# Engineering Risk Benefit Analysis

**1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721**

## DA 3. The Axioms of Rational Behavior

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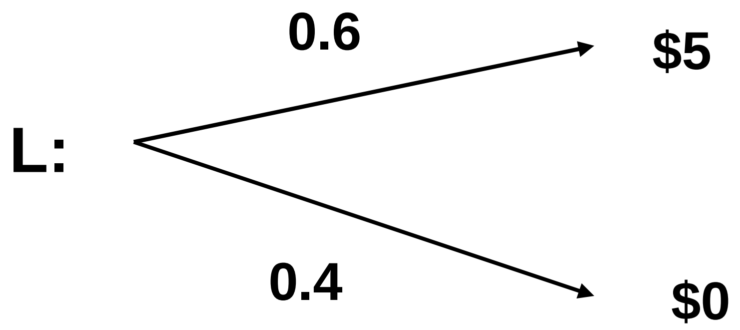
# Lotteries

- **A lottery is a probabilistic trial characterized by a set of mutually exclusive and exhaustive possible outcomes $C_1$, $C_2$, …, $C_m$, with respective probabilities $p_1$, $p_2$, …, $p_m$.**

- **$L(C_1, C_2, \ldots, C_m; p_1, p_2, \ldots, p_m)$**

**<u>Example:</u>** **$L(\$5, \$0; 0.6, 0.4)$**



# Preferences exist

- **For every pair of consequences $C_i$ and $C_j$, a DM will:**

➢ **prefer $C_i$ to $C_j$** $\Rightarrow$ $C_i \succ C_j$

➢ **be indifferent between $C_i$ and $C_j$** $\Rightarrow$ $C_i \sim C_j$

➢ **prefer $C_j$ to $C_i$** $\Rightarrow$ $C_i \prec C_j$

# Definition of $C^*$ and $C_*$

- **Define:**

➢ **$C^*$ a consequence that is <u>at least as</u> preferred as the most preferred of $C_1 \ldots C_m \Rightarrow C^* \succeq C_i$ for all i**

➢ **$C_*$ a consequence that is <u>at least as</u> low in preference as the least preferred of $C_1 \ldots C_m \Rightarrow C_* \preceq C_i$ for all i**

- **$C^*$ and $C_*$ need not be included in $C_1 \ldots C_m$**

# The desirability of a lottery

**It depends on:**

- **The probabilities**
- **The consequences**
- **The person's present wealth, needs, and attitude toward risk.**

# Axiom 1: Comparison of lotteries with identical consequences

**Given: $L_1 = L(C^*, C_* ; p_1, 1 - p_1)$, $L_2 = L(C^*, C_*; p_2, 1 - p_2)$ and $C^* \succeq C_i \succeq C_*$ for all i, then a Decision Maker will:**

**prefer $L_1$ over $L_2$ if $p_1 > p_2$,**

**be indifferent if $p_1 = p_2$**

**prefer $L_2$ over $L_1$ if $p_1 < p_2$.**

- **Given the same consequences, the DM prefers the lottery with the higher probability of achieving the most desirable consequence.**

# Axiom 2a: Quantification of preferences

**For each $C_i$, the DM can specify a number $\pi(C_i)$, with $0 \leq \pi(C_i) \leq 1$, such that the DM is indifferent between:**

**possessing $C_i$ with certainty**

**and**

**possessing the lottery L ($C^*$, $C_*$; $\pi(C_i)$, $1 - \pi(C_i)$)**

# Notes on Axiom 2a

- **The indifference probability (or "preference value") $\pi(C_i)$ is a measure of the preference of $C_i$ on a range of consequences from $C_*$ to $C^*$.**

- **This axiom provides the basis for the development of the metric of "utility" ("preference value").**

- **From Axiom 1, the DM will prefer $C_i$ for sure over the lottery $L\ (C^*, C_*\ ; p\ ,\ 1 - p)$, if $p < \pi(C_i)$.**

# Axiom 2b: Quantification of uncertainty

**Let R be any event. For each R, the DM has a quantity p(R), with $0 \leq p(R) \leq 1$, such that the DM is indifferent between**

- **the lottery L ($C^*$ , $C_*$ ; p(R) , 1 - p(R))**
- **a lottery as a result of which the DM will obtain $C^*$ if R occurs and $C_*$ if R does not occur.**

# Notes on Axiom 2b

- Judgmental probabilities exist for a rational DM.

- This axiom provides the means for finding the DM's probability of R. All the DM has to do is adjust p(R) until he/she is indifferent between the two lotteries.

# Axiom 3: Transitivity of preferences

**If $C_1$, $C_2$ and $C_3$ are consequences, then:**

**$C_1 \sim C_2$ and $C_2 \sim C_3$ implies $C_1 \sim C_3$**

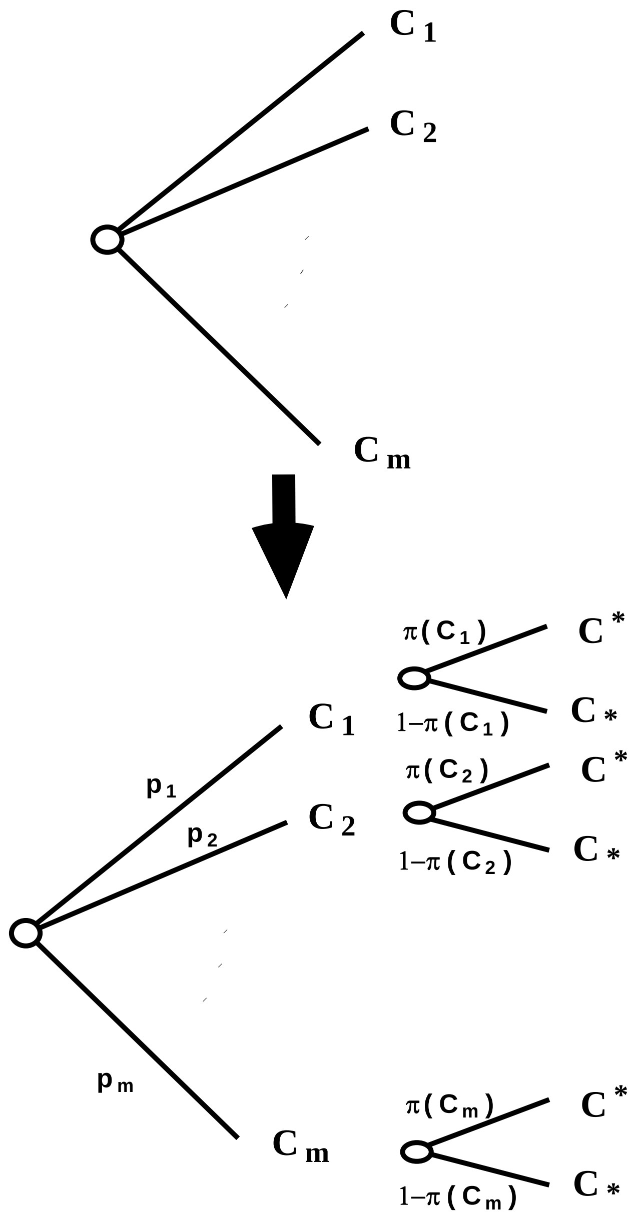
**and**

**$C_1 \succeq C_2$ and $C_2 \succeq C_3$ implies $C_1 \succeq C_3$**

# Axiom 4: Substitution of consequences

**If $C_1 \sim C_2$, then the DM is indifferent between two decision problems which are identical except that $C_1$ in the first problem has been substituted by $C_2$ in the second.**

**[If a DM is indifferent between two consequences, the DM's solution to a decision problem cannot be affected by substitution of one of these consequences for the other.]**



C 1
C 2
C m
π(C 1)
C *
1−π(C 1)
C *
C 1
p 1
π(C 2)
C *
C 2
p 2
1−π(C 2)
C *
p m
π(C m)
C *
C m
1−π(C m)
C *

# Axiom 5: Equivalence of preferences for actual and conjectural situations

**Let $C_1$ and $C_2$ be any two consequences which are possible if only some chance event R occurs. After it is known that R did indeed occur, the DM should have the same preference between $C_1$ and $C_2$ that the DM had before (s)he knew whether or not R occurred.**

**[A DM's preferences among consequences of a decision should not be affected by knowledge as to whether (s)he merely <u>may</u> or (s)he <u>certainly will</u> have to make that decision.]**

# Summary of Axioms

- <u>**Axiom 0:**</u> **Preferences exist**
- <u>**Axiom 1:**</u> **Two simple lotteries, each with same prize and penalty: choose lottery with higher probability of prize**
- <u>**Axiom 2a:**</u> **Quantification of preferences ("indifference probability" or "preference value")**
- <u>**Axiom 2b:**</u> **Quantification of uncertainty**
- <u>**Axiom 3:**</u> **Transitivity of preferences**

# Summary of Axioms (cont'd)

- **Axiom 4:** **Substitution of consequences**
- **Axiom 5:** **Equivalence of preferences for actual and conjectural situations**

**A DM who satisfies these axioms is rational or coherent.**