Logistical and Transportation Planning Methods

1.203J/6.281J/13.665J/15.073J/16.76J/ESD.216J

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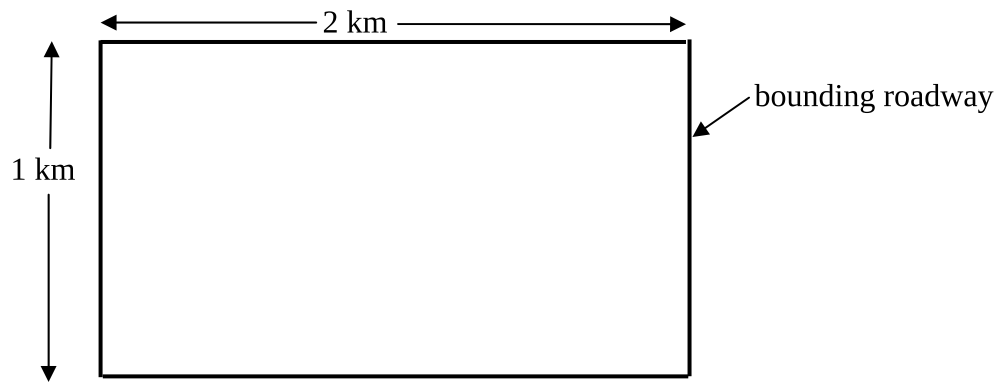
Quiz #1
October 25, 2006

OPEN BOOK
TWO HOURS
4 PAGES, 3 PROBLEMS

PLEASE SHOW ALL YOUR WORK!

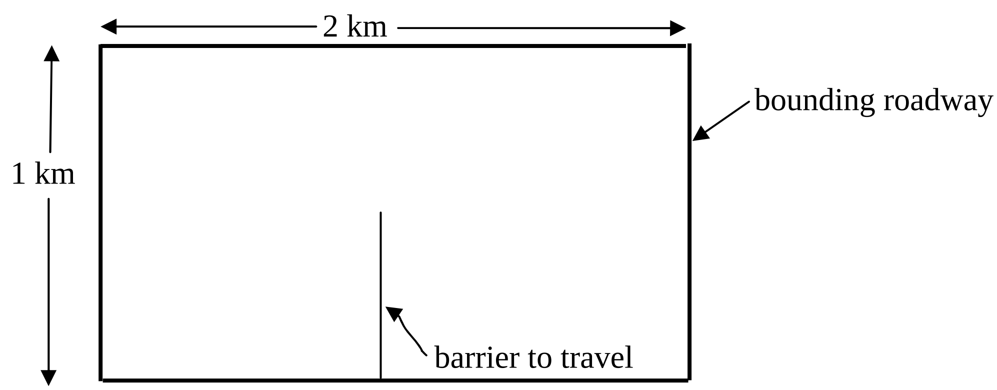
Problem 1: Patrolling Police Car (40 points)

A patrolling police car is assigned to the rectangular sector shown in the figure. The sector is bounded on all four sides by a roadway that requires 50% of the police car's patrolling time. The other 50% of the time, the car patrols the inner rectangular part of the sector. Thus, at a random time when the police car is available for dispatch, the police car's location is equally likely to be drawn from a uniform distribution over the bounding roadway or by a uniform distribution over the rectangular part of the sector inside the bounding roadway. Travel is right-angle or the Manhattan metric, with directions of travel parallel to the sides of the rectangle. 911 calls for service are also distributed randomly over the rectangular sector, in the same way as the police car and independently of the location of the police car. That is, 50% of the 911 calls are uniformly distributed over the bounding roadway and 50% uniformly distributed over the inner rectangular part of the sector. Given a random call for service at a give location, the police car will follow a minimum distance right-angle path from its current location to the location of the call. Thus, we assume that the police car can exit the bounding



roadway at any point, and – as is usual with the right angle metric, we ignore the complication of 'city blocks', assuming instead an infinitely divisible right-angle travel space within the region.

(a) Find the mean distance traveled by the police car in response to a random call for service.
(b) Now assume we have a barrier to travel that rises vertically from the midway point on the lower boundary of the sector to 1/2 km inside the sector. (See figure on next page.) The barrier does not stop travel along the bounding roadway near the barrier. It only prohibits east-west and west-east travel through the barrier within the interior of the sector. Now find the mean distance traveled in the presence of the barrier.



Problem 2: Halloween Treat (30 points)

Halloween pumpkins are planted in parallel straight-line rows on a very big field on a farm in New England. The field is so big that we ignore any boundary effects. The parallel rows are two meters apart. Research has shown that the planter of pumpkin seeds was a sort of random fellow, and that we can accurately model the spatial distribution of pumpkins along any row as a homogeneous spatial Poisson process with parameter $\gamma = 1$ pumpkin per meter. Any given pumpkin has one chance in ten of becoming a giant pumpkin for Halloween. A giant pumpkin weighs more than 30 pounds! Otherwise it is a regular sized pumpkin, weighing less than or equal to 30 pounds. The sizes of pumpkins are mutually independent.

(a) If we walk along any given row of pumpkins, what is the probability that in a walk of 100 meters we will have passed 3 or more giant pumpkins on that row?
(b) If we are standing at the location of a random giant pumpkin, what is the probability that at least 3 additional giant pumpkins are within a right-angle distance of 10 meters from our location? (This question includes pumpkins in near-by parallel rows.)

Problem 3: Strange Bureaucrat (30 points)

Consider the operation of a government office with a single clerk. For all practical purposes, the office constitutes a queueing system with infinite queue capacity (there are lots of chairs to accommodate all those waiting for service). This operation can be modeled as a M/M/1 system with one complication. The clerk, a capricious person, keeps a record of the time when every person seeking service arrives at the office. Whenever there are two or more people waiting at the time when a service is completed, the clerk selects the next person to be served in the following way: with probability 0.6 she selects the person with the *earliest* arrival time (i.e., the person who has been waiting the longest) and with probability 0.4 the one with the *latest* arrival time (i.e., the one who arrived most recently). The arrival rate at the office, λ, is equal to 4 per hour and the service rate, μ, is 6 per hour.

(a) Please draw the state transition diagram for this queueing system, with the state of the system indicating the number of people in the office (waiting or being served).

(b) Consider the expected values L, W and B (respectively, the average number of persons at the office, the average total time spent there, and the average length of a busy period for the clerk). Which, if any, of these three quantities for this queueing system is different from the corresponding quantities for a first-come, first-served (FCFS) M/M/1 system? Please explain briefly.

(c) How would the variance of W compare between this office and the FCFS M/M/1 system? Please explain briefly.

Numerical answers should be provided to questions (d) – (f) with brief explanations.

(d) Person A arrives to find exactly four other persons present at the office, including the one being served (but not counting the clerk). What is the probability that A will be the next person to be served?

(e) Person B arrives to find only one other person at the office, i.e., the person being currently served by the clerk. What is the probability that B will be the next person to be served by the clerk?

(f) For a randomly chosen person, C, who enters this office (and assuming steady state conditions), what is the probability that he will be the next customer served? [HINT: Consider, all the possible states that C may find the office in, and think of what will happen for each state.]