Introduction to Computers and Engineering Problem Solving

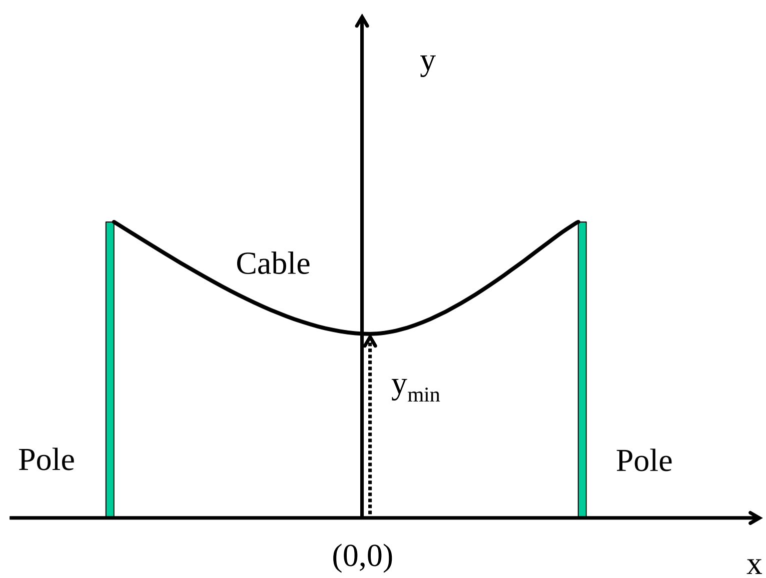
Spring 2005

Problem Set 2: Telephone cable height

Due: 12 noon, Session 8

1. Problem statement

Telephone, electrical and cable TV cables are hung from utility poles in most areas. The figure below shows an example:



The cable has a uniform weight w (Newtons/meter or N/m or kg/sec^2) per unit length. The differential equation, derived from force balance in the x and y directions, for the height y of the cable as a function of x is:

$$\frac{d^2 y}{dx^2} = \frac{w}{T}\sqrt{1+\left(\frac{dy}{dx}\right)^2} \qquad (1)$$

where T= tension force along the cable (Newtons).

The solution to the differential equation is:

$$y = \frac{T}{w}\cosh\left(\frac{wx}{T}\right) + y_{\min} - \frac{T}{w} \qquad (2)$$

where the hyperbolic cosine, cosh(x), is defined as

$$\cosh(x) = 0.5(e^{x} + e^{-x}) \qquad (3)$$

2. Problem objective

Your program will compute the tension T in a cable that is strung between two poles at positions +x and –x, each having a height of y. Since the geometry is symmetric, we only need to consider the pole at +x; the solution for x<0 will be the same. Your program must compute T from the following equation, obtained by rewriting equation (2) above:

$$f(T) = y - \frac{T}{w}\cosh\left(\frac{wx}{T}\right) - y_{\min} + \frac{T}{w} \qquad (4)$$

By finding a zero or root of f(T) (The value of T for which f(T) == 0), we will find a value of T that is consistent with the input values of x, y, w and y_{min}. Details are given in section 3 below; you will use the bisection method to find the root.

After you've found the value of T, you will use equation 2 to output the value of y for all x from 0 to the pole location, in increments of one meter.

This is a simple version of a program that is used to compute tensions and pole heights for placing actual telephone, electrical or cable TV cables.

3. Approach

Your program must do the following:

1. Accept inputs for w (N/m) and y_{min} (m)
2. Accept inputs for the height y (m) and location x (m) of the right-hand pole. The left hand pole is symmetric; its location is –x and its height is also y.
3. Compute T, the tension in the cable.
4. Compute and output the height y at each x, in one meter increments, from x=0 to the right-hand pole

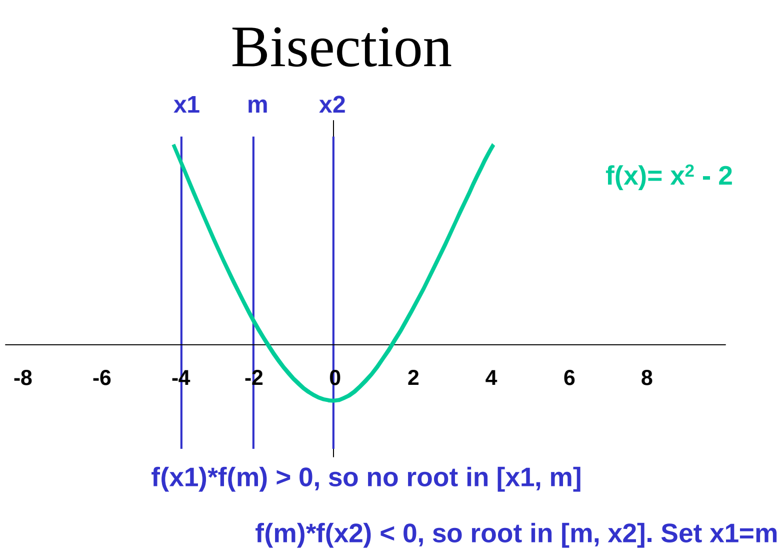
Your program must have the following methods:
1. `bisect()`, which will compute the value of T
2. `f()`, which will compute the value of equation (4)
3. `cosh()`, which will compute the hyperbolic cosine
4. `printHeight()`, which will compute and print the height of the cable at each x, in one meter increments, from x= 0 to the right hand pole

Each method must have appropriate arguments and return values. You will be passing a lot of arguments in this homework, so that you understand scope and argument passing. When we begin to write full-featured classes in homework 3 and beyond, we will have more flexible ways of sharing information (arguments) among methods.

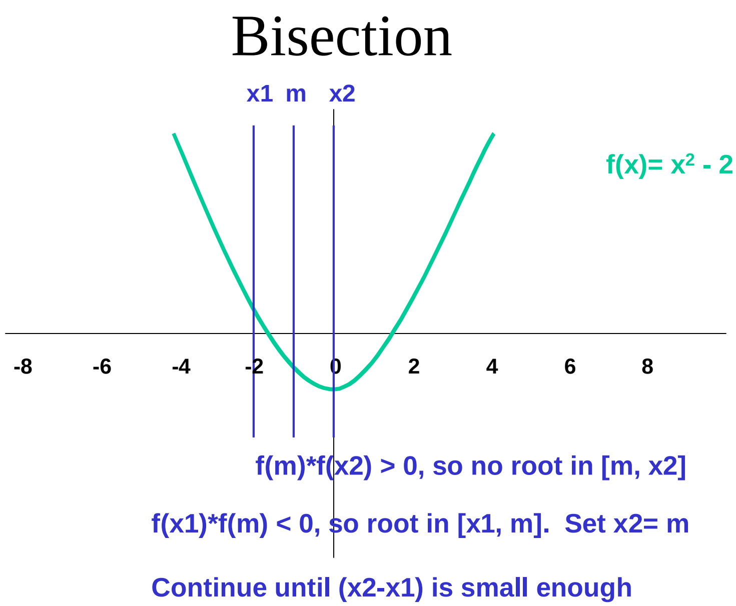
Bisection works as follows:

1. You have a function f(T), with one unknown, T, in our case.
2. Establish a minimum and maximum possible value for the unknown. In our case, use T_{min}= w and T_{max}= w*y*x.
3. Establish the tolerance for the accuracy of finding T. In our case, use epsilon (or tolerance) = 0.0001. T will thus be known to +/- 0.0001 Newtons.
4. Set T_1= T_{min} and T_2= T_{max}
5. Now loop as follows:
    a. Compute the midpoint m= 0.5*(T_1 + T_2)
    b. Compute f(T_1) and f(m)
    c. If f(T_1)*f(m) < 0, we know that f() crosses the x-axis between T_1 and m. This is where we want to search further for the root of f(T)=0, so we set T_2=m and loop again.
    d. If f(T_1)*f(m)>= 0, f() doesn't cross the x-axis between T_1 and m, but it does between m and T_2. Set T_1=m and loop again.
6. End the loop when (T_2-T_1) <= epsilon. You have found T.

The figures below show the bisection logic graphically for f(x)= $x^2 - 2$.



Assume/analyze only a single root in the interval (e.g., [-4.0, 0.0])



Remember that in this homework, your single unknown is T.

Hints and comments:
1. You may assume that there is one and only one root of T between T_{min} and T_{max}.
2. You will need to pass multiple arguments to most of your methods. For example, `bisect()` will need T_{min}, T_{max}, epsilon, w, x, y, and y_{min}. `f()` will need multiple arguments, as will `printHeight()`.
3. Your methods must be `public` and `static`.
4. Consider the return value of each method carefully.
5. You do not need to check any inputs for validity.
6. For w = 10, y_{min} = 5, x = 50 and y = 15, use T = 1266, to check your program.
7. You don't need to use equation 1 in the program.
8. Output the tension, the height of the cable at each x in one meter increments, w and y_{min}.

Later in the semester in class, and in homework 8, we will solve nonlinear equations more formally and more flexibly.

Turn In

1. Place a comment with your full name, MIT server username, section, TA name and assignment number at the beginning of all `.java` files in your solution. In this homework, you will have just one `.java` file.

2. Place all of the files in your solution in a single zip file. In this homework, you will have just a single file to put in your zip file.
    a. Do not turn in electronic or printed copies of compiled byte code (`.class` files) or backup source code (`.java~` files)
    b. Do not turn in printed copies of your solution.
3. Submit this single zip file.
4. Your solution is due at noon. Your uploaded files should have a timestamp of no later than noon on the due date.

Penalties

- 30% off if you turn in your problem set after Friday noon but before noon on the following Monday. You have one no-penalty late submission per term for a turn-in after Friday noon and before Monday noon.
- No credit if you turn in your problem set after noon on the following Monday.