Homework Set #6

Problem 1

In planning a building, the number of elevators is chosen on the basis of balancing initial costs versus the expected delay times of the users. These delays are closely related to the number of stops the elevator makes on a trip. If an elevator runs full (n people) and there are k floors, we want to find the expected number of stops R the elevator makes on any trip. Assuming that the passengers act independently and that any passenger chooses a floor with equal probability $1/k$. Show that:

$$E[R] = k\left[1 - \left(1 - \frac{1}{k}\right)^n\right]$$

Hint: It is often useful to define "indicator random variables" as follows: Let $X_i = 1$ if the elevator stops at floor i, and $X_i = 0$ if it does not. Then observe that $R = \sum_{i=1}^{k} X_i$. Find the expected value of X_i after finding the probability that $X_i = 0$.

Problem 2

In the very preliminary planning of some harbor island developments, there was discussion regarding the cost estimates of building four bridges. There had as yet not been a preliminary soil survey in the harbor area, and it was recognized that the bridge costs are highly dependent on soil conditions. Therefore, there was great uncertainty in the preliminary cost estimates. A spokesman, however, made this statement; "I recognize the uncertainty on the cost estimate of any bridge. But I am much more confident of our estimate for the total cost of all four bridges because the likelihood that a high estimate on one will be balanced by a low estimate on another."

Discuss this statement. Use your knowledge, simply, of the means and variances of sums of random variables to support your comments. If your initial intuition lies with the spokesman, be sure you resolve in your own mind why it is inconsistent. Compare both the variance and the coefficients of variation of the total cost versus those of an individual cost. If the four bridge sites are relatively close to one another, soil conditions, although unknown, are probably similar. What implications does this observation have for your analysis?

Problem 3

You finally succumbed to temptation! You are going to invest $1000 in the stock market, but do not know how to best allocate the money between two stocks you especially like. Here are the facts: historically, both stocks have had a mean annual gain of 10%, with a standard deviation of 5% for the first stock and 10% for the second stock and a correlation coefficient $\rho = 0.2$. Your objective is to minimize the volatility of your portfolio by reducing as much as possible the variance of the value of your investment one year from now. How should you allocate your money between the stocks?

Problem 4

The top-floor displacement of a building D depends on wind speed V and the stiffness of the building K as:

$$D = KV^2$$
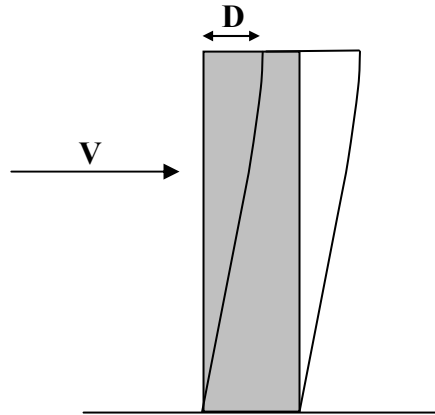
Suppose that K and V are independent random variables with second moment characteristics:

$$K \sim (m_K, \sigma_K^2); \qquad V \sim (m_V, \sigma_V^2)$$

(a) Find in approximation m_D and σ_D^2 using FOSM.

(b) Compare the FOSM mean value m_D with the exact mean value.

[Hint: If X_1 and X_2 are independent, then $E[g_1(X_1)g_2(X_2)] = E[g_1(X_1)].E[g_2(X_2)]$

(c) Under what conditions is the FOSM estimate accurate?