs. Continuous Compounding

- Effective rate of interest for more frequent compounding:
  - Annually: 12%
  - Semi-anually: 12.36%
  - Quarterly: 12.55%
  - Bi-monthly: 12.62%
  - Monthly: 12.68%
  - Daily: 12.75%

- In general, the effective rate i is a function of the nominal rate r and the frequency of compounding (M times per year):

$$i = [1 + (r/M)]^M - 1$$

Discrete vs. Continuous Compounding

- In the limit, as M approaches infinity, we get the following classic relationship:
- Limit of $(1+1/p)^p$ as p approaches infinity is e = 2.7128...!
- Hence:
  - $[F/P,\underline{r}\%,1] = e^r$
  - $[F/P,\underline{r}\%,N] = e^{rn}$

Discrete vs. Continuous Compounding

The basic relationships for continuous compounding are the same as those for discrete compounding, except for the following substitution:

$$e^{rn} = (1+i)^n$$

where

r is the nominal rate of interest and

i is the effective rate of interest

Discrete vs. Continuous Compounding

- Useful approximations:
  - If rn = 1, then e^{rn} = 2.718
  - If rn = 0.7, then e^{rn} = 2.013 ~ 2
  - If rn = 1.1, the e^{rn} = 3.004 ~ 3
  - If rn = 1.4, then e^{rn} = 4.055 ~ 4

Which is Worth More?

- $1000 invested at 10% for 7 years or

  $1000 invested at 6% for 10 years?

- $1000 invested at 5% for 14 years and reinvested at 7% for 10 years or

  $1000 invested at 6% for 24 years?

"What it comes down to is pieces of paper, numbers, internal rate of return, the net present value, discounted cash flows – that's what it's all about. ... Sure, we want to build quality and we want to build something that is going to be a statement, but if you can't do that and still have it financed and make a return, then why are we doing it?"

Terry Soderberg[1]

[1] Terry Soderberg was in charge of leasing a 50-story office tower for the Worldwide Plaza, quoted by Karl Sabbagh in **Skyscraper: The Making of a Building**, Penguin Books, NY, NY, 1991, p. 377

1.011 Project Evaluation
Spring 2011